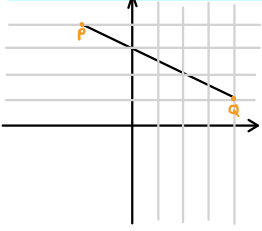


Aufgabe 6 Parametrisieren SS14

Strecke von $P = (-2/4)^T$ nach $Q = (4/1)^T$



$\Rightarrow$ Es gilt: $[0,1] \quad C(t) = P + t\overline{PQ} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} + t \begin{pmatrix} 4 - (-2) \\ 1 - 4 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} + t \begin{pmatrix} 6 \\ -3 \end{pmatrix} \rightarrow = C'(t)$

Potential bestimmen

$$f: \mathbb{R}^2 \rightarrow \mathbb{R}^2: (x_1, x_2)^T \mapsto \begin{pmatrix} 3x_2^2 \\ 6x_1x_2 + 1 \end{pmatrix}$$

$$U(x_1, x_2) = \int 3x_2^2 dx_1 = 3x_2^2 x_1 + C_1(x_2)$$

• $\frac{d}{dx_i}$ $\hat{=}$ 2te Zeile

$$\frac{d}{dx_2} (3x_2^2 x_1 + C_1(x_2)) = 6x_1 x_2 + 1$$

$$6x_1 x_2 + \frac{d}{dx_2} (C_1(x_2)) = 6x_1 x_2 + 1 \quad \Rightarrow \quad \frac{d}{dx_2} (C_1(x_2)) = 1 \quad \Rightarrow \quad C_1(x_2) = \int 1 dx_2 = x_2 + C_2$$

$$U(x_1, x_2) = 3x_2^2 x_1 + x_2$$

c) Berechne $\int_K f(x) \cdot dx$

$$\int_K f(x) \cdot dx = \int_0^1 f(C(t)) \cdot C'(t) dt$$

$$= 105$$

Aufgabe 9 Parametrisierung SS15

$$a) \quad C: [t_0, t_1] \rightarrow \mathbb{R}^2: t \mapsto \begin{pmatrix} t^2 + at + b \\ at^2 + bt + c \end{pmatrix}$$

Probe durch einsetzen $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ nach $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$ mit $a=0$

$$C(t_0) = \begin{pmatrix} 2 \\ 0 + 2b + c \end{pmatrix} \quad C(t_1) = \begin{pmatrix} 3 \\ 0 + 3b + c \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} t^2 \\ 2b + c \end{pmatrix} \quad \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 + 3b + c \end{pmatrix}$$

$$\text{I} \quad 3 = 2b + c \quad c = (3 - 2b) \text{ in II einsetzen}$$

$$\text{II} \quad 5 = 8b + c \quad \text{II}_c \quad 5 = 3b + 3 - 2b$$

$$\text{I}_{b=2}: 3 = 4 + c \quad c = -1$$

$$b) \quad C(t_0) = \begin{pmatrix} 0 \\ 0 + 0 + c \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \quad \text{I} \quad c = 4 \quad C(t_1) = \begin{pmatrix} 2 \\ 4a + 2b + 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad C(t_1) = \begin{pmatrix} 4 \\ 16a + 4b + 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

$$\text{II} \quad 4a + 2b + 4 = 0$$

$$\text{III} \quad 16a + 4b = 0$$

$$\underline{b = -4a} \text{ in II} \quad 4a + 2(-4a) + 4 = 0$$

$$-4a + 4 = 0$$

$$\underline{a = 1}$$

$$\underline{c = 4}$$

$$C: [0, 4] \rightarrow \mathbb{R}^2: t \mapsto \begin{pmatrix} t^2 \\ t^2 - 4t + 4 \end{pmatrix}$$

$$C'(t): t \mapsto \begin{pmatrix} 2t \\ 2t - 4 \end{pmatrix}$$

c) Bestimmen Sie ein Potential U zum Vektorfeld g

$$g: \mathbb{R}^2 \rightarrow \mathbb{R}^2: x \mapsto \begin{pmatrix} 2x_1 \\ 2x_2 \end{pmatrix}$$

$$\int 2x_1 dx_1 = x_1^2 + c_1(x_2) \quad U(x_1, x_2) = x_1^2 + x_2^2$$

$$\frac{d}{dx_2}(x_1^2 + c_1(x_2)) \stackrel{!}{=} 2x_2$$

$$\frac{d}{dx_2}(c_1(x_2)) = 2x_2$$

$$c_1(x_2) = \int 2x_2 \stackrel{!}{=} x_2^2$$

Berechnen Sie die Perabel aus b)

$$\int_0^4 g(c(t)) \cdot c'(t) dt = \int_0^4 g(z) \cdot z' \frac{dz}{z'} = \int_0^4 g(z) dz = \left[U(x_1, x_2) \right]_0^4 = \left[x_1^2 + x_2^2 \right]_0^4 = 16$$

Aufgabe 9 Parametrisierung SS16

$$f: \mathbb{R}^2 \rightarrow \mathbb{R}: \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mapsto x_1^2 + 6x_1x_2 + x_2^2 \quad \text{und Kurve } c: [0, \pi] \rightarrow \mathbb{R}^2: t \mapsto \begin{pmatrix} t \\ 2 + \cos(t) \end{pmatrix}$$

a) Anfangs- und Endpunkt: $c(0), c(\pi)$ $\nabla f \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2x_1 + 6x_2 \\ 2x_2 + 6x_1 \end{pmatrix}$ $U \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \stackrel{!}{=} x_1^2 + 6x_1x_2 + x_2^2$

$$c(0) = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \quad c(\pi) = \begin{pmatrix} \pi \\ 1 \end{pmatrix}$$

b) Kurvenintegral berechnen $\int_{\gamma} \text{grad } f(x) \cdot dx = \int_0^\pi \text{grad } f(c(t)) \cdot c'(t) dt$ $\int_{\gamma} \text{grad } f(x) \cdot dx = \left[U(x_1, x_2) \right]_0^\pi = U(c(\pi)) - U(c(0)) = \pi^2 + 6\pi + 1 - 9 = \pi^2 + 6\pi - 8$

$\int_{\gamma} \text{grad } f(x) \cdot dx = \left[U(x_1, x_2) \right]_0^\pi = U(c(\pi)) - U(c(0)) = \pi^2 + 6\pi + 1 - 9 = \pi^2 + 6\pi - 8$

$\int_{\gamma} \text{grad } f(x) \cdot dx = \left[U(x_1, x_2) \right]_0^\pi = U(c(\pi)) - U(c(0)) = \pi^2 + 6\pi + 1 - 9 = \pi^2 + 6\pi - 8$

$\int_{\gamma} \text{grad } f(x) \cdot dx = \left[U(x_1, x_2) \right]_0^\pi = U(c(\pi)) - U(c(0)) = \pi^2 + 6\pi + 1 - 9 = \pi^2 + 6\pi - 8$