

EXP $\sum_{k=0}^{\infty} \frac{(*)^k}{k!} = e^{(*)} \quad e^{(*)} \equiv \exp(*)$

speziell mit $*=1$ $\sum_{k=0}^{\infty} \frac{1}{k!} = e$

COS $\sum_{k=0}^{\infty} (-1)^k \cdot \frac{(*)^{2k}}{(2k)!} = \cos(*)$

SIN $\sum_{k=0}^{\infty} (-1)^k \cdot \frac{(*)^{2k+1}}{(2k+1)!} = \sin(*)$

GEO $\sum_{k=0}^{\infty} (**)^k = \frac{1}{1-(*')} \quad \text{falls } |*'| < 1 \quad (\text{die geometrische Reihe})$

HAR $\sum_{k=1}^{\infty} \frac{1}{k} = \infty \quad (\text{die harmonische Reihe divergiert})$

R 01 $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} (2\pi)^{2k+3} = \left[\sum_{k=0}^{\infty} (-1)^k \cdot \frac{(2\pi)^{2k}}{(2k)!} \right] \cdot (2\pi)^3 = \left[\cos(2\pi) \right] \cdot (2\pi)^3$ COS mit $*=2\pi$

R 02 $\sum_{k=0}^{\infty} \frac{5^k}{4^{2k}} = \left[\sum_{k=0}^{\infty} \left(\frac{5}{4^2}\right)^k \right] = \left[\frac{1}{1-\frac{5}{4^2}} \right] \quad \text{wobei } \left|\frac{5}{4^2}\right| < 1$ GEO mit $*= \frac{5}{4^2}$

R 03 $\sum_{n=0}^{\infty} \left(\frac{1}{3i}\right)^{n+1} = \left[\sum_{n=0}^{\infty} \left(\frac{1}{3i}\right)^n \right] \cdot \left(\frac{1}{3i}\right)^1 = \left[\frac{1}{1-\frac{1}{3i}} \right] \cdot \left(\frac{1}{3i}\right)^1 \quad \text{wobei } \left|\frac{1}{3i}\right| < 1$ GEO mit $*= \frac{1}{3i}$

R 04 $\sum_{n=0}^{\infty} \frac{2}{n!} = 2 \cdot \sum_{n=0}^{\infty} \frac{1}{n!} = 2 \cdot e$ $\sum_{n=0}^{\infty} \frac{1}{n!} = e$

R 05 $\sum_{k=0}^{\infty} \frac{1^k}{(-6)^k} = \left[\sum_{k=0}^{\infty} \left(-\frac{1}{6}\right)^k \right] = \left[\frac{1}{1-\left(-\frac{1}{6}\right)} \right] \quad \text{wobei } \left|-\frac{1}{6}\right| < 1$ GEO mit $*= -\frac{1}{6}$

R 06 $\sum_{k=0}^{\infty} \frac{(\ln(5))^k}{(k+1)!} \binom{k+1}{k} = \sum_{k=0}^{\infty} \frac{(\ln(5))^k}{(k+1)!} \cdot \frac{(k+1)!}{k! \cdot (k+1-k)!} = \left[\sum_{k=0}^{\infty} \frac{(\ln(5))^k}{k!} \right] = \left[e^{\ln(5)} \right]$ EXP mit $*= \ln(5)$

R 07 $\sum_{k=0}^{\infty} \frac{(-3)^{k+1}}{k!} = \left[\sum_{k=0}^{\infty} \frac{(-3)^k}{k!} \right] \cdot (-3)^1 = \left[e^{-3} \right] \cdot (-3)^1$ EXP mit $*= -3$

R 08 $\sum_{k=1}^{\infty} \frac{1}{5^k} = \left[\sum_{k=0}^{\infty} \left(\frac{1}{5}\right)^k \right] - \left(\frac{1}{5}\right)^0 = \left[\frac{1}{1-\frac{1}{5}} \right] - \left(\frac{1}{5}\right)^0 \quad \text{wobei } \left|\frac{1}{5}\right| < 1$ GEO mit $*= \frac{1}{5}$

R 10 $\sum_{k=0}^{\infty} \frac{(-1)^k \pi^{2k-1}}{(2k)!} = \left[\sum_{k=0}^{\infty} (-1)^k \cdot \frac{\pi^{2k}}{(2k)!} \right] \cdot \pi^{-1} = \left[\cos(\pi) \right] \cdot \pi^{-1}$ COS mit $*= \pi$

R 11 $\sum_{n=2}^{\infty} \frac{n+4}{2n^2} < \sum_{n=2}^{\infty} \frac{n+n}{2n^2} = \sum_{n=2}^{\infty} \frac{2n}{2n^2} = \sum_{n=2}^{\infty} \frac{1}{n} = \left[\sum_{n=1}^{\infty} \frac{1}{n} \right] - \frac{1}{1} = \left[\infty \right] - \frac{1}{1}$ HAR

R 11 $\sum_{n=2}^{\infty} \frac{n+4}{2n^2} > \sum_{n=2}^{\infty} \frac{n}{2n^2} = \sum_{n=2}^{\infty} \frac{1}{2n} = \frac{1}{2} \cdot \sum_{n=2}^{\infty} \frac{1}{n} = \frac{1}{2} \cdot \left(\left[\sum_{n=1}^{\infty} \frac{1}{n} \right] - \frac{1}{1} \right) = \frac{1}{2} \cdot \left(\left[\infty \right] - \frac{1}{1} \right)$ HAR

R 14 $\lim_{n \rightarrow +\infty} \sum_{k=0}^n \frac{2}{6k-5} \equiv \sum_{k=0}^{\infty} \frac{2}{6k-5} \quad \sum_{k=1}^{\infty} \frac{2}{6k-5} > \sum_{k=1}^{\infty} \frac{2}{6k} = \frac{2}{6} \cdot \left[\sum_{k=1}^{\infty} \frac{1}{k} \right] = \frac{1}{3} \cdot \left[\infty \right]$ HAR

R 15 $\lim_{n \rightarrow +\infty} \sum_{k=0}^n \left(\frac{5}{9}\right)^{k+1} \equiv \sum_{k=0}^{\infty} \left(\frac{5}{9}\right)^{k+1} = \left[\sum_{k=0}^{\infty} \left(\frac{5}{9}\right)^k \right] \cdot \left(\frac{5}{9}\right)^1 = \left[\frac{1}{1-\frac{5}{9}} \right] \cdot \left(\frac{5}{9}\right)^1 \quad \text{wobei } \left|\frac{5}{9}\right| < 1$ GEO mit $*= \frac{5}{9}$

R 16 $\left[\sum_{k=0}^{\infty} (4^{-1})^k \right] = \left[\frac{1}{1-4^{-1}} \right] \quad \text{wobei } |4^{-1}| < 1$ GEO mit $*= 4^{-1}$

$$\text{R 17} \quad \sum_{k=0}^{\infty} \frac{1}{(k+1)!} = \sum_{k=1}^{\infty} \frac{1}{k!} = \sum_{k=0}^{\infty} \frac{1}{k!} - \frac{1}{0!} = e - \frac{1}{0!}$$

$$\sum_{k=0}^{\infty} \frac{1}{k!} = e$$

$$\text{R 18} \quad -\sum_{k=0}^{\infty} \frac{(-1)^{k-1}}{4^k} = \left[\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \right] = \left[\frac{1}{1 - \left(-\frac{1}{4}\right)} \right] \quad \text{wobei} \quad \left| -\frac{1}{4} \right| < 1$$

$$\text{GEO mit } * = -\frac{1}{4}$$

$$\text{R 20} \quad \frac{7\pi}{2} \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \left(\frac{7\pi}{2}\right)^{2k} = \left[\sum_{k=0}^{\infty} (-1)^k \cdot \frac{\left(\frac{7\pi}{2}\right)^{2k+1}}{(2k+1)!} \right] = \left[\sin\left(\frac{7\pi}{2}\right) \right]$$

$$\text{SIN mit } * = \frac{7\pi}{2}$$

$$\text{R 21} \quad \sum_{k=2}^{\infty} \frac{2^k}{k!} = \left[\sum_{k=0}^{\infty} \frac{2^k}{k!} \right] - \frac{2^0}{0!} - \frac{2^1}{1!} = \left[e^2 \right] - \frac{2^0}{0!} - \frac{2^1}{1!}$$

$$\text{EXP mit } * = 2$$

$$\text{R 01} \quad 8\pi^3$$

$$\text{R 02} \quad \frac{16}{11}$$

$$\text{R 03} \quad -\frac{1}{10}(1 + 3i)$$

$$\text{R 04} \quad 2e$$

$$\text{R 05} \quad \frac{6}{7}$$

$$\text{R 06} \quad 5$$

$$\text{R 07} \quad -\frac{3}{e^3}$$

$$\text{R 08} \quad \frac{1}{4}$$

$$\text{R 09}$$

$$\text{R 10} \quad -\frac{1}{\pi}$$

$$\text{R 11} \quad \infty \quad (\text{divergent})$$

$$\text{R 12}$$

$$\text{R 13}$$

$$\text{R 14} \quad \infty \quad (\text{divergent})$$

$$\text{R 15} \quad \frac{5}{4}$$

$$\text{R 16} \quad \frac{4}{3}$$

$$\text{R 17} \quad e - 1$$

$$\text{R 18} \quad \frac{4}{5}$$

$$\text{R 19}$$

$$\text{R 20} \quad -1$$

$$\text{R 21} \quad e^2 - 3$$