

Divergenz- und Diff. Vektoren

$\text{div}(\mathbf{a}) = \sum_{i=1}^n \frac{\partial}{\partial x_i} a_i(\mathbf{x}) = \sum_{i=1}^n \nabla_{\mathbf{x}_i} a_i(\mathbf{x})$

Rotation

$\text{rot}(\mathbf{a}) = \begin{pmatrix} \frac{\partial a_3}{\partial x_1} - \frac{\partial a_2}{\partial x_3} \\ \frac{\partial a_1}{\partial x_3} - \frac{\partial a_3}{\partial x_1} \\ \frac{\partial a_2}{\partial x_1} - \frac{\partial a_1}{\partial x_2} \end{pmatrix}$

→ Vektor

Vektor $\mathbb{R}^1$ in $\mathbb{R}^3$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} g_1(x,y,z) \\ g_2(x,y,z) \\ g_3(x,y,z) \end{pmatrix}$

$\text{rot}(\mathbf{g}) = \begin{pmatrix} \frac{\partial g_3}{\partial x_1} - \frac{\partial g_2}{\partial x_3} \\ \frac{\partial g_1}{\partial x_3} - \frac{\partial g_3}{\partial x_1} \\ \frac{\partial g_2}{\partial x_1} - \frac{\partial g_1}{\partial x_2} \end{pmatrix}$

- wenn Vektor $\mathbf{g} = 0$

- Quellort: $\text{div} \mathbf{g} = 0$

- Sinkort: $\text{div} \mathbf{g} = 0$

Laplace-Operator

$\Delta = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

div $\mathbf{g} = 0$

$\mathbf{g} = \nabla \phi$ Skalar

Kommutativität

$\nabla \cdot (\mathbf{g} \cdot \mathbf{h}) = \mathbf{g} \cdot \nabla \mathbf{h} + \mathbf{h} \cdot \nabla \mathbf{g}$

$\nabla \cdot (\mathbf{g} \cdot \mathbf{h}) = \mathbf{g} \cdot \nabla \mathbf{h} + \mathbf{h} \cdot \nabla \mathbf{g}$

math. + Sinus: gegen Ungezogen

1D-Simultes Tangentialvektor

$\frac{c'(t)}{|c'(t)|}$

Zirkulation

$Z(\mathbf{g}, \mathbf{K}) = \int_{\mathbf{K}} \nabla \cdot \mathbf{g} = \int_a^b \nabla(\mathbf{g}(t)) \cdot \mathbf{c}'(t) dt$

$= \int_a^b \nabla(\mathbf{g}(t)) \cdot \mathbf{c}'(t) dt$

Ausfluss

$A(\mathbf{g}, \mathbf{K}) = \int_{\mathbf{K}} \nabla \cdot \mathbf{g} = \int_a^b \nabla(\mathbf{g}(t)) \cdot \mathbf{c}'(t) dt$

$= \int_a^b \nabla(\mathbf{g}(t)) \cdot (-\mathbf{c}'(t)) dt$

$T(\mathbf{g}, \mathbf{K}) = \nabla(\mathbf{g}(t)) \cdot \frac{\mathbf{c}'(t)}{|\mathbf{c}'(t)|}$

$N(\mathbf{g}, \mathbf{K}) = \nabla(\mathbf{g}(t)) \cdot \mathbf{n}(t)$

Feldlinien

wenn $\mathbf{g}(\mathbf{x})$ in jedem Punkt tangential zu $\mathbf{K}$

$\mathbf{c}'(t) = \mathbf{p}_t \mathbf{g}(\mathbf{c}(t))$

Wenn $\mathbf{g}(\mathbf{x})$ in jedem Punkt tangential zu $\mathbf{K}$

$\frac{dx}{dt} = \frac{g_1(x,y,z)}{g_2(x,y,z)}$

$\frac{dy}{dt} = \frac{g_2(x,y,z)}{g_3(x,y,z)}$

$\frac{dz}{dt} = \frac{g_3(x,y,z)}{g_1(x,y,z)}$

$\mathbf{c}^{\perp}(\mathbf{a})$ durch Rotation von $\mathbf{a}$ um $\pi/2$

$\mathbf{K}(\mathbf{a}) = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \rightarrow \mathbf{K}^{\perp}(\mathbf{a}) = \begin{pmatrix} -a_2 \\ a_1 \\ 0 \end{pmatrix}$

→ 1D-Magnetisches Feld für jedes Magnet

Potential, wenn:

$\int_{\mathbf{K}_1} \mathbf{g}(\mathbf{x}) \cdot d\mathbf{x} = \int_{\mathbf{K}_2} \mathbf{g}(\mathbf{x}) \cdot d\mathbf{x}$

- Menge einfach zusammenhängend (keine Orlöcher)

$\int_{\mathbf{K}} \mathbf{g}(\mathbf{x}) \cdot d\mathbf{x} = 0$