

Aufgabe 6 Partialbruchzerlegung

SS07

$$\frac{2(x^2+x+1)}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1} \quad \text{erweitern}$$

$$\frac{A \cdot (x^2+1)}{x(x^2+1)} + \frac{(Bx+C)x}{x(x^2+1)}$$

$$Ax^2+A+Bx^2+Cx=2(x^2+x+1)$$

$$Ax^2+A+Bx^2+Cx=2x^2+2x+2 \quad x=0$$

$$A=2$$

$$2x^2+2+Bx^2+Cx=2x^2+2x+2$$

$$Bx^2+Cx=2x \quad |:x$$

$$Bx+C=2 \quad x=0$$

$$C=2$$

$$Bx+2=2$$

$$Bx=0$$

$$B=0$$

$$\int_{\frac{1}{3}}^1 \frac{2(x^2+x+1)}{x^3+x} = \int_{\frac{1}{3}}^1 \frac{2}{x} + \frac{0+2}{x^2+1} = \left[2\ln(x) + \int \frac{2}{x^2+1} = 2 \cdot \arctan(x) \right]$$

$$\text{weil } \int \frac{1}{x^2+\beta x+\gamma} = \left[\frac{1}{\sqrt{\Delta}} \arctan(u) \right] \quad u = \frac{x - \frac{\beta}{2}}{\frac{\sqrt{\Delta}}{2}}$$

$$\Delta = \beta^2 - 4\gamma$$

Aufgabe 3 Integral-Partialbruchzerlegung

SS09

$$\int \frac{x^4+x^3-6x^2+5}{x^2+x-6} dx \quad \text{Grad im Zähler größer als im Nenner} \rightarrow \text{Polynomdivision erforderlich}$$

$$(x^4+x^3-6x^2+5) : (x^2+x-6) = x^2 + \frac{5}{x^2+x-6}$$

$$\frac{5}{x^2+x-6}$$

$$\int \left(x^2 + \frac{5}{x^2+x-6} \right) dx$$

Partialbruchzerlegung

nur einfache Nullstellen

Gleichsetzen

$$\frac{5}{x^2+x-6} = \frac{5}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3} = \frac{A \cdot (x+3) + B \cdot (x-2)}{(x-2)(x+3)}$$

Nullstellen bei $x_1=2$ $x_2=-3$

$$\int x^2 + \frac{1}{x-2} + \frac{-1}{x+3} = \left[\frac{1}{3}x^3 + \ln(x-2) - \ln(x+3) \right] \quad \int \frac{1}{x} = \ln(x)$$

$$I \quad A(x+3) + B(x-2) = 5$$

Setze $x=-3$

$$A(0) + B(-3) + B(-2) = 5$$

$$B = -1$$

Setze $x=2$

$$A(2) + A(3) + B(0) = 5$$

$$A = 1$$

Aufgabe 5 Polynomdivision und Partialbruchzerlegung

SS13

$$\frac{x^5-7x^3-18x+1}{x^2-9} \quad \text{da Grad Nenner < Grad Zähler Polynomdivision}$$

$$\frac{x^5-7x^3-18x+1}{x^2-9} : x^2-9 = x^3+2x$$

$$-(x^5-9x^3)$$

$$\frac{2x^3-18x+1}{-(2x^3-18x)}$$

$$\frac{1}{x^2-9} \text{ Rest}$$

$$x^3+2x + \frac{1}{x^2-9}$$

$$\frac{1}{x^2-9} = \frac{A}{x-3} + \frac{B}{x+3} = \frac{A(x+3) + B(x-3)}{(x-3)(x+3)}$$

$$I \quad 1 = A(x+3) + B(x-3) \quad \text{Setze Nullstellen } (x=-3, x=3) \text{ ein}$$

$$I_{x=3} \quad 1 = 6A + 0 \Leftrightarrow A = \frac{1}{6}$$

$$I_{x=-3} \quad 1 = 0 + (-6B)$$

$$B = -\frac{1}{6}$$

$$\frac{1}{x^2-9} = \frac{1}{6} \cdot \frac{1}{x-3} - \frac{1}{6} \cdot \frac{1}{x+3}$$

$$\int x^3+2x + \frac{1}{6} \cdot \frac{1}{x-3} - \frac{1}{6} \cdot \frac{1}{x+3} dx = \left[\frac{1}{4}x^4 + x^2 + \frac{1}{6} \ln|x-3| - \frac{1}{6} \ln|x+3| \right]$$

Aufgabe 6 Partialbruchzerlegung SS15

$$\frac{x^3 + x^2 + 4x + 2}{x(x^2 + 2)} = 1 + \frac{1}{x} + \frac{2x}{x^2 + 2}$$

$$x^3 + x^2 + 4x + 2 : (x^3 + 2x) = 1 + \frac{1}{x} + \frac{2x}{x^2 + 2x}$$

$$\begin{array}{r} x^3 + x^2 + 4x + 2 \\ -(x^3 + 2x) \\ \hline 0 \quad x^2 + 2x + 2 \\ \quad x^2 + 2x + 2 \\ \hline 0 \end{array}$$

$$\int 1 + \frac{1}{x} + \frac{2}{x^2 + 2} = \left[x + \ln|x| + \frac{1}{\sqrt{2}} \arctan\left(\frac{x}{\sqrt{2}}\right) \right]$$

$$\Delta = 2 - 0$$

$$u = \frac{x}{\sqrt{2}}$$

Aufgabe 7 Partialbruchzerlegung SS16

$$\frac{8x^2 + 9x + 18}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4} \Rightarrow$$

$$I \quad A(x^2+4) + (Bx+C)(x+2) \stackrel{!}{=} 8x^2 + 9x + 18 \quad (\text{setze Nullstelle ein}) \quad \underline{x = -2}$$

$$A(4+4) + 0 = 8 \cdot 4 + 9(-2) + 18$$

$$8A = 32 - 18 + 18$$

$$A = 4$$

$$II_{A=4} \quad 4x^2 + 16 + Bx^2 + 2Bx + Cx + 2C = 8x^2 + 9x + 18$$

$$I^* \quad 4x^2 + Bx^2 = 8x^2$$

$$4 + B = 8$$

$$B = 4$$

$$II^* \quad 16 + 2C = 18$$

$$2C = 2$$

$$C = 1$$

$$\frac{4}{x+2} + \frac{4x+1}{x^2+4}$$

Aufgabe 9 Partialbruchzerlegung SS17

$$\frac{x^3 + 2x^2 - 4x + 2}{x(x-2)(x^2+1)} = \frac{A}{x} + \frac{B}{x-2} + \frac{Cx+D}{x^2+1} = \frac{A(x-2)(x^2+1) + Bx(x^2+1) + (Cx+D)(x-2)x}{x(x-2)(x^2+1)}$$

$$I: x^3 + 2x^2 - 4x + 2 = A(x-2)(x^2+1) + Bx(x^2+1) + (Cx+D)(x-2)x \quad (\text{Nullstellen einsetzen}) \quad x_1 = 2 \quad x_2 = 0$$

$$I_{x=2}: 8 + 8 - 8 + 2 = 2B(4+1)$$

$$\left. \begin{array}{l} 10 = 10B \quad \Leftrightarrow \underline{B=1} \\ 2 = -2A \quad \Leftrightarrow \underline{A=-1} \end{array} \right\} \text{ in I einsetzen}$$

$$I_{x=0}: 2 = -2A$$

$$I^* \quad x^3 + 2x^2 - 4x + 2 = -(x-2)(x^2+1) + x(x^2+1) + (Cx+D)(x-2)x$$

$$x^3 + 2x^2 - 4x + 2 = -x^3 + 2x^2 + x + 2 + x^3 + x + Cx^3 - 2Cx^2 + Dx^2 - 2Dx$$

$$II \quad x^3 = Cx^3 \quad |:x^3$$

$$\underline{1 = C}$$

$$III \quad -4x = -2Dx \quad |:x$$

$$-4 = -2D$$

$$\underline{D=2}$$