

Aufgabe 1

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a)

i) $\underline{z} = -1 + j\sqrt{3}$ $z = a + j \cdot b$

$$|\underline{z}| = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

$$\varphi = \pm \arccos\left(\frac{a}{|\underline{z}|}\right)$$

$$\varphi = + \arccos\left(\frac{-1}{2}\right)$$

$$= \frac{2}{3}\pi$$

2/2

ii) $\underline{z} = \frac{-16 + j \cdot 4}{3 - j \cdot 5}$

$$|\underline{z}| = \frac{\sqrt{(-16)^2 + (4)^2}}{\sqrt{3^2 + (-5)^2}} = \frac{\sqrt{16 \cdot 17}}{\sqrt{24}} = \frac{4\sqrt{17}}{\sqrt{2 \cdot 12}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$$\underline{z} = \frac{-16 + j \cdot 4}{3 - j \cdot 5} \cdot \frac{3 + j \cdot 5}{3 + j \cdot 5}$$

$$= \frac{-68 - j \cdot 68}{34} = -2 - 2j$$

$$\varphi = -\arccos\left(\frac{-2}{2\sqrt{2}}\right) = -\frac{3}{4}\pi$$

$$= 2\pi - \frac{3}{4}\pi = \frac{5}{4}\pi$$

3/3

b) $\Omega = \frac{\omega}{\omega_0} \geq 0$

iii. $\underline{F}(j\Omega) = \frac{j \cdot \Omega}{1 + j\Omega}$

$$|\underline{F}(j\Omega)| = \frac{\Omega}{\sqrt{1 + \Omega^2}}$$

$$\varphi_F = \varphi(j\Omega) - \varphi(1 + j\Omega)$$

$$= \varphi(j\Omega) - \arctan(\Omega)$$

Betrag $|\underline{F}(j\Omega)| = \frac{\Omega}{\sqrt{1 + \Omega^2}}$

$$\Omega = 0 \Rightarrow 0$$

$$\Omega = 1 \Rightarrow \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$$

$$\Omega \rightarrow \infty \Rightarrow \lim_{\Omega \rightarrow \infty} \frac{\Omega}{\sqrt{1 + \Omega^2}} = 1$$

Winkel: $\varphi_F = \varphi(j\Omega) - \arctan(\Omega)$

$$\Omega = 0 \Rightarrow 0 - 0 = 0 \leftarrow \frac{\pi}{2} \text{ Hier}$$

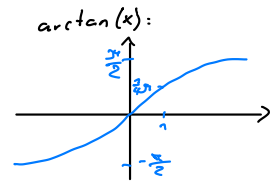
$$\Omega = 1 \Rightarrow \frac{\pi}{2} - \frac{\pi}{4} = \frac{1}{4}\pi$$

$$\Omega \rightarrow \infty \Rightarrow \lim_{\Omega \rightarrow \infty} (\varphi(j\Omega) - \arctan(\Omega))$$

$$\Rightarrow \frac{\pi}{2} - \frac{\pi}{2}$$

$$= 0$$

3/3



Ω	$ \underline{F}(j\Omega) $	$\varphi_{\underline{F}(j\Omega)}$
0	0	0
1	$\frac{1}{\sqrt{2}}$	$\frac{1}{4}\pi$
∞	1	0

iv

$$\underline{F}(j\Omega) = \frac{1}{1 + j\alpha\Omega + (j\Omega)^2}, \quad \alpha > 0$$

$$= \frac{1}{1 + j\alpha\Omega - \Omega^2}$$

$$|\underline{F}(j\Omega)| = \frac{1}{\sqrt{\alpha^2 \Omega^2 + (1 - \Omega^2)^2}} \quad \checkmark$$

$$\varphi_F = \varphi(1) - \varphi(1 - \Omega^2 + \alpha \Omega \cdot j)$$

$$= 0 - \arctan\left(\frac{\alpha \Omega}{1 - \Omega^2}\right)$$

Betrag: $|\underline{F}(j\Omega)| = \frac{1}{\sqrt{\alpha^2 \Omega^2 + (1 - \Omega^2)^2}}$

$$\Omega = 0 \Rightarrow |\underline{F}(j\Omega)| = 1$$

$$\Omega = 1 \Rightarrow |\underline{F}(j\Omega)| = \frac{1}{\sqrt{\alpha^2 + 0^2}} = \frac{1}{\alpha}$$

$$\Omega \rightarrow \infty \Rightarrow |\underline{F}(j\Omega)| = \lim_{\Omega \rightarrow \infty} \frac{1}{\sqrt{\alpha^2 \Omega^2 + (1 - \Omega^2)^2}}$$

$$\stackrel{(\infty)}{\lim_{\Omega \rightarrow \infty}} = \frac{1}{\sqrt{\Omega^4 \cdot \left(\frac{1}{\Omega^4} - \frac{2}{\Omega^2} + \frac{\alpha^2}{\Omega^2} + 1\right)}}$$

$$\stackrel{(\infty)}{\lim_{\Omega \rightarrow \infty}} = \frac{1}{\Omega^2} = 0 \quad \checkmark$$

Phase: $\varphi_F = -\varphi(1 - \Omega^2 + \alpha \Omega \cdot j)$

$$\Omega = 0 \Rightarrow \varphi_F = 0$$

$$\Omega = 1 \Rightarrow \varphi_F = -\arctan\left(\frac{\alpha}{1-1}\right) = -\frac{\pi}{2}$$

$$\Omega \rightarrow \infty \Rightarrow \varphi_F \stackrel{\lim_{\Omega \rightarrow \infty}}{=} -\pi + \arctan\left(\frac{\alpha \Omega}{1 - \Omega^2}\right)$$

$$\checkmark \quad = -\pi + \arctan(0) = -\pi$$

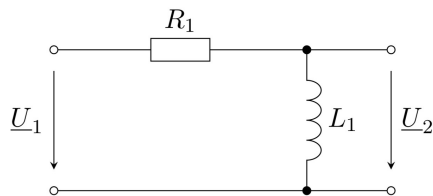
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Fallunterscheidung bei Phase fehlt
(Allgemein)

Aufgabe 2

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a)



(a)

$$Z_L = j\omega L$$

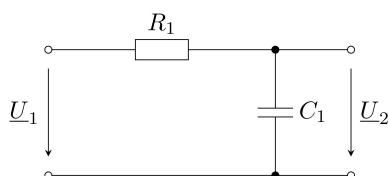
$$Z_C = \frac{1}{j\omega C}$$

$$U_2 = \frac{j\omega L_1}{R_1 + j\omega L_1} \cdot U_1$$

$$b = \frac{L_1}{R_1}$$

$$\Rightarrow \frac{U_2}{U_1} = \frac{j\omega L_1}{R_1 + j\omega L_1} = \frac{j\omega \frac{L_1}{R_1}}{1 + j\omega \frac{L_1}{R_1}} \triangleq \frac{j\omega b}{1 + j\omega b} = \underline{F}(j\omega)$$

b)

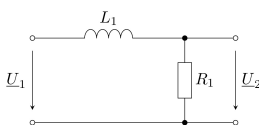


(b)

$$U_2 = \frac{\frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} \cdot U_1 = \frac{1}{R_1 j\omega C_1 + 1} \cdot U_1$$

$$\Rightarrow \frac{U_2}{U_1} = \frac{1}{1 + R_1 j\omega C_1} \triangleq \frac{1}{1 + j\omega a} = \underline{F}(j\omega)$$

c)



(c)

$$U_2 = \frac{R_1}{j\omega L_1 + R_1} \cdot U_1$$

$$\frac{U_2}{U_1} = \frac{1}{1 + j\omega \frac{L_1}{R_1}} \triangleq \frac{1}{1 + j\omega a} = \underline{F}(j\omega)$$

b) schaltungspaar b) und c)

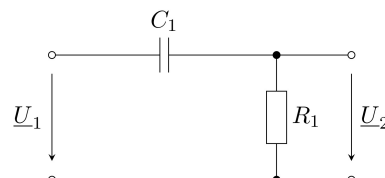
gleiches Klemmenverhalten, wenn:

$$\rightarrow R_1 C_1 = \frac{L_1}{R_1}$$

schaltungspaar d) und a)

gleiches Klemmenverhalten, wenn:

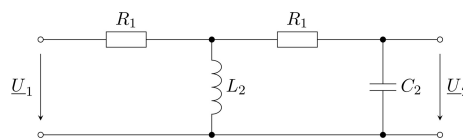
$$\rightarrow \frac{L_1}{R_1} = R_1 C_1$$



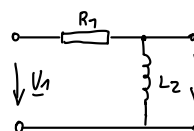
(d)

$$U_2 = \frac{R_1}{\frac{1}{j\omega C_1} + R_1} \cdot U_1$$

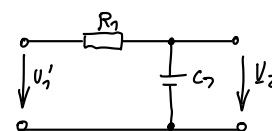
$$\Rightarrow \frac{U_2}{U_1} = \frac{j\omega C_1 R_1}{1 + j\omega C_1 R_1} \triangleq \frac{j\omega b}{1 + j\omega b} = \underline{F}(j\omega)$$



(e)



$$\frac{U'_2}{U_1} = \frac{j\omega \frac{L_1^2}{R_1}}{1 + j\omega \frac{L_1^2}{R_1}}$$



$$\frac{U_2}{U'_2} = \frac{1}{1 + j\omega C_2 R_1}$$

$$U'_2 \stackrel{!}{=} U_1$$

$$\Rightarrow U_2 = \frac{1}{1 + j\omega C_2 R_1} \cdot \frac{j\omega \frac{L_1^2}{R_1}}{1 + j\omega \frac{L_1^2}{R_1}} \cdot U_1$$

$$\Rightarrow \frac{U_2}{U_1} = \frac{j\omega \frac{L_1^2}{R_1}}{1 + j\omega \left(\frac{L_1^2}{R_1} + C_2 R_1 \right) + C_2 R_1 \frac{L_1^2}{R_1} (j\omega)^2}$$

$$\triangleq \frac{j\omega \cdot c}{1 + j\omega \cdot d + (j\omega \cdot e)^2} = \underline{F}(j\omega)$$

$$c = \frac{L_1^2}{R_1}$$

$$d = \left(\frac{L_1^2}{R_1} + C_2 R_1 \right)$$

$$e = \sqrt{C_2 \cdot L_2}$$