

Physics of molecular and biological Nanostructures

1. Introduction

$n = 10^{-9} \text{ m}$	e.g.	Cell	10 - 100 μm	ratio: 1 nm : 1 m	> 100 nm	light microscope
		bacteria	1 μm	1 nut : 1 earth	< 100 nm	electron microscope
		virus	10 - 100 nm			scanning probe
		DNA	1 nm			
		small molecules	1 nm			

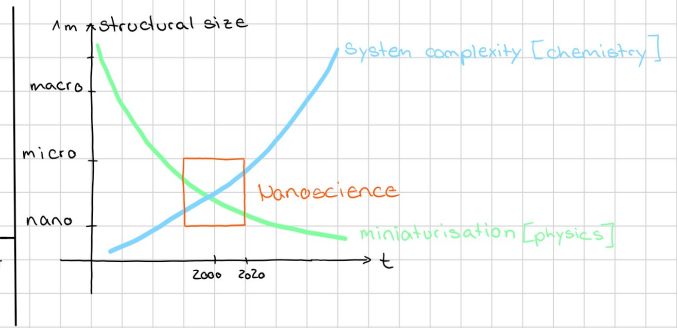
Nanotechnology:

- creation of new functional elements with defined control of atomic structures materials techniques

- atom/particle type, size, configuration (structure) $\rightarrow$ characteristics (phys, chem, bio) $\rightarrow$ application

3x fundamental characteristics

- Physics: Quantum mechanical behaviour
- Chemistry: increased/max. surface
- Biology: molecular recognition



Prokaryotic cells [bacteria]

- size $\approx 1 \mu\text{m}$
- propagation:
 - Flagellum: molecular motor $\rightarrow$ rotation of Flagellum
 - no Flagellum: utilize actin of host cell

Eukaryotic cells

- size $\approx 10 - 100 \mu\text{m}$
- propagation: actin polymerization [cytoskeleton]
 - actin filament: small helix $\varnothing 8 \text{ nm}$
 - Brownian motion $\rightarrow$ actin monomers attach to filament = growing filopodium $\rightarrow$ leading edge in direction of migration

2. Fundamentals of molecular biology

Life:

- characteristics:
 - metabolism
 - growth
 - reproduction

cell = smallest element of life compartment

virus is NOT life

hierarchy / assembly

atom → molecule → macromolecules → organelles → cell → tissue → organs → organism → populations → communities → ecosystem → biosphere
(functions) (mitochondria, nucleus, golgi) (bacteria, ...) (social) (subsystem)

history:

4000 Ma 1. Life → 3500 Photosynthesis → 2300 Ma Oxygen → bacteria → 530 Ma Cambrian explosion → 380 Ma 1. vertebrates → 2 Ma hominides

Cells

constitution:

90% H₂O
10% → 50% protein
15% carbohydrates (e.g. sugar)
15% nucleic acid
10% lipids
10% other

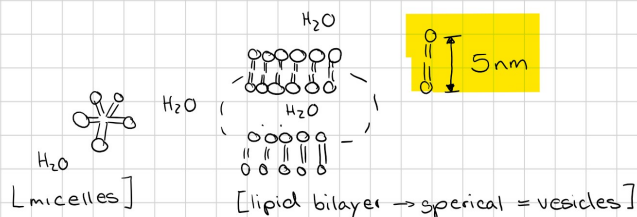
elements:

60% H
25% O
12% C (organic)
5% N

Phospho-Lipids

polar (Cholin) → hydrophilic
unpolar (Hydrocarbons) → hydrophobic

assembly in H₂O:



Lipid membranes:

- Form (T): crystal - gel - fluid
T_s << T_{ag}
structure >> chaos

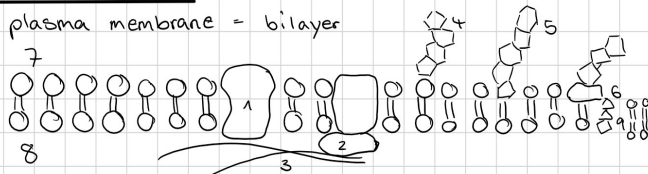
different physical attributes (e.g. elasticity...)

- Physics (A): stretching, bending, compressing

A₁ < A₂
A₁ = A₂ d₁ = d₂
d₁ > d₂] ΔE > 0

Cell membrane

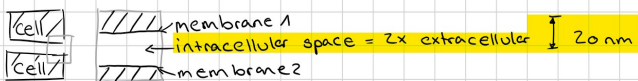
- plasma membrane = bilayer



- integrated protein
- peripheral protein
- filaments (cytoskeleton) → structure

- carbohydrates
- glycolipids
- glycoproteins
- cholesterol

- extracellular
- intracellular (cytoplasm)



Protein (proteios = erstrangig) =

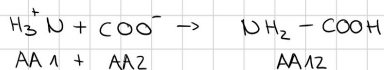
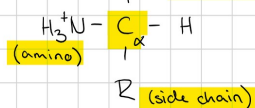
- function:
 - katalyze metabolism [enzymes]
 - transport [haemoglobin]
 - supporting structure [collagen = fibreprotein → bones and cartilage]
 - immunological [antibodies against pathogens]
 - sensors [signal conduction / receptor]
 - hormone [growth, insulin]
 - motor [myosin, actin, titin → (muscle) contraction → mechanical movement]

↳ important role in biological processes

structure:

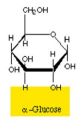
- primary: 2-D / linear AA sequence [peptide-bonds]
- secondary: 3-D / spacial arrangement [α-helix (1-strand), β-sheet (n-strands)]
- tertiary: 3-D / 2nd spacial arrangement [disulfid-, H-, ionic-, V.d.W. interactions] → function
- quaternary: multiple tertiary-proteins

20x AA: COO⁻ (carboxyl)

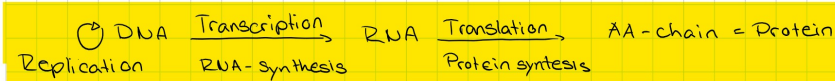


- hydrophobic / -philic, aliphatic
- acidic, basic, aromatic
- charged, neutral
- polar / unpolar

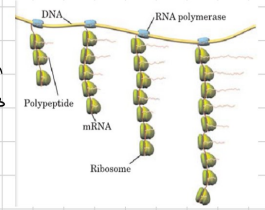
Carbohydrates



Central dogma [DNA $\rightarrow$ Protein]



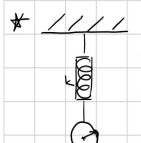
Simultaneous on
different locations
on the DNA



- AFM : measurement of 3D surface structure on atomic lv. (height as colour coordinates)
- atomic force microscopy - probe pushed into flexible bilayer $\rightarrow$ measurement of force required/resistance $\rightarrow$ rupture/break through
- mechanical (rupture, elasticity) information
- distinguish lipids: variations $\rightarrow$ constant measurement fluctuations (thermodynamics) $\rightarrow$ gauss distribution
- Micropipette : - pipette on vesicle $\rightarrow$ neg. pressure $\rightarrow$ membrane sucked into pipette
- Aspiration - stiffness membrane $\propto$ distance into pipette: linear

1. Scaling Effects

- weight of physical effects on nanoscale $\rightarrow$ scaling affects physical behavior

* 

$D \hat{=}$ diameter spring
 $d \hat{=}$ diameter spring wire
 $r \hat{=}$ $\frac{1}{2}$ diameter of sphere

$D \approx r$
 $d \approx \frac{1}{10} r$

$A \propto r^2$
 $Q \propto r^2$
 $m \propto r^3$
 $C \propto r^6$
 $E_e \propto \frac{1}{r^2}$

$k \propto r$

$F \propto r^2$
 $E \propto r^3$
 $f_0 \propto \frac{1}{r}$

$\frac{1}{m} \propto \frac{1}{r^3}$

2. Free Energy [F]

- quantity that drives processes and determines dynamics of a system

• isothermal $\Delta T = 0$

$$\Delta F = \Delta U - T \Delta S$$

• reversible: eq. formula

$$\Delta S = \frac{\Delta Q}{T}$$

• Irreversible: statistical formula

$$S = k_B \ln(\Omega)$$

F: free Energy

U: potential/inner Energy

T: Temp

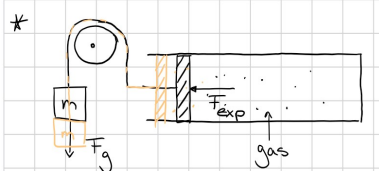
S: Entropy

] thermodynamics

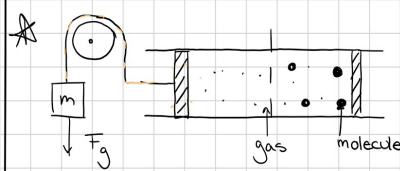
Ω : # of possible states per molecule

Ω^N : # " " for N molecules

N: # of molecules

* 

$F_g + F_{exp}$

* 

$F_{exp, gas}$ & $F_{exp, molecule}$

$$\Delta U < T \Delta S \quad \Delta F < 0$$

$$\Delta U > T \Delta S \quad \Delta F > 0$$

1) $\Delta U > 0$

2) $\Delta A_m > 0$
 $\Delta P_m < 0$
 $S > 0$

3) $\Delta F < 0$
 > 0

; movement direction stays changes

* irreversible (diaphrame), isothermal ($\Delta T = 0$), isolated System



$T_{gas} \propto \varnothing$ kinetic Energy; $\varnothing$ velocity

$$\Delta T = 0 \rightarrow \Delta U = 0$$

$$\Delta S > 0 \rightarrow \Delta F = \Delta U - T \Delta S < 0$$

$$\Delta S = S_B - S_A \quad \Omega: \text{right, left}$$

$$S_A = k_B \ln(1^N) = 0 \quad A[r] = 1$$

$$S_B = k_B \ln(2^N) \quad B[r \vee L] = 2$$

$$\Delta W = |-\Delta F| \quad W: \text{mechanical work}$$

$$pV = N k_B T \quad F: \text{Energy that can be gained from the System change } A \rightarrow B \text{ has to be put into}$$

* reversible (movable membrane)

- Δ kinetic Energy < 0 ; velocity $\rightarrow$ movement molecule membrane

$$\Delta T_{gas} < 0$$

; loss of energy

Systems

	matter	energy
- isolated	-	-
- closed	-	✓
- open	✓	✓

relevant energy for calc. of $\vec{F}_{interaction}$

$$\vec{F} = -\nabla U$$

$$\vec{F} = -\nabla \mu \quad (\text{if } \Delta T = 0)$$

2.1 Boltzmann statistics

- average distribution of the occupation of (non-interacting) energy states at $\Delta T = 0$

- probability of system to be in a state i with energy U_i :

$$p \propto e^{-\frac{U_i}{k_B T}}$$



$$p_2 = e^{-U_2/k_B T}$$

$$\frac{p_2}{p_1} = e^{-\frac{(U_2 - U_1)}{k_B T}} = e^{-\frac{\Delta U}{k_B T}}$$

$$p_1 = e^{-U_1/k_B T}$$

- the higher the state the lower the probability

Boltzmann averaging

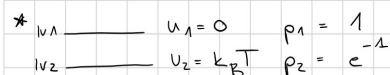
$$\langle U \rangle = \frac{\sum w_i p_i}{\sum p_i N_i} \rightarrow \text{normalization}$$

w_i : weight

N_i : often $N=1$

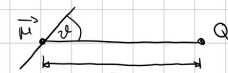
p_i : BMF

$\langle U \rangle$: average inner energy



$$\langle U \rangle = \frac{p_1 U_1 + p_2 U_2}{p_1 + p_2} = \frac{k_B T \cdot e^{-1}}{1 + e^{-1}} \approx 0.27 k_B T$$

* charged monopole - dipole interaction: fixed $T=0$



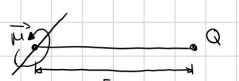
fixed dipole: $\theta = \text{const}$

$\vec{p}$: dipole moment

θ : angle $\hat{=}$ potential energy

$$w(r, \theta) = \frac{-Q \mu \cdot \cos(\theta)}{4 \pi \epsilon_0 r^2} \hat{=} \text{potential energy}$$

* charged monopole - dipole interaction: rotating $\Delta T \gg 0$



freely rotatable: $p_{\theta i} = e^{-\frac{w(r, \theta)}{k_B T}}$

$$U(r) = \langle w(r, \theta) \rangle_{\Omega, p} = \frac{\int d\Omega w(r, \theta) p_{\theta}}{\int d\Omega p_{\theta}} = \frac{-(Q \mu)^2}{3(4 \pi \epsilon_0)^2 k_B T r^4}$$

Ω : # configurations / angles

p : Boltzmann factor

$\langle \rangle$: average energy of Ω and their probability

$$F = \frac{C}{T}$$

$$U = -T^2 \left(\frac{\partial \frac{C}{T}}{\partial T} \right) = 2 \frac{C}{T} = 2F \quad (\text{Gibbs-Helmholtz eq})$$

$$F = \frac{1}{2} U$$

isolated system

U : inner/potential energy

isothermal process

F : free energy

Type of interaction	Interaction energy $w(r)$	only free energy
Covalent, metallic		
Charge-charge	$Q_1 Q_2 / 4 \pi \epsilon_0 r$ (Coulomb energy) $-Q u \cos \theta / 4 \pi \epsilon_0 r^2$	
Charge-dipole	Fixed dipole: will fluctuate since $\Delta T \neq 0 \rightarrow$ rotate / oscillate Freely rotating	$-Q^2 u^2 / 6(4 \pi \epsilon_0)^2 k_B T r^4$ $\rightarrow$ free (Helmholtz) internal process $\rightarrow$ inner isolated sys
Dipole-dipole	Fixed Freely rotating	$-u_1 u_2 [2 \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 \cos \phi] / 4 \pi \epsilon_0 r^3$ $-u_1^2 u_2^2 / 3(4 \pi \epsilon_0)^2 k_B T r^6$ (Keesom energy) r^6 : VdW
Charge-non-polar		$-Q^2 \alpha / 2(4 \pi \epsilon_0)^2 r^4$ $-u^2 \alpha (1 + 3 \cos^2 \theta) / 2(4 \pi \epsilon_0)^2 r^6$ interaction is comb of Keesom-Debye-London
Dipole-non-dipolar	Fixed Rotating	$-u^2 \alpha / (4 \pi \epsilon_0)^2 r^6$ (Debye energy) $\frac{3}{4} \frac{h \nu^2}{(4 \pi \epsilon_0)^2 r^6}$ (London dispersion energy)
Two non-polar molecules		
Hydrogen bond		Complicated, short range, energy roughly proportional to $-1/r^2$

Fig. 2.2. Common types of interactions between atoms, ions and molecules in vacuum. $w(r)$ is the interaction free energy (in J); Q : electric charge (C); u : electric dipole moment

3 DNA I Conformation

3.1 Molecular structure of DNA

- 3 D molecule
- highly negatively charged
- double helix
- backbone (Phosphate and deoxyribose) + bases (Nucleotides: Guanine, Cytosine, Adenine, Thymine)
- 2 linear polymers that are coupled with complementary base
 - Hydrogen bonds (G-C, A-T)
 - Planar/perpendicular to the strand
- Directionality (not symmetric) (opposing for the two strands)

3.2 Genome: DNA encodes information

- Gene = information that encodes one protein
 - n genes = n proteins

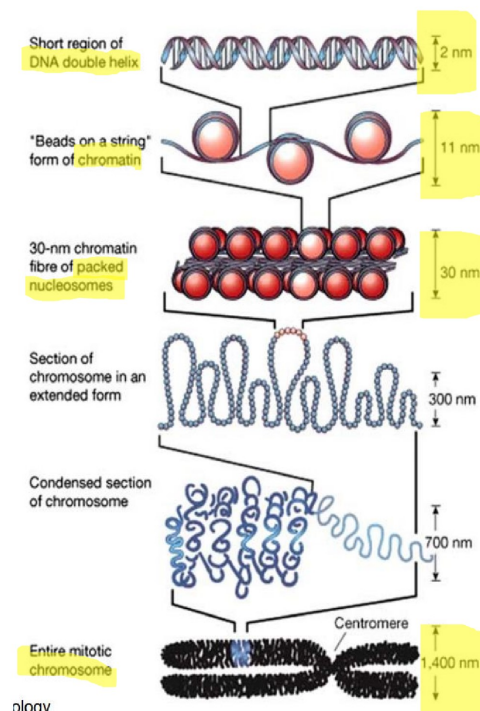
— n base pairs 4^n combinations

○ Virus:	5.000 base pairs	10 genes
○ Bacteria:	5 mio. (10^6)	4.000
○ Human:	3 bio (10^9)	25.000

- 20 AA: 3 base pairs = 1 AA $\rightarrow$ 20 AA = 4^3 combinations
- Why 3?: $4^3 = 64 > 20$ BUT $4^2 = 16 < 20$

3.3 Chromosomes

- DNA storage in cell by condensing: Length + diameter -
- DNA in cells present in form of chromosomes: 1 Chromosome = 1 DNA molecule
 - DNA wrapped around Histone proteins $\rightarrow$ Fibre $\rightarrow$ Filaments



- Bacteria: 1 chromosome
- Human: 46 chromosomes

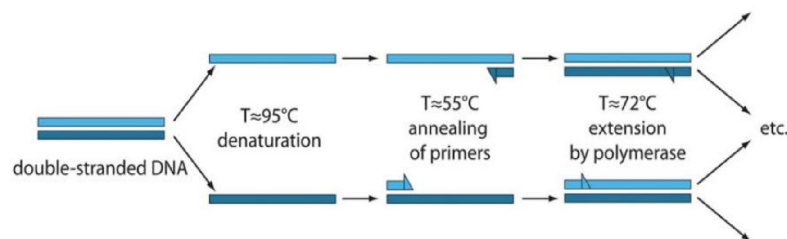
4 DNA II Computers

4.1 Gel electrophoresis

- Voltage $\rightarrow$ neg DNA moves through medium to Electrode $\rightarrow$ pores in medium $\rightarrow$ length/size separation

4.2 Polymer-chain-reaction [PCR]

- DNA replication [5'-3']
- Thermocycle
 - o Denaturation: breaking of H-H bonds between complementary nucleotides
 - o Annealing: start-codon for Polymerase
 - o Extension: Polymerase attachment
- n : cycles $\rightarrow$ # DNA = 2^n



5 DNA III Packaging

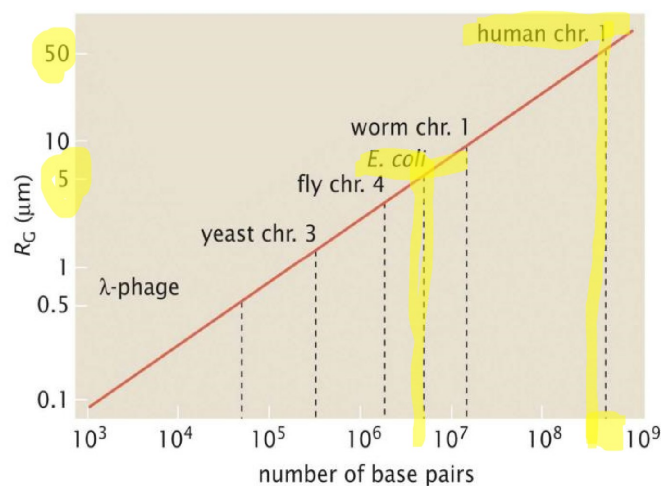
- Size in solution

$$N_{bp} \approx 4.6 \times 10^6 \text{ base pairs}$$

$$\rightarrow R_G \approx 5 \mu\text{m}$$

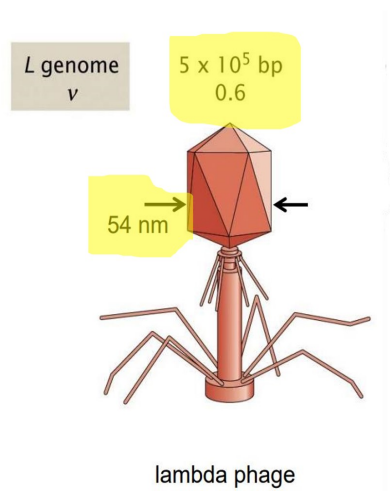
- o Bacteria [E. coli]

Size of genomic DNA in solution

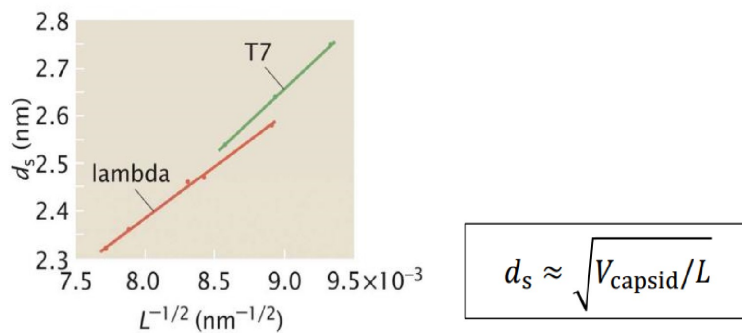


- Virus DNA packaging rate

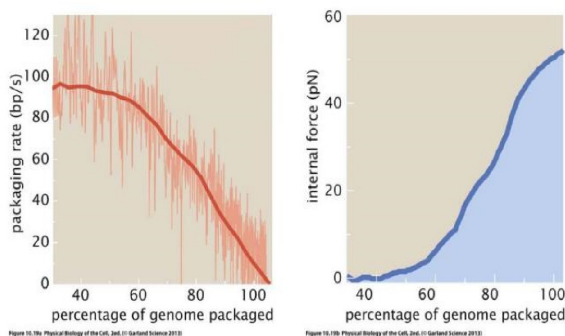
— Virus [Bacteriophage T4] DNA packaging



- Life cycle ~ 30 min
- DNA packaging [motor molecule using ATP]
 - Hierarchy: Orderly stored in circles on the outer layer = energetically favourable



The longer the tighter packed



The later the more force is needed to pack the genome (because of the bending force of the molecules)

— Retro virus [HIV]

- Replication:
 - RNAvirus $\rightarrow$ Replication [enzymesvirus] DNAvirus $\rightarrow$ Intergration DNAhost $\rightarrow$ Proteinsvirus

3. DNA I: Conformation of linear Polymeres

- Model: uniform worm-like-chain (WLC) [2-D]
- lowest energy state $\theta = 0^\circ$
- thermal energy fluctuations $\rightarrow$ const. conf. change (bending) to maintain thermal equilibrium

Bending energy: $\Delta U = \frac{EI}{2L} \theta^2$

E : Young's modulus $\sim$ elasticity $E_{cell} \sim kPa$ $E_{DNA} = 300 MPa$
 I : area moment of inertia $\sim$ shape: $I = \frac{\pi}{4} r^4$

propability for configuration θ : $w(\theta) \propto e^{-\frac{EI}{2L} \theta^2}$

- normalized $w(\theta) = \sqrt{\frac{EI}{2\pi L}} e^{-\frac{EI}{2L} \theta^2}$ more bending = more energy = less likely

Persistence length: $L_p = \frac{EI}{k_B T}$ = $\frac{\text{mechanical properties}}{\text{thermal energy properties}}$ = l , at which polymere loses original orientation (overlapping beginin)

* $l_1 = 2 \times l_2$

$\Delta U_1 < \Delta U_2$

$\Delta U_1 > \Delta U_2$

Autocorrelation fct (ACF)

- correlation of position L & L'

$ACF = e^{-\frac{|L-L'|}{2L_p}} = \langle t(L) \cdot t(L') \rangle_{RMT} = \langle \cos(\theta)_{LL'} \rangle_{RMT}$

$L_{contour} \ll L_{persistence} \rightarrow L-L'$ close $\rightarrow$ related $\rightarrow$ short & straight

$L_{contour} \gg L_{persistence} \rightarrow L-L'$ apart $\rightarrow$ unrelated $\rightarrow$ long & folded

L_p in cells:

- related to structural properties & function

microtubuli
 $r \approx 20 nm$
 $L_p \approx 10 nm \gg L$
(stiff)

actin
 $r \approx 4 nm$
 $L_p \approx 15 \mu m \approx L$
(semiflexible)

DNA
 $r \approx 1.5 nm$
 $L_p \approx 50 nm \ll L$
(flexible)

L = size of a cell = (10 - 100) μm

* DNA: $T = 25^\circ C$, $L_p = 58 nm$
shape: circular $r = 1 nm$
 $E = 300 MPa$

* Pen: $r = 3 nm$, $L_p = 10^{20} m$
 $E = 10 GPa$
 $\rightarrow$ won't bend unless $10^{20} m$ long

Stretching

high Entropie
high # states

low Entropie
1 state

- Entropie can only be decreased by Force

- refolding if $F=0$

* relevance of L_p :

$L_p: r$; $E = 5 MPa$
 $1: 100$ or smaller

$r \leq 3 \sqrt{\frac{4 \cdot 100 \cdot k_B T}{\pi E}} = 4,7 nm$

$\rightarrow L_p$ only becomes relevant at nm radius

* $L_p = 10 nm$
 $L = 1000 nm$

$\rightarrow \Delta R = 2 \cdot \sqrt{10 \cdot 1000 nm^2} = 200 nm$
closer to smaller number

End-to-end distance $\vec{R}$

$\vec{R} = \int_0^L t(s) ds$ $\langle \vec{R} \rangle = 0$

$\langle \vec{R}^2 \rangle = \langle \vec{R} \cdot \vec{R} \rangle = 4Lp (1 - \frac{2Lp}{L} (1 - e^{-\frac{L}{2Lp}}))$

$L \gg L_p$ $\langle \vec{R}^2 \rangle = 4LpL$

$\Delta R = \sqrt{\langle \vec{R}^2 \rangle} = 2\sqrt{LpL}$ $\Delta R \hat{=} RMS$

$\langle \vec{R}^2 \rangle_{50} = 2Lp (1 - \frac{Lp}{L} (1 - e^{-\frac{L}{Lp}})) = \frac{1}{2} \langle \vec{R}^2 \rangle_{20}$

$L \gg L_p$ $\langle \vec{R}^2 \rangle_{50} = 2LpL = \frac{1}{2} \langle \vec{R}^2 \rangle_{20}$

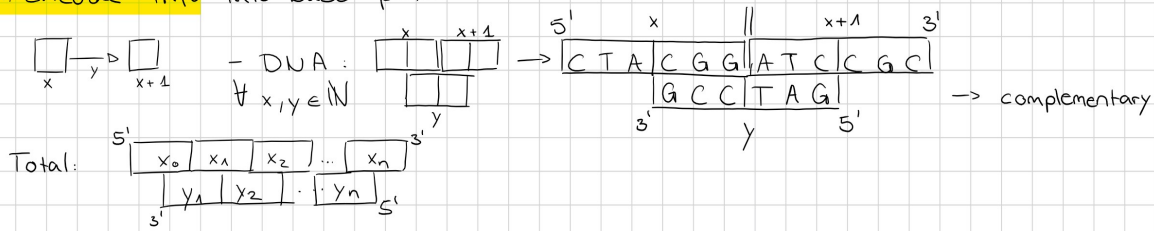
$\Delta R_{50} = \sqrt{2LpL}$

arithmetic mean: $\frac{a+b}{2}$
geometric mean: $\sqrt{ab}$

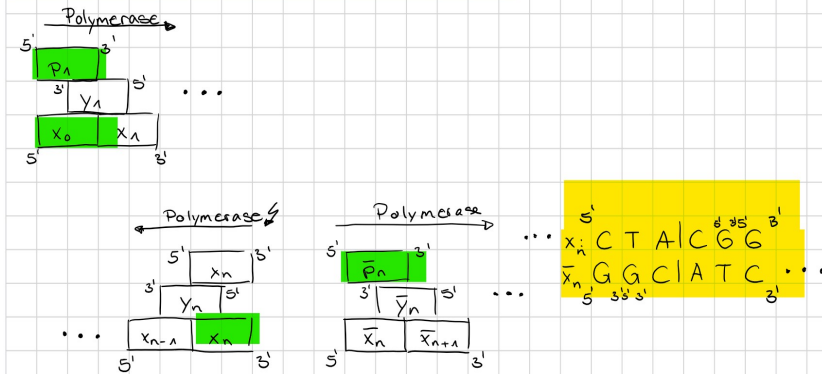
4. DNA II: Computing

- Computing info. process
- DNA info. storage

1. Encode info into base pairs



2. Amplify specific x_0, x_n [PCR]



3. Extract [Gel electrophoresis]

- = bestimmte Länge

$x_0 x_1 x_2 x_3 x_4 x_5$

4. Extract type [Tandem affinity purification]

- Prime magnetic beads



- extract $x_0 \rightarrow$ extract $x_1 \rightarrow \dots$ extract x_n

$\rightarrow$ length made up out of different x_i

5 DNA III: Packaging

- molecule "size" in solution

$$\Delta R_{3D} = \sqrt{2LpL}$$

$$L \approx 0,34 \text{ nm} \cdot \# \text{ base pairs}$$

L : contour length

$$R_g = \sqrt{\frac{1}{N} \cdot \sum_{i=1}^N \langle |\vec{r}_i - \vec{r}_{CM}|^2 \rangle} = 0,4 \Delta R_{3D} \text{ radius of gyration}$$

average distance of monomers to the polymer's CM

$$\vec{r}_{CM} = \frac{1}{N} \sum_{i=1}^N \vec{r}_i$$

$$\# \text{ bacteria} : \# \text{ base pairs } 5 \cdot 10^6$$

$$L_p = 50 \text{ nm}$$

$$\rightarrow R_g = 5 \mu\text{m}$$

6. Elektrohydrodynamics

6.1 electric double layer: Debye layer

- Maxwell

$$\vec{\nabla} \times \vec{E} = 0$$
$$\vec{\nabla} \cdot \vec{D} = \rho_{el}$$
$$\vec{D} = \epsilon \epsilon_0 \vec{E}$$
$$\vec{E} = -\vec{\nabla} \Phi$$

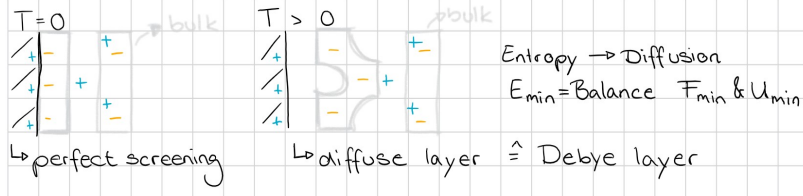
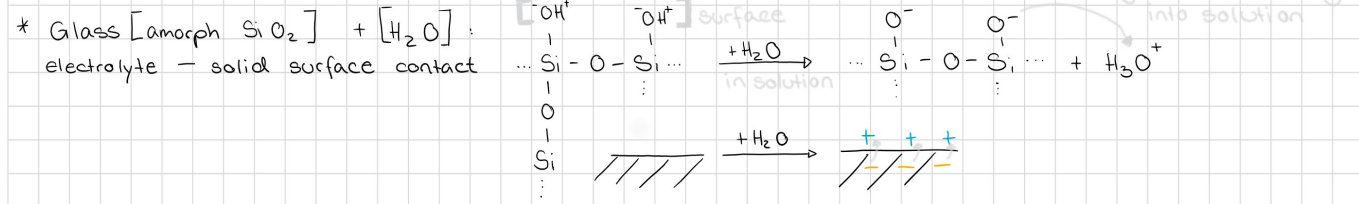
- Poisson

$$\vec{\nabla}^2 \Phi = -\frac{1}{\epsilon \epsilon_0} \rho_{el}$$

$\vec{\nabla}$: x rot
 $\vec{\nabla} \cdot$: div.
 $\vec{E}$: el. field
 $\vec{D}$: el. displacement field
 ρ : el. charge density
 ϵ/ϵ_0 : relative/vacuum dielectr. permeability
 Φ : potential
 $\vec{\nabla}^2$: Laplace operator

- Spatial distribution of free ions in solution (diffusion)

Situation: - salt solution ("electrolyte") [NaCl] [KCl]
→ complete ion dissociation
→ charge neutral bulk solution (far away from surface)



Boltzmann concentration: $c_{\pm} = c_0 e^{-\frac{\Delta E_{pot\pm}}{k_B T}} = c_0 e^{\pm \frac{ze}{k_B T} \Phi}$

$\frac{\text{\#ions}}{[m^3]}$ z [unit charges]

$$\rho_{el} = ze [c_+ - c_-] = -2ze c_0 \sinh\left(\frac{ze}{k_B T} \Phi\right)$$

$\sinh(x) = \frac{1}{2}(e^x - e^{-x})$

Debye-Hückel approximation:
 $ze\Phi \ll k_B T \rightarrow \sinh(x) \approx x$
Debye length

$$\vec{\nabla}^2 \Phi = \frac{1}{\lambda_D^2} \Phi$$
$$\lambda_D^2 = \frac{\epsilon \epsilon_0 k_B T}{z(z e)^2 c_0}$$

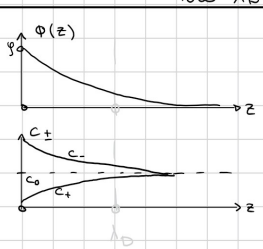
* $RT; Z=1 \rightarrow \Phi \ll \frac{1}{40} V \approx 26 mV$
[0,1 molar NaCl] $T_{RT} \rightarrow \lambda_D \approx 0,96 nm$
low $\lambda_D \hat{=}$ better screening

boundary cond:

$$\Phi(\infty) = 0 \quad c_{\pm}(\infty) = c_0$$
$$\Phi(0) = \varphi$$

* planar surface: $z=0$

$$\vec{\nabla}^2 \Phi(z) = \frac{1}{\lambda_D^2} \Phi(z) = \Phi(z) = \varphi e^{-\frac{z}{\lambda_D}}$$
$$\rho_{el} = -\frac{\epsilon \epsilon_0}{\lambda_D^2} \Phi(z)$$
$$c_{\pm} = c_0 \left[1 \pm \frac{ze \varphi}{k_B T} e^{-\frac{z}{\lambda_D}} \right]$$



* Infinite parallel plate channel: $z = \pm \frac{h}{2}$

BC: $\Phi(z) = \varphi \frac{\cosh(\frac{z}{\lambda_D})}{\cosh(\frac{h/2}{\lambda_D})}$



* cylindrical channel

- Bessel DEQ of order 0

$$\nabla^2 \Phi(r) = \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right] \Phi(r)$$
$$\Phi(r) = \varphi \frac{I_0(\frac{r}{\lambda_D})}{I_0(\frac{a}{\lambda_D})}$$
$$\rho_{el} = -\frac{\epsilon \epsilon_0}{\lambda_D^2} \Phi(r)$$

* derivation:

planar surface $z=0$ BC: $\Phi(z) = a e^{bz}$
 $\Phi(0) = \varphi$
 $\Phi(\infty) = 0$

$$\frac{d^2 \Phi(z)}{dz^2} = b^2 a e^{bz} = \frac{1}{\lambda_D^2} \Phi(z) = \frac{1}{\lambda_D^2} a e^{bz}$$
$$\rightarrow \Phi(z) = \varphi e^{-\frac{1}{\lambda_D} z}$$

$a = \varphi$
 $bz = -\infty$
 $b^2 = \frac{1}{\lambda_D^2}$
 $b = -\frac{1}{\lambda_D}$

- surface charge:

$$A q_{surf} = \epsilon \epsilon_0 E A = q_{surf} \cdot A$$

$$q_{surf} = -\epsilon \epsilon_0 \frac{\partial}{\partial z} \Phi(0) = \frac{\epsilon \epsilon_0}{\lambda_D} \varphi$$

$$q_{aq} = \int_0^\infty \rho(z) dz = \frac{\epsilon \epsilon_0 \varphi}{\lambda_D^2} \int_0^\infty e^{-\frac{z}{\lambda_D}} dz = -q_{surf}$$

- surface conductivity

Debye layer $\hat{=}$ charge neutral capacitor

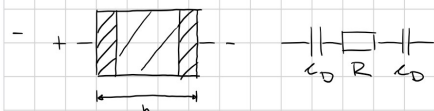
$$C_D = \frac{q_{surf}}{\varphi} = \frac{\epsilon \epsilon_0}{\lambda_D}$$

$A q_{surf}$: surface charge density
 q_{surf} : surface charge
 q_{aq} : debye layer charge
 C_D : capacitance per unit area ($\frac{q}{V}$ per unit area)

- RC - t - constance (t-until Debye layer is formed) :

* φ -potential of electrolyte if 10V applied at surface [metal]?

- $\varphi = 10V$: applied $V \hat{=}$ zeta potential



$$\tau_{RC} = R \cdot C = R \cdot \frac{1}{2} C_D = \left(\frac{h}{\sigma_{el} \cdot A} \right) \cdot \frac{1}{2} \left(\frac{\epsilon \epsilon_0}{\lambda_D} \cdot A \right) = \frac{\epsilon \epsilon_0}{2 \sigma_{el}} \cdot \frac{h}{\lambda_D}$$

$f \gg \frac{1}{\tau_{RC}}$: Debye layer does not form

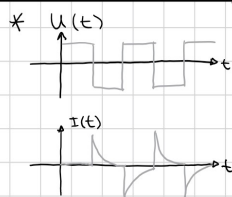
* $\lambda_D = 9,6 \text{ nm}$

$h = 100 \mu\text{m}$

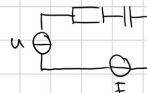
$\epsilon = 78$

$\epsilon = 10^{-3} \frac{\text{S}}{\text{m}}$

$\tau_{RC} = 3,6 \text{ ms}$



- HP-Filter,
low f are filtered out



- Assumptions [P-B-theory]

• solution in continuum
(reality: atoms)

• no near surface effects
(reality: stern layer)

• ions are continuous
(reality: ion diameter with hydration shell: 0,3-0,6nm)

• ideal surface
(reality: surface movement)

• only Coulombic interactions
(reality: hydration forces...)

6.2 Navier-Stokes-hydrodynamics

- Navier-Stokes eq.
- Newtons 2. law: $\Sigma \vec{F} = m \ddot{x}$ for fluids

$$\rho = \frac{m}{V} \rightarrow \Sigma \vec{F} = \rho \ddot{x}$$

$$\rho \left[\frac{d\vec{v}}{dt} + (\vec{v} \cdot \nabla) \vec{v} \right] = - \nabla P_{\text{ext}} + \underbrace{\eta \nabla^2 \vec{v}}_{\text{friction}} - \underbrace{\rho_{\text{el}} \nabla \Phi_{\text{ext}}}_{\text{electrical force}}$$

! non-linear!

solution: small velocities

$\frac{d}{dt} \hat{=}$ material derivative; total differential
 $\frac{\partial}{\partial t} \hat{=}$ partial differentiation

$$\frac{dv(x,y,z,t)}{dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z}$$

P_{ext} external pressure

η : viscosity

ρ : density

v_{eo} : velocity electroosmosis: $(\varphi, \eta, \vec{E})$

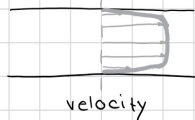
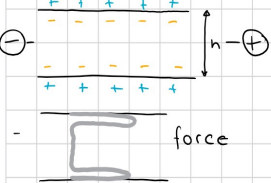
6.3 Electroosmosis [EO]

-  channel walls; neg. charged solution; pos. charged $\rightarrow$ ion flow = fluid flow

- Assumptions: ideal EO-flow

- φ is constant along the surface
- $\vec{E}$ is homogenous
- flow in steady-state
- $\lambda_D \ll$ channel radius/width h

* Flow between 2 infinite parallel plates ($z = \pm \frac{h}{2}$)



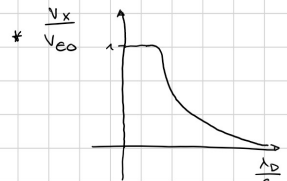
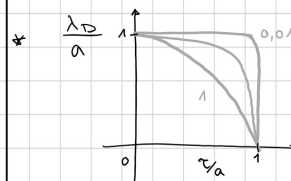
$$v_x(z) = \left[1 - \frac{\cosh\left(\frac{z}{\lambda_D}\right)}{\cosh\left(\frac{h/2}{\lambda_D}\right)} \right] v_{\text{eo}}$$

$$v_{\text{eo}} = \frac{\epsilon \epsilon_0}{\eta} \varphi \vec{E}$$

- cations on surface experience force
- $\rightarrow$ dragging of H_2O molecules
- $\rightarrow$ acceleration of whole mass
- $\rightarrow$ friction at surface

* $\lambda_D \ll h$ $\vec{v} = v_{\text{eo}} \cdot \vec{e}_x$ v_{aq} of $\forall \text{ aq} = \text{const}$

$$Q_{\text{eo}} = \int_{-h/2}^{h/2} \int_0^w dy v_x(y,z) = v_{\text{eo}} \cdot w \cdot h \hat{=} \text{flow rate over time}$$



- $\frac{\lambda_D}{a} = 0,01 \rightarrow v_x(r) =$

* cylindrical channel

$$v_x(r) = \left[1 - \frac{I_0\left(\frac{r}{\lambda_D}\right)}{I_0\left(\frac{a}{\lambda_D}\right)} \right] v_{\text{eo}}$$

$$v_{\text{eo}} = \frac{\epsilon \epsilon_0}{\eta} \vec{E} = \frac{\epsilon \epsilon_0}{\eta} \frac{\Delta \varphi}{L}$$

$$Q_{\text{eo}} = \int_0^{2\pi} d\theta \int_0^a dr r v_x(r,\theta) = v_{\text{eo}} \pi a^2$$

* v_{eo} for applied $U = 1V$ over a length of 1 cm^2

- H_2O : $\epsilon = 78$; $\epsilon_0 = 8,85 \cdot 10^{-12}$; $\eta = 10^{-3} \frac{\text{kg}}{\text{ms}}$
 $\varphi = 100 \text{ mV}$

$$v_{\text{eo}} = 7 \frac{\mu\text{m}}{\text{s}}$$

Streaming potential

- movement of aq
- $\rightarrow$ aq dragging of surface charges
- $\rightarrow$ charge concentration increase in direction of velocity
- $\rightarrow$ potential difference between capillary ends

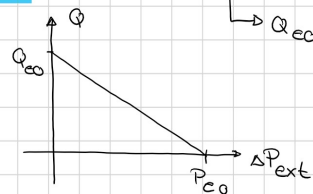
- Ideal EO-flow with back-pressure
- pressure generated by resistance (e.g. gravity)

* cylindrical channel

$$v_x(r) = \underbrace{\left[1 - \frac{I_o(r/\lambda_D)}{I_o(a/\lambda_D)}\right] v_{eo}}_{EO} - \underbrace{\left[\frac{a^2 - r^2}{4\eta}\right] \frac{1}{L} \frac{\Delta P}{L}}_{\text{Hagen-Poiseuille}}$$

$$Q = Q_{eo} - Q_p = \frac{\pi a^2 \epsilon \epsilon_0}{\eta L} \gamma \Delta V - \frac{\pi a^4}{8\eta} L \cdot \Delta P = \pi a^2 v_{eo} - \frac{1}{R_{hyd}} \Delta P$$

↳ Q_{eo} at free flow ↳ electroosmotic pressure at 0 flow



$$Q_{eo} = \frac{\gamma \epsilon \epsilon_0}{\eta L} \Delta V \cdot \pi a^2$$

$$P_{eo} = \frac{\gamma \epsilon \epsilon_0}{8} \Delta V \cdot \frac{1}{a^2}$$

$$\rightarrow Q_{eo} \propto \frac{1}{P_{eo}} \propto a^2$$

generate high pressure for small channels

Typical values:

- $\gamma = 100 \text{ mV}$
 - $a = 10 \mu\text{m}$
 - $L = 100 \mu\text{m}$
 - $\eta = 10^{-3} \frac{\text{kg}}{\text{ms}}$
 - $\epsilon = 78$
- $\frac{Q_{eo}}{\Delta V} = 0,21 \frac{\text{nL}}{\text{sV}}$
- $\frac{P_{eo}}{\Delta V} = 5,5 \frac{\text{Pa}}{\text{V}}$

- relatively low numbers
- will be given in exam!



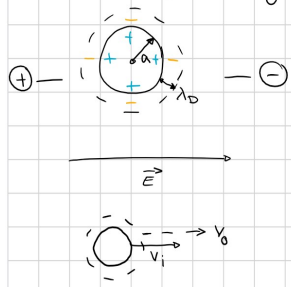
max. back pressure (one closed end)
with 0 net flow at end

Effects of channel dimension

	Electric field	Current	Backpressure	Flow rate
Length	↑	↑	-	↑
	↓	↓	-	↓
Radius	-	↑	↑	↑
	↑	↓	↓	↓

6.4. Electrophoresis

- motion of charged particles in electrolyte because of $\vec{E}$



- $\vec{E}, \lambda_D$ unaffected by each other
- $\varphi \ll$
- v_i : particle velocity
- v_o : debye layer velocity
- relative to the bulk water

Sedimentation potential

- charged particle moves in electrolyte $\rightarrow$ Induction of $\vec{E}$

- gravitational field ; centrifuge
- $\rightarrow v_o < v_i$
- $\rightarrow$ dipole moment
- $\rightarrow \vec{E}$ dipole moment - sedim. pot.

System i [particle] System o [debye layer]

radius [r] a a + λ_D

$\rightarrow Q = 4\pi \cdot r^2 \cdot q_{surf}$ $F_{el,i} = -F_{el,o}$

el. force [F_{el}] $Q\vec{E}$ $-Q\vec{E}$

$\rightarrow$ Stokes law: $F_{drag} = 6\pi\eta \cdot r \cdot v$

viscous drag force [F_{drag}] $6\pi\eta a \cdot v_i$ $6\pi\eta (a + \lambda_D) v_o$

$\rightarrow F_{el} = F_{drag}$ $v = \frac{F_{el}}{F_{drag}}$

- $v_{resulting}$ $v_i = \frac{QE}{6\pi\eta a}$ $v_o = \frac{-QE}{6\pi\eta (a + \lambda_D)}$

$v_{particle}$ relative to bulk water

$$v_{cp} = v_i + v_o = \frac{QE}{6\pi\eta a} \left(1 - \frac{1}{1 + \frac{\lambda_D}{a}}\right) = \frac{QE}{6\pi\eta a} \cdot \frac{\lambda_D}{a} = \frac{2 \epsilon \epsilon_0 \vec{E}}{3 \eta} \varphi$$

$\lambda_D \ll a$

$$v_{cp} = \begin{cases} v_{eo} & \lambda_D \ll a \\ \frac{2}{3} v_{eo} & \lambda_D \gg a \end{cases}$$

- v_{cp} independent of size

Reynolds number

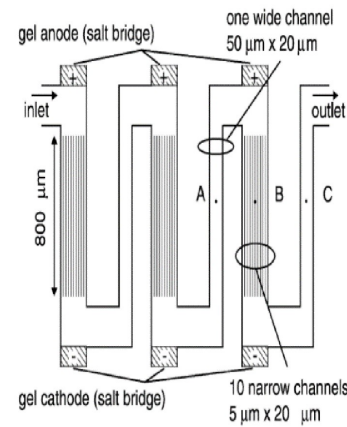
$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho a v}{\eta}$$

- $Re \gg$: turbulence

6 Electrohydrodynamic: Micro- and Nanofluidic Applications

6.1 EO: Low voltage (10 V) cascade pump

- Construct micropump with decent flow rate and still have high pressure?
 - Pump large volumes over large distances.
- Parallel channels and different diameter:
 - Down channel made up of single parallel channels
→ high pressure in parallel components → Total: voltage constant for all BUT current increases → EO flow.
 - Up single channel → small pressure and EO-flow small → Pressure driven flow.
 - → Total: Generation of high flow rate Q
- Series channel combination
 - up-down channel in series → pressure addition
 - → Generation of high pressure P

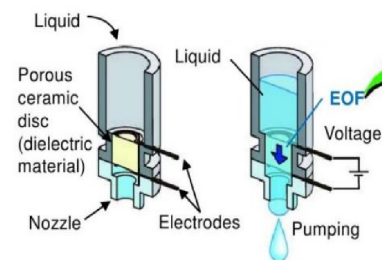


6.2 EO: dehydration

- Damp proofing: generate EO flow to drive water out of surfaces → drying.

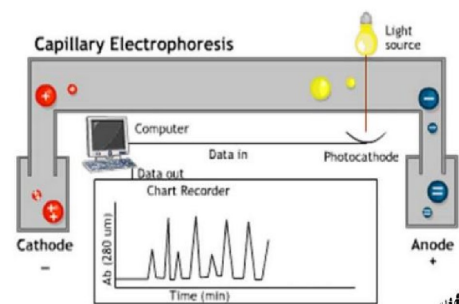
6.3 EO: Pumps

- Frit = sand in electrolyte liquid
- Used for driving EO circuits; cooling processors.
- Pump without moving parts → silent.



6.4 EP: Capillary

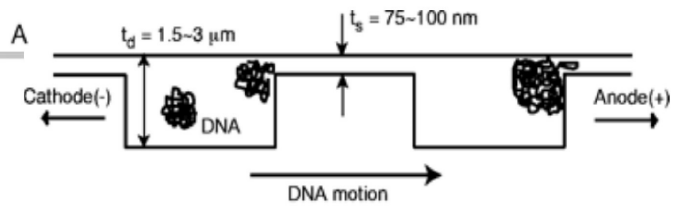
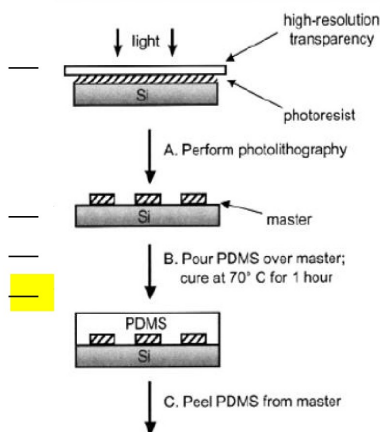
- Cathode and anode: Sample movement through gel → size separation
- Measurement: using light source
- Advantage: little dispersion (sharper peaks)



6.5 Sieving of samples in nanofluidic channels

- Separation by sample size

— DNA in liquid templated form:

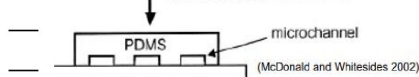


oping

portional to velocity, because they get stuck for shorter time
at a channel goes into channel and then pulls the rest of the

ent on size

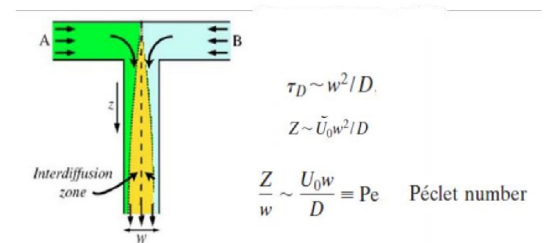
6.6 R



— Used for Filter:

- Separate particles using the different Pe-number
- Extraction of the smaller Pe-particles

— More contact area = better mixing



$$\tau_D \sim w^2 / D$$

$$Z \sim \tilde{U}_0 w^2 / D$$

$$\frac{Z}{w} \sim \frac{U_0 w}{D} \equiv \text{Pe} \quad \text{Péclet number}$$

6.7 Re: Components separators

- Separation based on size

- Big particles in centre
- Small particles outer region

- Centrifuge

- Cell free blood = plasma

- Zweifach-Fung effect

- Different flow in two channels [Diameter]
 - Big objects pulled into capillary with larger flow
 - Large diameter = large flow = large particles

- External fields

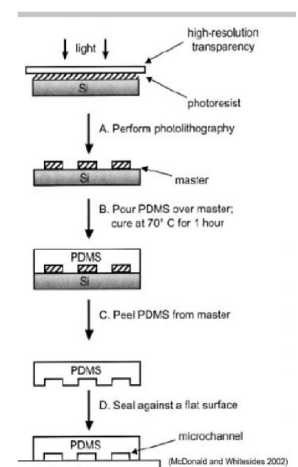
- Reaction of some particles to external force

6.8 Re: Components valves

- Only open into one direction (opening and closing; active and passive)

6.9 Re: Microfabrication method PDMS

- Make microstructure of choice



Tests: Physics of molecular and biological nanostructures

Test 03:

Scaling laws 1

How does the capacitance of a sphere-capacitor scale with the sphere radius R ?

☐ R^{-1} ☐ R^0 ☐ R^2 ☐ R^1

Scaling laws 2

The geometrical size of a spring-mass system is changed in each spatial dimension by a factor of 1.5.

- a) By which factor does its resonant frequency change? (for fractional answers give 2 decimal places)
- b) By which factor does the restoring force of the spring change when the mass is extended by a fixed distance d_e (for fractional

Free energy

During an irreversible process in a closed system, the free energy of that system

☐ increases ☐ decreases.

Test 04:

Boltzmann Factor 1

What is the thermal energy $k_B T$ at room temperature (25°C) in units of Joule? Tipp: The temperature T must be in units of Kelvin and not Celsius.

☐ 3.45×10^{-22} ☐ 1.38×10^{-23} ☐ 4.11×10^{-21} ☐ 00

Boltzmann Factor 2

Suppose a molecule can be in 2 states. The potential energy is 10 eV in state 1 and 12 eV in state 2. In thermal equilibrium, the probability that the molecule is in state 1 is p_1 and that it is in state 2 is p_2 . What is the probability the molecule is either in state 1 or in state 2?

☐ 0.37 ☐ 0 ☐ 0.5 ☐ 01

Boltzmann Factor 2 (follow-up question)

What is the ratio p_2/p_1 ?

Free energy

The free energy of a system equals its inner energy if (but not only if)

☐ $T = 0^\circ\text{C}$ ☐ the system is open ☐ the temperature is infinity ☐ the system is closed.

Boltzmann Averaging

Suppose a molecule can be in one of 2 different energy levels with potential energies 0 (level 1), $k_B T$ (level 2). In thermal equilibrium, what is the average inner energy of the molecule in units of $k_B T$?

Test 05:

Persistence length 1

Bring these polymers in the order of increasing persistence length:

☐ actin filament ☐ microtubule ☐ DNA

Persistence length 2

When the diameter of a linear polymer (using the WLC model) is increased by a factor of 2, its persistence length increases by a factor of

☐ 2 ☐ 8 ☐ 16 ☐ 4

Boltzmann Averaging reloaded

Suppose a molecule can be in one of 3 different energy levels with potential energies 0 (level 1), $k_B T$ (level 2), and $2 k_B T$ (level 3). In thermal equilibrium, what is the average inner energy of the molecule in units of $k_B T$?

Autocorrelation function

The autocorrelation of unit tangent vectors along a WLC polymer in thermal equilibrium

☐ increases exponentially with increasing distance between the vectors,

☐ is independent of the distance between the vectors.

☐ decreases exponentially with increasing distance between the vectors,

☐ decreases exponentially with decreasing distance between the vectors.

Test 06:

What is the complementary DNA sequence of GCCATT? Tipp: DNA sequences are read in the direction from the 5' to 3' ends, and the opposing strand is in the opposite direction.

- ☐ AATGGC ☐ CGGTAA ☐ TTACCG ☐ GCCATT

When the contour length of a WLC polymer is doubled, its RMS end-to-end distance increases by a factor of

- ☐ $\sqrt{2}$ ☐ 2 ☐ 4 ☐ $2\sqrt{2}$

When a WLC polymer is stretched between its ends by applying an external force,

☐ the force diverges (towards infinity) when the stretched length approaches the contour length of the polymer.

☐ the force becomes 0 when the stretched length approaches the contour length of the polymer.

☐ the force diverges (towards infinity) when the stretched length becomes half the contour length of the polymer.

☐ the force diverges (towards minus infinity) when the stretched length becomes half the contour length of the polymer.

What is the geometric mean of 100 and 200?

Test 08:

When the valency of a salt in aqueous solution is changed from 1 to 2 and all other parameters are kept constant, the Debye length changes by a factor of

- ☐ $\frac{1}{2}$ ☐ $\frac{1}{\sqrt{2}}$ ☐ $\frac{1}{\sqrt{2}}$ ☐ 2

The electrostatic potential in a weak aqueous electrolyte (at neutral pH) close to a glass surface is

- ☐ positive and depends on the temperature.
☐ negative and does not depend on the temperature.
☐ negative and depends on the temperature.
☐ positive and does not depend on the temperature.

The Debye-Hückel approximation of the Poisson Boltzmann equation

- ☐ gets rid of the non-linearity in the differential equation.
☐ leads to the definition of the Zeta potential.
☐ is only valid at low temperatures.
☐ is necessary to solve the Poisson-Boltzmann equation.

The Poisson equation

- ☐ depends on thermodynamic phenomena.
☐ predicts the presence of a Debye layer.
☐ allows to directly calculate the electric potential distribution in a system of diffusing ions, if their spatial positions are known.
☐ is valid only in electrolytes.

Test 09:

What is the electrical conductivity of a physiological salt solution (0.9% w/v NaCl) at room temperature (25°C)?

Consider an electrical circuit, consisting of an electrolyte between two electrodes (such as in the past lecture). Which of the following statements is correct?

- ☐ A change in voltage causes the Debye length to change
- ☐ A sudden change in voltage causes the current to drop to 0,
- ☐ A change in voltage causes the total charge in the Debye layer to change
- ☐ Increasing the conductivity of the electrolyte increases the RC time constant

A cylindrical channel is filled with an electrolyte and has walls at the zeta potential. How large is the radius of the cylinder (in multiples of the Debye length), if the potential at the center of the cylinder is half the zeta potential?

- ☐ ~ 2
- ☐ $\sim \frac{1}{2}$
- ☐ ~ 1
- ☐ Depends on the temperature.

What is $I_0(3)$, where I_0 is the modified Bessel function of order 0?

Test 10:

The EO velocity increases when the

- ☐ the Debye length is decreased.
- ☐ fluid's viscosity is decreased.
- ☐ electrical field is decreased.
- ☐ Zeta potential of the walls is decreased.

For the case of an arbitrary Debye length, where in a cylindrical channel is the EO flow velocity the highest?

- ☐ In the center.
- ☐ Depends on the temperature.
- ☐ Close to the walls but further away from the center.
- ☐ At the walls.

The Navier-Stokes equation describes

- ☐ something else.
- ☐ the motion of a fluid under the influence of forces.
- ☐ the motion of ions under the influence of electrical forces.
- ☐ the intermolecular forces between fluid atoms.

Which of the following is NOT an assumption for ideal electroosmotic flow?

- ☐ No inhomogeneities of the electrical field.
- ☐ No flow transients.
- ☐ A channel wall of only one material.
- ☐ A sufficiently high ionic strength.

Test 11:

Electroosmotic flow results from

- ☐ an electric force on the counter ions.
- ☐ the viscosity of the liquid.
- ☐ thermodynamic diffusion of the counter ions.
- ☐ the dielectric polarizability of the liquid.

Which of the following statements is incorrect?

- ☐ Back-pressure can arise from pumping against gravity
- ☐ Back-pressure increases the pumping rate.
- ☐ Back-pressure can revert the flow direction.
- ☐ Back-pressure causes a Hagen Poiseuille-type flow in the channel that is superposed with the EO-type flow.

Consider an ideal EO flow with back pressure in a cylindrical channel, where the total flow rate is 0. At which distance to the channel's center (in units of the channel radius) is the flow velocity zero?

- ☐ $\frac{1}{2}$ ☐ $\frac{1}{\sqrt{3}}$ ☐ $\frac{1}{\sqrt{2}}$ ☐ $\frac{1}{4}$

What is the electroosmotic velocity in $\mu\text{m/s}$ for an aqueous solution of 100 mM NaCl in a channel with a Zeta potential of 50 mV and an applied electric field of 1000 V/m? (calculate to 2 digits of precision; assume 1.00 mPa s for the viscosity and 81 for the dielectric polarizability of the solution)

Test 12:

Reynolds number:

Which Reynolds number approximate describes the system when you swim (at 3km/h)?

Streaming potential:

0 when moving an electrolyte through a channel whose walls are charged.

0 when charged particles are moved through an electrolyte.

0 when uncharged particles are moved through an electrolyte.

0 when moving an electrolyte through a channel whose walls are uncharged.

Electrohydrodynamic effects:

Assign the Terms to the correct definitions

1. Electric-field-driven motion of an electrolyte in a channel with charged walls
2. Electric-field-driven motion of a (surface-) charged particle in an electrolyte
3. Creation of an electric field by moving an electrolyte along a charged wall
4. Creation of an electric field by moving an (surface-) charged particles through an electrolyte

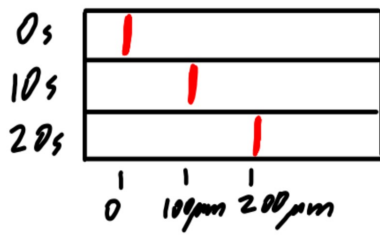
a. electrophoresis

b. electroosmosis

c. Streaming potential

d. Sedimentation potential

Sample exam question #1



A colored region of an aqueous electrolyte moves in a channel ($\Psi = 100 \text{ mV}$), driven by EO. How large is the applied el. field?

- a) $14 \frac{\text{V}}{\text{m}}$
- b) $140 \frac{\text{V}}{\text{m}}$
- c) $1,400 \frac{\text{V}}{\text{m}}$
- d) $14,000 \frac{\text{V}}{\text{m}}$

$$v_{EO} = \frac{\epsilon \epsilon_0 \Psi}{\eta} E ; E = \frac{\eta}{\epsilon \epsilon_0 \Psi} v_{EO} ; v_{EO} = \frac{100 \mu\text{m}}{10 \text{ s}} = 10^{-5} \frac{\text{m}}{\text{s}}$$

$$= 140 \frac{\text{V}}{\text{m}}$$

Sample exam question #2



Consider EO flow in a channel with a homogenous electrolyte and a " el. field. What effect can explain this observation (s. schematic)?

- a) turbulence in the liquid
- b) thermal gradient in the liquid
- c) different zeta potential of the walls ✓
- d) different Debye length at the "

Test 13:

The geometrical size of a spring-mass system is changed in each spatial dimension by a factor of 1.5 .

By which factor does the restoring force of the spring change when the mass is extended by a fixed distance d ? (for fractional answers give 1 decimal place)

Two freely rotatable dipoles (closed system, contact to a heat bath) in thermodynamic equilibrium are isothermally approached to each other. In doing so, mechanical work is carried out, e.g., which can be stored in an elastic spring (theoretically). The work carried out is the larger ...

☐ the larger the temperature.

☐ the smaller the temperature.

☐ the smaller the approach velocity.

☐ the larger the approach velocity.