

Aufgabe 2 Integralrechnung SS07

$$a) \int \underbrace{2x}_u \underbrace{\sqrt{x-1}}_{v'} dx = \left[\underbrace{2x}_{u \cdot v} \cdot \underbrace{\left(\frac{2}{3}(x-1)^{3/2}\right)}_{\frac{4}{3}(x-1)^{3/2}} \right] - \int \underbrace{2}_{u'} \cdot \underbrace{\left(\frac{2}{3}(x-1)^{3/2}\right)}_v dx$$

$$= \left[\frac{4x}{3}(x-1)^{3/2} \right] - \left[\frac{2 \cdot 4}{3 \cdot 5/2} (x-1)^{5/2} \right] = \left[\frac{4x}{3}(x-1)^{3/2} - \frac{8}{15}(x-1)^{5/2} \right]$$

$$b) \int \underbrace{\cos(3x)}_{\frac{1}{3}} \underbrace{(\sin(3x))^2}_{t} dx = \int \cos(3x) \cdot t^2 \cdot \frac{dt}{3\cos(3x)} = \int \frac{t^2}{3} dt = \left[\frac{1}{3} + \frac{1}{3} \right]$$

resub: $\frac{1}{9}(\sin(3x))^3$

$$c) \int_0^{\pi/2} \cos(3x)(\sin(3x))^2 dx$$

$$\left[\frac{1}{9}(\sin(3x))^3 \right]_0^{\pi/2} = \frac{1}{9}(\sin(3 \cdot \frac{\pi}{2}))^3 - \frac{1}{9}(0) = -\frac{1}{9}$$

Aufgabe 7 Stammfunktionen SS08

$$a) \int \frac{-2\sin(x)\cos(x)}{5 + (\cos(x))^2} dx$$

Sub: $\begin{cases} \cos(x) = z \\ -\sin(x) = z' \end{cases}$

$$\int \frac{-2\sin(x)z}{5 + (z)^2} \cdot \frac{dz}{-\sin(x)} = \int \frac{2z}{5 + (z)^2} dz$$

Sub: $\begin{cases} 5 + (z)^2 = t \\ 2z = t' \end{cases}$

$$\int \frac{t'}{t} \cdot \frac{dt}{2z} = \int \frac{1}{t} dt = [\ln(t)] = [\ln(5 + z^2)] = [\ln(5 + \cos(x)^2)]$$

$$b) \int \frac{-\sqrt{5}\sin(x)}{5 + (\cos(x))^2} dx$$

Sub: $\begin{cases} z = \cos(x) \\ z' = -\sin(x) \end{cases}$

$$\int \frac{-\sqrt{5}\sin(x)}{5 + (z)^2} \cdot \frac{dz}{-\sin(x)} = \int \frac{\sqrt{5}}{5 + (z)^2} dz$$

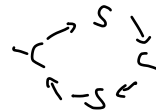
$$\sqrt{5} \int \frac{1}{5 + (z)^2} dz$$

Es gilt bei $\int \frac{1}{z^2 + a^2} = \left[\frac{1}{a} \cdot \arctan\left(\frac{z}{a}\right) \right] \rightarrow \sqrt{5} \int \frac{1}{z^2 + \sqrt{5}^2} = \left[\sqrt{5} \cdot \frac{1}{\sqrt{5}} \cdot \arctan\left(\frac{z}{\sqrt{5}}\right) \right]$

Aufgabe 6 Integrale SS09

Partielle Integration: $\int_a^b f'(x)g(x)dx = [f(x) \cdot g(x)]_a^b - \int_a^b f(x) \cdot g'(x)dx$

$$\begin{aligned} a) \int (2x-1) \sin(x+3) dx &= \left[\underbrace{(2x-1)}_{g(x)} \underbrace{(-\cos(x+3))}_{f'(x)} \right] - \int \underbrace{2}_{g'(x)} \cdot \underbrace{(-\cos(x+3))}_{f(x)} dx \\ &= \left[(2x-1)(-\cos(x+3)) \right] - [2(-\sin(x+3))] = \left[(2x-1)(-\cos(x+3)) + 2\sin(x+3) \right] \end{aligned}$$



$$\begin{aligned} b) \int \sinh(\sin(x)) \cos(x) dx & \quad \begin{array}{l} f(x) = \sin(x) = z \\ f'(x) = \cos(x) = z' \end{array} \quad dx = \frac{dz}{z'} \\ \int \sinh(z) \cos(x) \frac{dz}{\cos(x)} &= \int \sinh(z) dz = [\cosh(z)] = [\cosh(\sin(x))] \end{aligned}$$

$$c) \int z^{x-1} dx = \left[\frac{z^{x-1}}{\ln(z)} \right]$$

Aufgabe 7 Integrale SS09

$$a) \int 2xe^{-x^2} dx \stackrel{\text{sub}}{=} \int 2xe^z \frac{dz}{-2x} = \int -e^z dz = [-e^z] = [-e^{-x^2}] \quad \begin{array}{l} \text{result} \\ f(x) = -x^2 = z \\ f'(x) = -2x = z' \end{array} \quad dx = \frac{dz}{z'}$$

$$b) \int_{-\infty}^{\infty} 2xe^{-x^2} dx = [-e^{-x^2}]_{-\infty}^{\infty} = \left[\frac{1}{-e^{x^2}} \right]_{-\infty}^{\infty} = \frac{1}{\infty} - \frac{1}{-\infty} = 0 + \frac{1}{e}$$

Aufgabe 5 Integrale SS10

$$a) \int xe^{\frac{x}{2}} dx = \left[\underbrace{2xe^{\frac{x}{2}}}_{g(x)f'(x)} \right] - \int \underbrace{2e^{\frac{x}{2}}}_{g'(x)f(x)} = \left[2xe^{\frac{x}{2}} - 4e^{\frac{x}{2}} \right] = \left[e^{\frac{x}{2}}(2x-4) \right]$$

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$$\begin{aligned} b) \int (1+2(\cos x)^3) dx &= \int 1 dx + 2 \int \cos^3(x) dx = [x] + 2 \int (1-\sin^2(x)) \cos(x) dx \\ &= [x] + \int (2 - 2\sin^2(x)) \cos(x) dx \\ &= [x] + \int 2\cos(x) dx - \int 2\sin^2(x) \cos(x) dx \\ &= [x + 2\sin(x)] - \int 2z^2 \cos(x) \frac{dz}{\cos(x)} \\ &= [x + 2\sin(x) - \frac{2}{3}(\sin(x))^3] \end{aligned}$$

$$c) \int \frac{x-3}{x^2-3x+2} dx \quad x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{3 \pm \sqrt{9-4 \cdot 2}}{2} = \frac{3 \pm 1}{2} \quad \begin{array}{l} x_1 = 2 \\ x_2 = 1 \end{array}$$

$$\frac{x-3}{x^2-3x+2} = \frac{A}{(x-2)} + \frac{B}{(x-1)} = \frac{A(x-1)}{(x-2)(x-1)} + \frac{B(x-2)}{(x-2)(x-1)} \quad \text{Gleichsetzen} \quad \begin{array}{l} A(x-1) + B(x-2) = x-3 \\ \text{Für } x=1 \\ \text{Für } x=2 \end{array}$$

$$\int \frac{-1}{(x-2)} + \frac{2}{(x-1)} = [-\ln(x-2) + 2(\ln(x-1))]]$$

$$\begin{array}{l} -B = -2 \\ B = 2 \end{array} \quad \begin{array}{l} A = -1 \end{array}$$

Aufgabe 7 Integral SS11

Partielle Integration

$$\int \underbrace{x}_{f'} \cdot \underbrace{e^{-2x}}_{g'} dx = \left[\underbrace{x}_{f'} \cdot \underbrace{\left(-\frac{1}{2}e^{-2x}\right)}_{g'} \right] - \int \underbrace{-\frac{1}{2}e^{-2x}}_{f'} \cdot \underbrace{1}_{g'} dx = \left[x \cdot \left(-\frac{1}{2}e^{-2x}\right) \right] + \left[\frac{1}{4}e^{-2x} \right]$$

$$\left[-\frac{1}{2}x e^{-2x} + \frac{1}{4}e^{-2x} \right] = \left[e^{-2x} \left(-\frac{1}{2}x + \frac{1}{4} \right) \right]$$

$$\int_0^{+\infty} x e^{-2x} dx = \left[-\frac{1}{2}x e^{-2x} + \frac{1}{4}e^{-2x} \right]_0^{+\infty} = \left[\cancel{\frac{1}{2}x} \left(-\frac{1}{2}x + \frac{1}{4} \right) \right] - \frac{1}{4}$$

$$\int 3x^2 e^x \cos(x^3 e^x) + x^3 e^x \cos(x^3 e^x) dx = \int \cos(x^3 e^x) \left(\underline{3x^2 e^x + x^3 e^x} \right) dx$$

$$= \int \cos(z) \cdot \frac{dz}{z'} \cdot \frac{dz}{z'} = \int \cos(z) dz = \left[\sin(x^3 e^x) \right] \quad \begin{matrix} z = x^3 e^x \\ z' = 3x^2 e^x + x^3 e^x \end{matrix}$$

Aufgabe 5 Integral SS12

$$\int_{-\infty}^{-2} \frac{1}{x^4} dx = \int_{-\infty}^{-2} x^{-4} dx = \left[-\frac{1}{3}x^{-3} \right]_{-\infty}^{-2}$$

$$\int \underbrace{2x}_{f(x)} \exp(\underbrace{3x}_{g(x)}) = \left[\frac{2}{3}x \exp(3x) \right] - \int \frac{2}{3} \exp(3x) dx$$

$$= \left[\frac{2}{3}x \exp(3x) - \frac{2}{3} \cdot \frac{1}{3} \exp(3x) \right] = \left[\frac{2}{3} \exp(3x) \left(x - \frac{1}{3} \right) \right]$$

$dx = \frac{dz}{3}$

$$\int \underbrace{6x}_{z'} \sin(\underbrace{3x^2}_{z}) dx = \int \cancel{6x} \sin(z) \frac{dz}{\cancel{6x}} = \int \sin(z) dz = \left[-\cos(3x^2) \right]$$

$$\int_1^7 \frac{1}{x} = \left[\frac{1}{\ln(x)} \right]_1^7 = \left[\frac{1}{\ln(7)} - \frac{1}{\ln(1)} \right] = \frac{1}{\ln(7)} = \left[\frac{1}{\ln(7)} \right]$$

Aufgabe 4 Integral SS13

$$\int \cosh(x)^2 dx = \left[\frac{1}{2}(x + \sinh(x)\cosh(x)) \right]$$

$$\int (\cosh(x)^2 + 2) dx = \int \cosh^2(x) + \int 2 = \left[\frac{1}{2}(x + \sinh(x)\cosh(x)) + 2x \right] = \left[\frac{5}{2}x + \sinh(x)\cosh(x) \right]$$

$$\int \frac{\sin(x)\cos(x)}{\sqrt{2 + (\sin(x))^2}} dx = \int \frac{\sin(x)\cos(x)}{\sqrt{z}} \cdot \frac{dz}{2 \cdot \sin(x)\cos(x)} = \int \frac{1}{2\sqrt{z}} dz = \int \frac{z^{-\frac{1}{2}}}{2} = \frac{z^{\frac{1}{2}}}{2 \cdot \frac{1}{2}} = \left[\sqrt{z} \right] = \left[\sqrt{2 + (\sin(x))^2} \right]$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin(x)\cos(x)}{\sqrt{2 + (\sin(x))^2}} dx = \left[\sqrt{2 + (\sin(x))^2} \right]_0^{\frac{\pi}{2}} = \sqrt{2 + \sin^2\left(\frac{\pi}{2}\right)} - \sqrt{2} = \sqrt{3} - \sqrt{2}$$

Aufgabe 7 Integralrechnung SS14

$$\int \frac{1}{4\sqrt{x^3}} dx = \int x^{-\frac{3}{2}} = \left[x^{-\frac{3}{2}+1} \cdot \frac{1}{-\frac{3}{2}+1} \right] = -\frac{1}{2}x^{-\frac{1}{2}}$$

$$\int \underbrace{(x-2)}_{f(x)} \underbrace{e^{-x}}_{g(x)} dx = \left[\underbrace{(x-2)}_{f(x)} \underbrace{e^{-x}}_{g(x)} \right] - \int \underbrace{-e^{-x}}_{f'(x)g(x)} = \left[(x-2)e^{-x} - (-e^{-x}) \right] = \left[e^{-x}(-x+2+1) \right]$$

$$\int \frac{1}{x^2 + 3x + 2} dx = \frac{-3 \pm \sqrt{9-4 \cdot 2}}{2} = \frac{-3 \pm 1}{2} \quad \begin{matrix} x_1 = -1 \\ x_2 = -2 \end{matrix}$$

$$\frac{1}{x^2 + 3x + 2} dx = \frac{A}{(x+1)} + \frac{B}{(x+2)} = \int \frac{1}{(x+1)} - \frac{1}{(x+2)} = \left[\ln|x+1| - \ln|x+2| \right] = \left[\ln \left| \frac{x+1}{x+2} \right| \right]$$

$$I: 1 = A(x+2) + B(x+1)$$

$$A = 1 \quad B = -1$$

Kostenlos heruntergeladen von



Aufgabe 5 Integralrechnung SS15

$$\int e^x \sin(2e^x) dx = \int \cancel{e^x} \sin(z) \frac{dz}{\cancel{2e^x}} = \int \sin(z) \cdot \frac{1}{2} dz = \left[-\frac{1}{2} \cos(2e^x) \right]$$

$$\int_0^{\ln(\pi)} e^x \sin(2e^x) dx = \left[-\frac{1}{2} \cos(2e^x) \right]_0^{\ln(\pi)} = -\frac{1}{2} \cos(2\pi) - \left(-\frac{1}{2} \cos(2) \right) = -\frac{1}{2} + \frac{1}{2} \cos(2)$$

$$\int x^4 \ln(x) dx = \left[\frac{1}{5} x^5 \ln(x) \right] - \int \frac{1}{5} x^5 \cdot \frac{1}{x} dx \stackrel{\text{gcm} \cdot \text{fcm}}{=} \int \frac{1}{5} x^4 dx = \left[\frac{1}{5} x^5 \ln(x) - \frac{1}{25} x^5 \right]$$

$$\int_0^1 x^4 \ln(x) dx = \left[\frac{1}{5} x^5 \ln(x) - \frac{1}{25} x^5 \right]_0^1 = \frac{1}{5} \ln(1) - \frac{1}{25} - (0) = -\frac{1}{25}$$

Aufgabe 6 Integrale SS16 Sub

$$\int \frac{\sin(\ln(x^2))}{3x} dx = \boxed{\begin{matrix} z = \ln(x^2) \\ z' = \frac{2}{x} \end{matrix}} = \int \frac{\sin(z)}{3x} \frac{dz}{\frac{2}{x}} = \int \frac{\sin(z)}{3} \frac{x}{2} dz = \frac{1}{6} \int \sin(z) dz = \left[-\frac{1}{6} \cos(\ln(x^2)) \right]$$

$$\int \frac{5x^4 + 15x^2 + 4x}{x^5 + 5x^3 + 2x^2 + 1} dx = \frac{f'}{f} = [\ln|f|] = [\ln|x^5 + 5x^3 + 2x^2 + 1|]$$

Aufgabe 5 Integrale SS17

$$\int \frac{x \cos(2x^2)}{\sin(2x^2)} dx = \begin{matrix} z = \sin(2x^2) \\ z' = 4x \cos(2x^2) \end{matrix} = \frac{1}{4} \int \frac{f'}{f} = \left[\frac{1}{4} \ln|f| \right] = \left[\frac{1}{4} \ln|\sin(2x^2)| \right]$$

$$\begin{aligned} \int \underbrace{\cos(x)}_{f(x)} \underbrace{\cosh(x)}_{g'(x)} dx &= \left[\underbrace{\cos(x) \sinh(x)}_{f(x) g(x)} \right] - \int \underbrace{-\sin(x)}_{f'(x)} \underbrace{\sinh(x)}_{g(x)} dx = \left[\cos(x) \sinh(x) \right] - \left(\left[-\sin(x) \cosh(x) \right] - \int -\cos(x) \cosh(x) dx \right) \\ &= \left[\cos(x) \sinh(x) + \sin(x) \cdot \cosh(x) \right] - \int \cos(x) \cosh(x) dx = \int \cos(x) \cosh(x) dx \\ \left[\cos(x) \sinh(x) + \sin(x) \cdot \cosh(x) \right] &= 2 \int \cos(x) \cosh(x) dx \\ \frac{1}{2} \left[\cos(x) \sinh(x) + \sin(x) \cdot \cosh(x) \right] &= \int \cos(x) \cosh(x) dx \end{aligned}$$

$$\int_1^2 \frac{x}{x-3} dx = \left[5 \ln|x-3| \right]_1^2 = 5 \ln|2-3| - 5 \ln|1-3| = -5 \ln(2)$$

Aufgabe 3 Integral SS 18

$$\int_1^2 2x \, dx = [x^2]_1^2 = 4 - 1 = 3$$

$$\int \frac{-3}{(x+3)^2+1} \, dx = -3 \int \frac{1}{(x+3)^2+1} = [-3 \arctan(x+3)]$$

$\uparrow$ sub = u

$$\int e^{4x} \cosh(e^{2x}) \, dx \stackrel{\text{Sub}}{=} \int e^{\frac{2x}{1}} \cosh(u) \frac{du}{2e^{2x}} = \frac{1}{2} \int u \cosh(u) \, du \stackrel{u=e^{2x}}{=} \frac{1}{2} [u \sinh(u)] - \frac{1}{2} \int \cosh(u) \, du \stackrel{\text{mit } u=e^{2x}}{=} \frac{1}{2} [u \sinh(u) - \cosh(u)]$$

$$\int \left(\frac{5x}{5x^2+1} - \frac{1}{x} \right) dx = \frac{5}{10} \int \frac{10x}{5x^2+1} - \int \frac{1}{x} = \frac{5}{2} \cdot \ln|5x^2+1| - [\ln|x|]$$

$\uparrow$
2, 1, 0

$$\int_1^{+\infty} \left(\frac{5x}{5x^2+1} - \frac{1}{x} \right) dx = \left[\frac{1}{2} \ln(5x^2+1) - \ln|x| \right]_1^{+\infty} = \lim_{n \rightarrow \infty} \frac{1}{2} \ln(5a^2+1) - \ln(a) - \frac{1}{2} \ln(5+1) = \ln(\sqrt{5a^2+1}) - \ln a - \frac{1}{2} \ln(6)$$

$$= \ln\left(\frac{a\sqrt{5+\frac{6}{a^2}}}{a}\right) - \frac{1}{2} \ln(6) = \frac{1}{2} \ln\left(\frac{5}{6}\right)$$