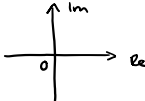


Aufgabe 6

a)

$$z = a + jb = |z| [\cos(\varphi) + j \sin(\varphi)] = |z| e^{j\varphi}$$

$$\bar{z} = a - jb$$

$$|z| = \sqrt{a^2 + b^2}$$


i) $z = -1 + j\sqrt{3}$

$$|z| = \sqrt{4} = 2$$

$$\varphi = \pi - \arctan(\sqrt{3}) = \pi - \frac{\pi}{3} = \frac{2}{3}\pi \rightarrow$$

ii) $z = \frac{-16 + j4}{3 - j5} \cdot \frac{3 + j5}{3 + j5}$

$$= \frac{-48 - j68 - 20}{9 + 25} = \frac{-68 - j68}{34}$$

$$= -2 - j2$$

$$|z| = \sqrt{8} = 2\sqrt{2}$$

$$\varphi = \pi + \arctan(1) = \pi + \frac{\pi}{4} = \frac{5}{4}\pi$$

iii) $F(j\Omega) = \frac{j\Omega}{1 + j\Omega}$

$$|F(j\Omega)| = \frac{\Omega}{\sqrt{1 + \Omega^2}}$$

$$\begin{array}{l} \Omega = 0 \\ \Omega = 1 \\ \Omega = \infty \end{array} \quad |F(j\Omega)| = \begin{array}{l} 0 \\ 1/\sqrt{2} \\ 1 \end{array}$$

$$\angle F(j\Omega) = \angle j\Omega - \angle (1 + j\Omega)$$

$$= \angle j\Omega - \arctan(\Omega)$$

$$\begin{array}{l} \Omega = 0 \\ \Omega = 1 \\ \Omega = \infty \end{array} \quad \angle F(j\Omega) = \begin{array}{l} 0 \\ \pi/2 - \pi/4 = \pi/4 \\ \pi/2 - \pi/2 = 0 \end{array}$$

iv) $F(j\Omega) = \frac{1}{1 + ja\Omega + (j\Omega)^2}$ $a > 0$

$$|F(j\Omega)| = \frac{1}{\sqrt{(1 - \Omega^2)^2 + (a\Omega)^2}}$$

$$= \frac{1}{\sqrt{1 - 2\Omega^2 + \Omega^4 + a^2\Omega^2}}$$

$$= \frac{1}{\sqrt{1 + \Omega^2(-2 + \Omega^2 + a^2)}}$$

$$\begin{array}{l} \Omega = 0 \\ \Omega = 1 \\ \Omega = \infty \end{array} \quad |F(j\Omega)| = \begin{array}{l} 1 \\ 1/a \\ 0 \end{array}$$

$$\angle F(j\Omega) = \angle 1 - \angle (1 - \Omega^2 + ja\Omega)$$

$$= 0 - \angle (1 - \Omega^2 + ja\Omega)$$

$$\begin{array}{l} \Omega = 0 \\ \Omega = 1 \\ \Omega = \infty \end{array} \quad \angle F(j\Omega) = \begin{array}{l} 0 \\ -\pi/2 \\ -\pi + \lim_{\Omega \rightarrow \infty} \arctan\left(\frac{a\Omega}{1 - \Omega^2}\right) \\ = -\pi + \arctan(0) = -\pi \end{array}$$

b) $j\omega = x$

$$F(x) = \frac{a}{1 + xb + \frac{1}{xc}} \cdot \frac{xc}{xc}$$

$$= \frac{xac}{xc + x^2bc + 1}$$

$$ac = \omega_0 k \quad a = k$$

$$\ell\omega_0^2 = 1$$

$$\ell = bc$$

$$\omega_0 = c = \frac{1}{\sqrt{bc}}$$

$$F\left(\frac{x}{\omega_0}\right) = \frac{k}{1 + \ell\left(\frac{x}{\omega_0} + \frac{\omega_0}{x}\right)} \cdot \frac{x\omega_0}{x\omega_0}$$

$$= \frac{x\omega_0 k}{x\omega_0 + \ell x^2 + \ell \cdot \omega_0^2}$$

Aufgabe 7 a

$$\underline{F}(x) = \frac{\underline{u}_2}{\underline{u}_1} \quad x = j\omega$$

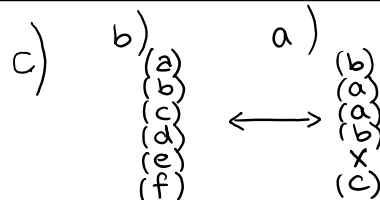
- a) (a) Tiefpass
(b) Hochpass
(c) Bandpass
(x) Bandstopp

$$b) (a) \frac{\underline{u}_2}{\underline{u}_1} = \frac{xL}{R + xL} \hat{=} \frac{x b}{1 + x b}$$

$$(b) \frac{\underline{u}_2}{\underline{u}_1} = \frac{1}{R + \frac{1}{xC}} = \frac{1}{RxC + 1} \hat{=} \frac{1}{1 + x a}$$

$$(c) \frac{\underline{u}_2}{\underline{u}_1} = \frac{R}{xL + R} \hat{=} \frac{1}{1 + x a}$$

$$(d) \frac{\underline{u}_2}{\underline{u}_1} = \frac{R}{\frac{1}{xC} + R} = \frac{xCR}{1 + xCR} \hat{=} \frac{x b}{1 + x b}$$



$$(e) \text{---} \parallel \text{---} : \frac{1}{xC} + R \quad \frac{1}{\frac{1}{a} + b} = \frac{1}{1 + ab}$$

$$\text{---} \parallel \text{---} : \frac{1}{xC + \frac{1}{R}}$$

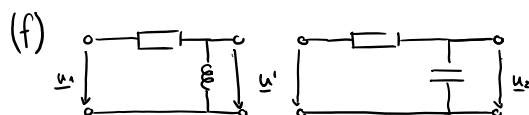
$$\frac{\underline{u}'_2}{\underline{u}_1} = \frac{1}{xC + \frac{1}{R_1}} \cdot \frac{1}{xC + \frac{1}{R_1} + \frac{1}{xC} + R_1} = \frac{1}{1 + \left(xC + \frac{1}{R_1}\right)\left(\frac{1}{xC} + R_1\right)} = \frac{1}{3 + xCR_1 + \frac{1}{xCR_1}}$$

$$= \frac{xCR_1}{3xCR_1 + (xCR_1)^2 + 1}$$

$$\frac{\underline{u}''_2}{\underline{u}_1} = \frac{R_2}{2R_2 + R_2} = \frac{1}{3}$$

$$\frac{\underline{u}_2}{\underline{u}_1} = \frac{\underline{u}'_2 - \underline{u}''_2}{\underline{u}_1} = \frac{1}{3xCR_1 + (xCR_1)^2 + 1} - \frac{1}{3} = \frac{3xCR_1 - 3xCR_1 - (xCR_1)^2 - 1}{3[3xCR_1 + (xCR_1)^2 + 1]}$$

$$= \frac{(xCR_1)^2 + 1}{(-3)[3xCR_1 + (xCR_1)^2 + 1]} \hat{=} f \cdot \frac{1 + (xg)^2}{1 + xh + (xg)^2}$$



$$\frac{\underline{u}_2}{\underline{u}_1} = \frac{1}{xCR + 1}$$

$$\frac{\underline{u}_2}{\underline{u}_1} = \frac{\underline{u}_2}{\underline{u}'} \cdot \frac{\underline{u}'}{\underline{u}_1} = \frac{1}{xCR + 1} \cdot \frac{xL}{xL + R} = \frac{xL}{x^2CRL + x(CR^2 + L) + R} \hat{=} \frac{x c}{1 + x d + (x e)^2}$$

d) niedrige Frequenz & $R=0$

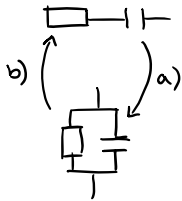
$(f) = (a)$ (a) ist die einfachere

oder:

$$\begin{aligned} (a) = (d) \quad \left[\begin{array}{l} L = R^2 C \\ \frac{L}{R} = CR \end{array} \right] \quad \left[\begin{array}{l} (b) = (c) \\ RC = \frac{L}{R} \end{array} \right] \quad \left[\begin{array}{l} C = L/R^2 \\ L = CR^2 \end{array} \right] \end{aligned}$$

(b); (d)
ist einfacher,
da Kondensator

e)



$$\omega = 2\pi f \quad X_C = \frac{1}{2\pi f C} \quad R = 1 \cdot 10^3 \Omega \quad C = 10 \cdot 10^{-12} F \quad f = 10 \cdot 10^6 1/s \quad [\Omega = \frac{V}{A}]$$

$$a) \quad |Z| = \sqrt{R^2 + X_C^2} = 1879,6 \Omega$$

$$Z = R + \frac{1}{j2\pi f C} \Big|_{-j}$$

$$= R + \frac{j}{2\pi f C} = 1 k\Omega - j \cdot 1,6 k\Omega$$

$$Z' = Z$$

$$\frac{1}{Z'} = R + j2\pi f C$$

$$\frac{1}{Z'} + j2\pi f C' = \frac{1 k\Omega + j1,6 k\Omega}{(1^2 + 1,6^2) k^2 \Omega}$$

$$= \frac{1 \Omega + j1,6 \Omega}{3,56 k\Omega}$$

$$R' = 3,56 k\Omega$$

$$C' = \frac{1,6 \Omega}{2\pi f \cdot 3,56 k\Omega} = 7,15 \cdot 10^{-12} F$$

$$b) \quad Z = \frac{R}{1 + j2\pi f CR} \Big|_{\frac{1 - j2\pi f CR}{1 - j2\pi f CR}}$$

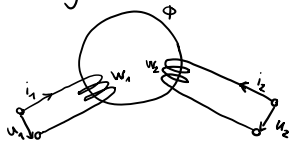
$$= \frac{R - j2\pi f CR^2}{1 + (2\pi f CR)^2} = \frac{1 k\Omega - j628 \Omega}{1,394} = 717 \Omega - j448,1 \Omega$$

$$Z' = R' + \frac{1}{j2\pi f C'} = 717 \Omega - j448,1 \Omega$$

$$R' = 717 \Omega$$

$$C' = \frac{1}{2\pi f \cdot 448,1 \Omega} = 35,5 \cdot 10^{-12} F$$

Aufgabe 8



a) $u_i(t) = u_i(i_1(t), i_2(t)) \quad i = 1, 2$

$$\begin{aligned} u_1 &= L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \\ u_2 &= M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \end{aligned}$$

$\underline{u}_i = \underline{u}_i(\underline{I}_1, \underline{I}_2) \quad i = 1, 2$

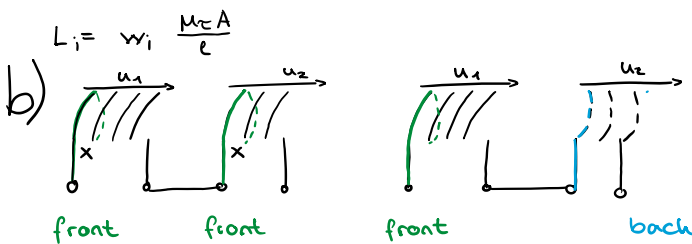
$\underline{u}_1 = j\omega(L_1 \underline{I}_1 + M \underline{I}_2)$

$\underline{u}_2 = j\omega(L_2 \underline{I}_2 + M \underline{I}_1)$

$L_1 = w_1^2 G$

$L_2 = w_2^2 G$

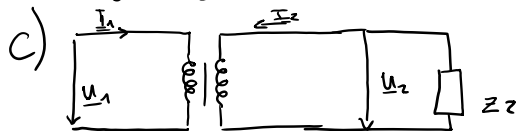
$M = \sqrt{L_1 L_2} = w_1 w_2 G$



$L_{ges} = L_1 + L_2 + 2M$

$L_{ges} = L_1 + L_2 - 2M$

da $u_{ges} = L_{ges} \frac{di}{dt} = \underline{u}_1(t) + \underline{u}_2(t)$

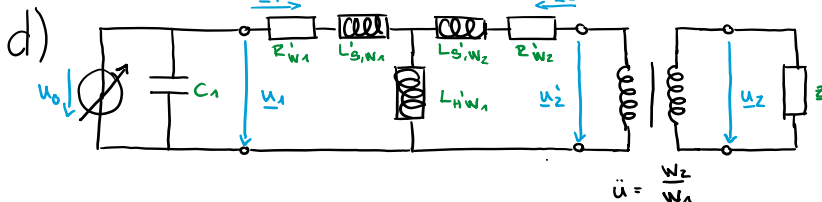


$\frac{u_1}{u_2} = \bar{u} = \frac{I_2}{I_1} \quad Z_1 = \frac{u_1}{I_1} = \bar{u}^2 \frac{u_2}{-I_2} = -\bar{u}^2 Z_2$

oder: aus Skript S. 18

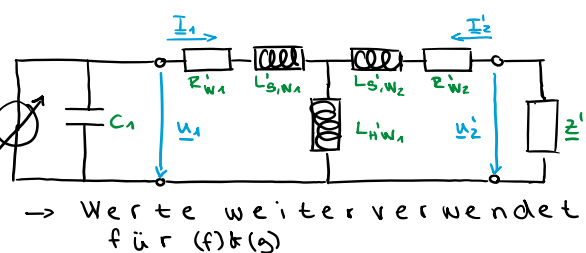
(2-60) mit $R_1 = R_2 = 0$

$Z_1 = \frac{u_1}{u_2} = j\omega L_1 + \frac{w^2 M^2}{j\omega L_2 + Z_2}$

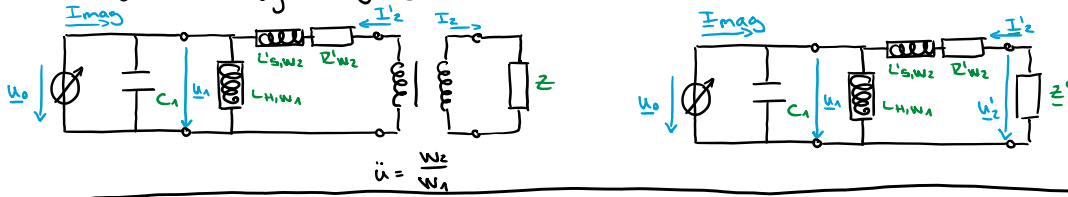


e)

$$\begin{aligned} R'_{w2} &= \frac{1}{\bar{u}^2} R_{w2} & L'_{h,w2} &= \frac{1}{\bar{u}^2} L_{h,w2} \\ L'_{s,w2} &= \frac{1}{\bar{u}^2} L_{s,w2} & Z' &= \frac{1}{\bar{u}^2} Z \\ L'_{h,w1} &= \frac{1}{\bar{u}^2} L_{h,w1} & \bar{u} &= \frac{L_{h,w2}}{L_{h,w1}} \\ L'_{h,w1} &= L_{h,w1} = L'_{h,w2} \\ L'_{s,w1} &= L_{s,w1} \end{aligned}$$



f) ges: $\underline{I}_{\text{mag}}$ geg: $R_1 = 0 \Omega$; $L_{S, W_1} = 0$



$$\ddot{u} = \frac{W_2}{W_1}$$

$$\underline{I}_{\text{mag}} = \underline{I}_0 = \frac{U_0}{\underline{Z}_{\text{mag}}}$$

$$\frac{1}{\underline{Z}_{\text{mag}}} = \frac{\ddot{u}^2}{\underline{Z} + R_{W_2} + j\omega L_{S, W_2}} + \frac{1}{j\omega L_{H, W_1}} + j\omega C$$

$$\underline{I}_{\text{mag}} = \left[\frac{\ddot{u}^2}{\underline{Z} + R_{W_2} + j\omega L_{S, W_2}} + \underbrace{\frac{1}{j\omega L_{H, W_1}} + j\omega C}_{= k} \right] U_0$$

$$\underline{Z}'' = \frac{1}{\ddot{u}^2} \underline{Z} \cdot \underline{Z}$$

$$\underline{I}'_{\text{mag}} = \frac{\ddot{u}^2 U_0}{\underline{Z} \underline{Z} + R_{W_2} + j\omega L_{S, W_2}} + k \cdot U_0$$

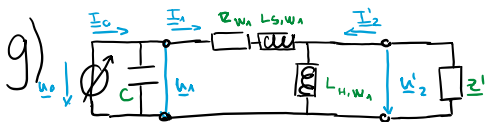
$$2\underline{Z} + R_{W_2} + j\omega L_{S, W_2} > \underline{Z} + R_{W_2} + j\omega L_{S, W_2}$$

$$\underline{I}'_{\text{mag}} < \underline{I}_{\text{mag}}$$

$$\underline{I}_{\text{mag}} = \underline{I}_1 + \underline{I}'_2$$

$$= \frac{U_1}{j\omega L_{H, W_1}} = U_1 \cdot j\omega C = \frac{U_0}{j\omega L_{H, W_1}} = U_0 \cdot j\omega C$$

u. a. von $\underline{Z}$, $2 \times \underline{Z}$ keine Auswirkung



$$\frac{U_2'}{U_0} = \frac{U_2'}{U_1} \cdot \frac{U_1}{U_0} = \frac{\underline{Z}' \cdot j\omega L_{H, W_1}}{j\omega L_{H, W_1} + \underline{Z}'} \cdot \frac{j\omega C}{R_{W_1} + j\omega L_{S, W_1} + \frac{\underline{Z}' \cdot j\omega L_{H, W_1}}{\underline{Z}' + j\omega L_{H, W_1}}}$$

$$= \frac{\underline{Z}' j\omega L_{H, W_1}}{(R_{W_1} + j\omega L_{S, W_1})(\underline{Z}' + j\omega L_{H, W_1}) + \underline{Z}' \cdot j\omega L_{H, W_1}}$$

$$\underline{U}_2' = \frac{\underline{Z} j\omega L_{H, W_1}}{(R_{W_1} + j\omega L_{S, W_1})(\underline{Z} + \ddot{u}^2 j\omega L_{H, W_1}) + \underline{Z} \cdot j\omega L_{H, W_1}} \cdot U_0$$

$$\underline{U}_2 = \ddot{u} \underline{U}_2' = \frac{\underline{Z} j\omega L_{H, W_1} \ddot{u}}{(R_{W_1} + j\omega L_{S, W_1})(\underline{Z} + \ddot{u}^2 j\omega L_{H, W_1}) + \underline{Z} j\omega L_{H, W_1}} \cdot U_0$$