

Aufgabe 7 Grenzwerte SS07

a) $\lim_{x \rightarrow 0} \left(\frac{1}{\sin(x)} + \frac{1}{x} \right) = 0 + 0$

b) $\sum_{n=1}^{\infty} \frac{1}{2^n} = \left(\frac{1}{2} \right)^1 \quad |q| < 1 \quad \sum_{n=0}^{\infty} q^n \rightarrow \frac{1}{1-q} \quad \left| \sum_{n=1}^{\infty} q^n \rightarrow \frac{1}{1-q} - 1 \right. \quad \left. \begin{array}{l} \text{für } n=0 \text{ wird Reihe} \\ q^0 = 1 \end{array} \right| = \frac{1}{1-\frac{1}{2}} - 1 = 2 - 1 = 1$

c) $\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x^2} = \frac{0}{0}$ Problem! (Hospital) $\frac{d}{dx}(\dots)$
 $\lim_{x \rightarrow 0} \frac{\sin(x)}{2x} = \frac{\cos(x)=1}{2} = \frac{1}{2}$

Aufgabe 3 Grenzwertberechnung SS08

a) $\sum_{k=0}^{\infty} \left(\frac{1}{3} \right)^k$ geometrische Reihe $|q| < 1$ gilt $\sum_{j=0}^{\infty} q^j = \frac{1}{1-q}$

$\frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$ konvergiert

b) $\lim_{x \rightarrow \infty} \frac{\sqrt{1+9x^4}}{(1+x)^2} = \frac{\sqrt{9x^4 \left(\frac{1}{9x^4} + 1 \right)}}{\left(x \left(\frac{1}{x} + 1 \right) \right)^2} = \frac{3x^2 \sqrt{\frac{1}{9x^4} + 1}}{x^2 \left(\frac{1}{x} + 1 \right)^2} = 3 \quad \left[p = \frac{1}{3} \right]$

c) $\lim_{x \rightarrow 0} e^{\frac{1}{x}} = \left[e^{\frac{1}{0}} \right] \rightarrow$ Funktion existiert nicht divergent

d) $\lim_{x \rightarrow 0} \frac{x^2 - x - 1}{e^x - x - 1} = \frac{0}{0} = \frac{2x}{e^x - 1} = \frac{2}{e^x} = \underline{\underline{2}}$

Aufgabe 5 Grenzwerte SS09

a) $\lim_{x \rightarrow 3} \frac{x-1}{x+2} = \frac{3-1}{3+2} = \frac{2}{5}$

b) $\lim_{x \rightarrow +\infty} \frac{1+x^2}{2-x+x^2} = \frac{x^2 \left(\frac{1}{x^2} + 1 \right)}{x^2 \left(\frac{2}{x^2} - \frac{1}{x} + 1 \right)} = -1$

c) $\lim_{x \rightarrow 0} \frac{1 - e^{3x}}{2x} = \frac{1-1}{0} = \frac{0}{0} = \frac{\frac{d}{dx}(1 - e^{3x})}{\frac{d}{dx}(2x)} = \frac{-e^{3x} \cdot 3}{2} = \frac{-3}{2}$

Aufgabe 9 Grenzwert SS09

a) $\lim_{n \rightarrow \infty} \frac{n}{\sqrt[3]{n^2 + n}} = \frac{n}{(n^2 + n)^{\frac{1}{3}}} = \underline{\underline{\text{divergent}}}$

b) $\sum_{k=1}^{\infty} \frac{-1}{(k-1)!}$ Formelsammlung $\sum_{k=1}^{\infty} \frac{-k}{k!} = \underline{\underline{-e}}$

c) $\sum_{k=0}^{\infty} \left(\frac{3}{4} \right)^{k+2} = \sum_{k=0}^{\infty} \left(\frac{3}{4} \right)^k \cdot \left(\frac{3}{4} \right)^2 = \sum_{k=0}^{\infty} \left(\frac{3}{4} \right)^k \cdot \left(\frac{9}{16} \right) = \left(\frac{9}{16} \right) \cdot \sum_{k=0}^{\infty} \left(\frac{3}{4} \right)^k = \left(\frac{9}{16} \right) \cdot \frac{1}{1-\frac{3}{4}} = \left(\frac{9}{16} \right) \cdot \frac{4}{1} = \underline{\underline{\frac{9}{4}}}$

Aufgabe 2 Grenzwerte SS10

$$\lim_{n \rightarrow +\infty} \sum_{k=0}^n \frac{2}{4k-3} = \text{divergent} \quad \text{--- harmonische Reihe}$$

$$\lim_{n \rightarrow +\infty} \frac{2n + (-1)^n}{3n} = \frac{1(2 + \frac{(-1)^n}{n})}{3} = \frac{2}{3}$$

$$\lim_{x \rightarrow 1} \frac{\ln(x)}{1-x^5} = \frac{\ln(1)}{1-1} = \frac{0}{0} = \frac{\frac{d}{dx}(\ln(x))}{\frac{d}{dx}(1-x^5)} = \frac{\frac{1}{x}}{-5x^4} = \lim_{x \rightarrow 1} \frac{1}{-5x^5} = -\frac{1}{5}$$

$$\lim_{n \rightarrow +\infty} \sum_{k=3}^n \frac{3^k}{(k-3)!} \Rightarrow \sum_{n=0}^{\infty} \frac{2^n}{n!} = e^2$$

$$\sum_{k=0}^n \frac{3^{k+3}}{k!} = \frac{3^k \cdot 3^3}{k!} = \underline{\underline{27e^3}}$$

Aufgabe 5 Grenzwerte SS11

$$f: \mathbb{R} \setminus \{-3, 1, 3\} \rightarrow \mathbb{R}: x \mapsto \frac{2x^3 - 4x^2 - 2x + 4}{3(x^2 - 9)(x-1)}$$

$$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{2x^3 - 4x^2 - 2x + 4}{3(x^2 - 9)(x-1)} = \lim_{x \rightarrow +\infty} \frac{x^3(2 - \frac{4}{x} - \frac{2}{x^2} + \frac{4}{x^3})}{3x^3(1 - \frac{9}{x^2})(1 - \frac{1}{x})} = \frac{2}{3}$$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{2x^3 - 4x^2 - 2x + 4}{3(x^2 - 9)(x-1)} = \frac{2 \cdot 0^3 - 4 \cdot 0^2 - 2 \cdot 0 + 4}{3(0^2 - 9)(0-1)} = \frac{4}{3 \cdot 9} = \frac{4}{27}$$

$$\lim_{x \rightarrow -3+0} f(x) = \lim_{x \rightarrow -3+0} \frac{2x^3 - 4x^2 - 2x + 4}{3(x^2 - 9)(x-1)} = \lim_{x \rightarrow -3+0} \frac{2(-3+\frac{1}{n})^3 - 4(-3+\frac{1}{n})^2 + 2(-3+\frac{1}{n}) + 4}{3((-3+\frac{1}{n})^2 - 9)(-3+\frac{1}{n}-1)}$$

$$\lim_{x \rightarrow 1} f(x) = \frac{2x^3 - 4x^2 - 2x + 4}{3(x^2 - 9)(x-1)} = \frac{2-4-2-4}{3(8)(0)} = \frac{0}{0} \stackrel{\text{L'H}}{=} \frac{\frac{d}{dx}(2x^3 - 4x^2 - 2x + 4)}{\frac{d}{dx}(3(x^2 - 9)(x-1))} = \lim_{x \rightarrow 1} \frac{6x - 8x - 2}{3(3x^2 - 2x - 9)} = \frac{6-8-2}{3(3-2-9)} = \frac{-4}{-24} = \frac{1}{6}$$

Aufgabe 3 Grenzwerte / Ableitung SS12

$$f: \mathbb{R} \rightarrow \mathbb{R}: x \mapsto 2 \cosh(\sqrt{2}x) - 2$$

$$g: \mathbb{R} \rightarrow \mathbb{R}: x \mapsto 1 - \cos(x)$$

$$\cosh(0) = 1 \\ \sinh(0) = 0$$

$$\textcircled{1} \lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \lim_{x \rightarrow 0} \frac{2 \cosh(\sqrt{2}x) - 2}{1 - \cos(x)} = \frac{0}{0} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{2 \cdot \sqrt{2} \sinh(\sqrt{2}x)}{\sin(x)} = \frac{0}{0} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{2 \cdot 2 \cosh(\sqrt{2}x)}{\cos(x)} = \frac{4}{1}$$

$$\textcircled{2} \lim_{x \rightarrow \infty} \sqrt{2} e^{f(x)} = \lim_{x \rightarrow \infty} \sqrt{2} e^{(2 \cosh(\sqrt{2}x) - 2)} = \sqrt{2} e^{(2-2)} = \underline{\underline{\sqrt{2}}}$$

$$\textcircled{3} \frac{d}{dx}(f(g(x))) = \frac{d}{dx} f \circ g = \frac{d}{dx} (2 \cosh(\sqrt{2}(1 - \cos(x))) - 2) = 2 \sinh(\sqrt{2} - \sqrt{2} \cos(x)) \cdot \sqrt{2} \sin(x) - 2$$

Aufgabe 3 Grenzwerte SS14

$$\sum_{k=0}^{\infty} \frac{1}{(-4)^k} = \left(\frac{1}{-4}\right)^k \Rightarrow \text{geo. Reihe } |q| < 1, \text{ dann gilt } \frac{1}{1-q} \Rightarrow \frac{1}{1 - (-\frac{1}{4})} = \frac{1}{\frac{5}{4}} = \frac{4}{5}$$

$$\lim_{x \rightarrow +\infty} \frac{7x^3 + x}{x^4 + 2} = \lim_{x \rightarrow +\infty} \frac{x^4(\frac{7}{x} + \frac{1}{x^4})}{x^4(1 + \frac{2}{x^4})} \left\{ \text{geht gegen } \frac{0}{1} = 0 \right\}$$

$$\lim_{x \rightarrow 2} \frac{\sin(x^2 - 2x)}{x^2 - x - 2} = \frac{\sin(4-4)}{4-2-2} = \frac{0}{0} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 2} \frac{\cos(x^2 - 2x)(2x-2)}{2x-1} = \frac{\cos(0)(-2)}{4-1} = -\frac{2}{3}$$

$$\lim_{x \rightarrow +\infty} \frac{(4x+3)^2}{\sqrt{1+4x^4}} = \frac{(x(4+\frac{3}{x}))^2}{\sqrt{x^4(\frac{1}{x^4}+4)}} = \frac{x^2(4+\frac{3}{x})^2}{x^2 \sqrt{\frac{1}{x^4}+4}} = \frac{4^2}{\sqrt{4}} = 8$$

Aufgabe 2 Grenzwerte SS15

$$\lim_{x \rightarrow +\infty} \frac{1+3x-2x^2}{4-\sqrt{2}x+9x^2} = \frac{x^2 \left(\frac{1}{x^2} + \frac{3}{x} - 2 \right)}{x^2 \left(\frac{4}{x^2} - \frac{\sqrt{2}}{x} + 9 \right)} = \frac{-2}{9}$$

$$\sum_{k=0}^{\infty} \frac{(k+1)(k+2)}{(k+2)!} = \frac{(k+1)(\cancel{k+2})}{(\cancel{k+2})(k+1)!} = \frac{1}{k!} = e$$

$$\lim_{x \rightarrow 3} \frac{x^3-27}{\ln(x-2)} = \frac{0}{0} = 3x^2 : \frac{1}{(x-2)} = 27$$

Aufgabe 7 Grenzwerte SS18

$$\lim_{x \rightarrow +\infty} \frac{3x^3-5x^2+2x}{7x^2(3x-2)} = \frac{x^3 \left(3 - \frac{5}{x} + \frac{2}{x^2} \right)}{7x^3 \left(3 - \frac{2}{x} \right)} = \frac{3}{21} = \frac{1}{7}$$

$$\sum_{k=3}^{\infty} \frac{\ln(k) - \ln(k-1)}{4 \ln(k) \ln(k-1)} = \sum_{k=3}^{\infty} \frac{\ln(k)}{4 \ln(k) \ln(k-1)} - \frac{\ln(k-1)}{4 \ln(k) \ln(k-1)} = \sum_{k=3}^{\infty} \frac{1}{4 \ln(k-1)} - \frac{1}{4 \ln(k)} = \frac{1}{4 \ln(2)} - 0$$

Teleskopsumme nur 1. und letztes Glied kürzt sich nicht

$$\lim_{x \rightarrow 3} \frac{4 \cos\left(\frac{x\pi}{9}\right) - 2}{x^2 - 9} = \frac{4 \cos\left(\frac{3\pi}{9}\right) - 2}{9 - 9} = \frac{0}{0} = \lim_{x \rightarrow 3} \frac{-4 \sin\left(\frac{x\pi}{9}\right) \cdot \frac{\pi}{9}}{2x} = \frac{-4 \sin\left(\frac{1}{3}\pi\right) \cdot \frac{\pi}{9}}{6} = \frac{-2\sqrt{3} \cdot \frac{\pi}{9}}{6} = -\sqrt{3} \cdot \frac{\pi}{9} \cdot \frac{1}{6} = -\frac{\sqrt{3}\pi}{27}$$