

Aufgabe 2 Konvergenzradius SS06

$$\sum_{n=0}^{\infty} e^n x^n = e^n (x - x_0)^n \quad \text{mit } \sqrt[n]{a_n} \quad \sqrt[n]{e^n} = e$$

a_n Ursprung $\rho = \frac{1}{e}$

$$\lim_{x \rightarrow 0} \frac{\cos(2x) - \cos(x)}{x^2} = \frac{\cos(0) - \cos(0)}{0^2} = \frac{0}{0} \quad \text{1'Hospital}$$

Ableiten einzeln
Nenner und Zähler

$\begin{matrix} \text{sin} \\ \text{cos} \\ -\text{sin} \\ \text{cos} \end{matrix}$

$$= \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(\cos(2x) - \cos(x))}{\frac{d}{dx}(x^2)} = \lim_{x \rightarrow 0} \frac{-2\sin(2x) + \sin(x)}{2x} = \frac{0}{0}$$

$$\frac{\frac{d}{dx}(-2\sin(2x) + \sin(x))}{\frac{d}{dx}(2x)} = \lim_{x \rightarrow 0} \frac{-4\cos(2x) + \cos(x)}{2} = \frac{-4(1) + 1}{2} = \boxed{-\frac{3}{2}}$$

Aufgabe 8 Potenzreihen SS07

$$\sum_{n=0}^{\infty} \frac{x^{n+1}}{n!} = \sum_{n=0}^{\infty} \frac{x^n \cdot x}{n!} = x \sum_{n=0}^{\infty} \frac{x^n}{n!} = x e^x \quad \text{exp Reihe} \rightarrow \rho = \infty$$

ein geschlossener Ausdruck $\boxed{x e^x}$

Aufgabe 2 Komplexe Potenzreihe SS08

a) $\sum_{k=0}^{\infty} \frac{z^{2k}}{5^k}$ [Wurzelkrit] $\sqrt[k]{|a_n|} = \sqrt[k]{\frac{1}{5^k} (z^{2k})} = \sqrt[k]{\frac{1}{5^k}} = \frac{1}{\sqrt{5}}$

$z_0 = 0 \quad \rho = \frac{1}{\sqrt{5}}$

b) $\sum_{k=0}^{\infty} (-1)^k k! (4+z)^k \rightarrow$ Wurzelkrit $\sqrt[k]{|a_n|} = \lim_{k \rightarrow \infty} \sqrt[k]{(-1)^k \cdot k!} = \lim_{k \rightarrow \infty} \sqrt[k]{k!} = \infty \quad \boxed{\rho = 0}$

$z_0 = -4 \quad \rho = 0$

c) $\sum_{k=1}^{\infty} \frac{(z+2-i)^k}{k^k}$ $\sqrt[k]{|a_n|} = \lim_{k \rightarrow \infty} \sqrt[k]{\frac{1}{k^k} (z+2-i)^k} = \lim_{k \rightarrow \infty} \left(\frac{1}{k}\right) = 0$

$z_0 = -2+i \quad \rho = \infty$

Aufgabe 2 Potenzreihe (Radius) SS09

a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{3^n - 2} x^n$ EP: 0 $a_n = \frac{(-1)^n}{3^n - 2}$ $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \frac{(-1)}{\sqrt[n]{3^n - 2}} = \frac{(-1)}{\sqrt[n]{3^n(1 - \frac{2}{3^n})}} = \frac{-1}{3} \quad \boxed{\rho = 3}$

b) $\sum_{n=0}^{\infty} 7e^{-2n} x^{2n}$ EP: 0 $a_n = 7e^{-2n}$ $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \sqrt[n]{7e^{-2n}} = \frac{1}{e} \quad \boxed{\rho = e}$

Aufgabe 2 Potenzreihen SS11

$$\sum_{k=0}^{\infty} \frac{2^k}{\sqrt{k+1}} x^k = \lim_{k \rightarrow \infty} \sqrt[k]{|a_n|} = \lim_{k \rightarrow \infty} \sqrt[k]{\frac{2^k}{\sqrt{k+1}}} = \frac{2}{\sqrt[k]{k+1}} = \frac{2}{1} \quad \rho = \frac{1}{2} = \frac{1}{2}$$

$$\sum_{k=0}^{\infty} -\frac{3^k}{4} x^{2k} = \lim_{k \rightarrow \infty} \sqrt[k]{\left| -\frac{3^k}{4} \right|} = \sqrt[k]{\frac{3^k}{4}} = \frac{\sqrt[k]{3}}{\sqrt[k]{4}} = \frac{1}{1} \quad \rho = \frac{1}{3}$$

$\lim_{k \rightarrow \infty} \sqrt[k]{4} = 1$

Aufgabe 2 Komplexe Potenzreihen SS12

$$\sum_{n=3}^{\infty} \left(\frac{z-2}{n+2} \right)^n = \frac{1}{(n+2)^n} (z-2)^n \quad z_0 = 2$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \sqrt[n]{\frac{1}{(n+2)^n}} = \lim_{n \rightarrow \infty} \frac{1}{n+2} = 0 \quad \rho = +\infty$$

$$\sum_{m=3}^{\infty} \frac{(-1)^m}{\sqrt{m+1}} (z+3-i)^m \quad z_0 = -3+i$$

$$\lim_{m \rightarrow \infty} \sqrt[m]{\left| \frac{(-1)^m}{\sqrt{m+1}} \right|} = \frac{1}{\sqrt{m+1}} = 1 \quad \rho = 1$$

$$\sum_{n=3}^{\infty} 3^n \left(\frac{1}{2} z - i \right)^n = \sum_{n=3}^{\infty} 3^n \left(\frac{1}{2} \left(z - \frac{i}{2} \right) \right)^n = \sum_{n=3}^{\infty} 3^n \cdot \frac{1}{2^n} (z - \frac{i}{2})^n \quad z_0 = \frac{i}{2}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\left(\frac{3}{2} \right)^n} = \frac{3}{2} \quad \rho = \frac{2}{3}$$

Aufgabe 3 Komplexe Potenzreihen SS13

$$\sum_{k=2}^{\infty} \frac{(z-12)^k}{7^k} \quad z_0 = 12 \quad \lim_{k \rightarrow \infty} \sqrt[k]{|a_k|} = \sqrt[k]{\frac{1}{7^k}} = \frac{1}{7} \quad \rho = 7$$

$$\sum_{k=1}^{\infty} \left(\frac{7+(-1)^k}{5} \right)^k (z+2i)^k \quad z_0 = -2i \quad \lim_{k \rightarrow \infty} \sqrt[k]{\left| \frac{(7+(-1)^k)^k}{(5)^k} \right|} = \lim_{k \rightarrow \infty} \left| \frac{7+(-1)^k}{5} \right| = \frac{7+1}{5} = \frac{8}{5} \quad \rho = \frac{5}{8}$$

$$\sum_{k=0}^{\infty} 3^k \left(\frac{1}{4} z - i \right)^k = \sum_{k=0}^{\infty} 3^k \left(\frac{1}{4} (z - 4i) \right)^k = \sum_{k=0}^{\infty} 3^k \cdot \frac{1}{4^k} \quad \lim_{k \rightarrow \infty} \sqrt[k]{\left(\frac{3}{4} \right)^k} = \frac{3}{4} \quad \rho = \frac{4}{3}$$

Aufgabe 2 Komplexe Potenzreihen SS14

$$\sum_{k=1}^{\infty} \left(-\frac{1}{9} \right)^k (z+3)^k = \lim_{k \rightarrow \infty} \sqrt[k]{|a_k|} = \lim_{k \rightarrow \infty} \sqrt[k]{\left| \frac{1}{9} \right|} = \frac{1}{9} \quad \rho = 9$$

$$\sum_{k=2}^{\infty} \left(\frac{1}{9} (z^2 + 4iz - 4) \right)^k \dots \quad z^2 + 4iz - 4 = 0 \quad z_0 = \frac{-4i \pm \sqrt{(4i)^2 - 4 \cdot (-4)}}{2} = \frac{-4i \pm \sqrt{-16 + 16}}{2} = \frac{-4i \pm 0}{2} = -2i \Rightarrow (z+2i)^2$$

$$\sum_{k=2}^{\infty} \left(\frac{1}{9} \right)^k (z+2i)^{2k} = \lim_{k \rightarrow \infty} \sqrt[k]{\left| \frac{1}{9} \right|} = \sqrt[k]{\frac{1}{9}} = \frac{1}{3} \quad \rho = 3$$

$$\sum_{k=3}^{\infty} \left(\frac{z+(-1)^k}{3e^4} \right)^k (z+5i-2)^k = \lim_{k \rightarrow \infty} \sqrt[k]{\left| \frac{z+(-1)^k}{3e^4} \right|} = \lim_{k \rightarrow \infty} \left| \frac{z+(-1)^k}{3e^4} \right| = \frac{1}{e^4} \quad \rho = e^4$$

Aufgabe 8 Potenzreihen SS15

$$\sum_{k=1}^{\infty} \left(\frac{z+5i}{15} \right)^k = \sum_{k=1}^{\infty} \frac{1}{(15)^k} (z+5i)^k = \sqrt[k]{\frac{1}{15^k}} = \frac{1}{15} \quad \rho = 15$$

$$\sum_{n=0}^{\infty} (\sqrt{3} + (-1)^n)^n (z + \sqrt{7})^n = \sqrt[n]{|(\sqrt{3} + (-1)^n)^n|} = (\sqrt{3} + 1) \quad \rho = \frac{1}{\sqrt{3}+1}$$

$$\sum_{j=2}^{\infty} 4^j \left(\frac{1}{7} z - 2i + 1 \right)^j = 4^j \left(\frac{1}{7} \right)^j (z - 14i + 7)^j \Rightarrow \sqrt[j]{\left(\frac{4}{7} \right)^j} = \frac{4}{7} \quad \rho = \frac{7}{4}$$

Aufgabe 1 Potenzreihen SS16

$$\sum_{k=2}^{\infty} \frac{1}{(k+i)^k} (z-i)^k = \lim_{k \rightarrow \infty} \sqrt[k]{\left| \frac{1}{(k+i)^k} \right|} = \left| \frac{1}{1+i} \right| = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \quad \rho = \sqrt{2}$$

$$\sum_{j=0}^{\infty} \left(\frac{1}{2} (z-2i+2) \right)^j = \left(\frac{1}{2} \right)^j (z-2i+2) = \lim_{k \rightarrow \infty} \sqrt[k]{1} = \frac{1}{2} \quad \rho = 2$$

$$\sum_{n=4}^{\infty} (\sqrt{2}+i)^n \left(\frac{z+3}{1+2i} \right)^n = \sum_{n=4}^{\infty} (\sqrt{2}+i)^n \left(\frac{1}{1+2i} (z+3) \right)^n = \sum_{n=4}^{\infty} (\sqrt{2}+i)^n \left(\frac{1}{1+2i} \right)^n = \lim_{n \rightarrow \infty} \sqrt[n]{\left| \frac{(\sqrt{2}+i)^n}{(1+2i)^n} \right|} = \left| \frac{\sqrt{2}+i}{1+2i} \right| = \frac{|\sqrt{2}+i|}{\sqrt{5}} = \frac{\sqrt{2}+1}{\sqrt{5}} \quad \rho = \frac{\sqrt{5}}{\sqrt{2}+1}$$

Aufgabe 2 Potenzreihen SS17

$$\sum_{k=1}^{\infty} \frac{1}{(k+3i)^k} (z-2i)^k = \sqrt[k]{\left| \frac{1}{(k+3i)^k} \right|} = \left| \frac{1}{1+3i} \right| = \frac{1}{\sqrt{10}} \quad \rho = \sqrt{10}$$

$$\sum_{j=0}^{\infty} 3^j \left(\frac{5}{2} z - i + 2 \right)^j = \sum_{j=0}^{\infty} 3^j \left(\frac{5}{2} \right)^j \left(z - \frac{2i}{5} + \frac{4}{5} \right)^j = \sqrt[j]{\left(\frac{15}{2} \right)^j} = \frac{15}{2} \quad \rho = \frac{2}{15}$$

$$\sum_{n=3}^{\infty} \frac{(6+(-1)^n)^n}{n} (z+2)^n = \sqrt[n]{\left| \frac{(6+(-1)^n)^n}{n} \right|} = \frac{16+(-1)^n}{\sqrt[n]{n}} = \frac{7}{1} \quad \rho = \frac{1}{7}$$

Aufgabe 5 Potenzreihen SS18

$$\sum_{k=1}^{\infty} 5^{k+1} (z+7)^k = \sum_{k=1}^{\infty} 5^{k+1} (z+7)^k = \sum_{k=1}^{\infty} 5^k \cdot 5 (z+7)^k = \lim_{k \rightarrow \infty} \sqrt[k]{5^k \cdot 5} = 5 \cdot \sqrt[5]{5} = 5 \quad \rho = \frac{1}{5}$$

$$\sum_{k=0}^{\infty} \frac{3^k}{k^2+1} (\sqrt{2}z+3)^k = \sum_{k=0}^{\infty} \frac{3^k}{k^2+1} \cdot \sqrt{2}^k \left(z + \frac{3}{\sqrt{2}} \right)^k = \lim_{k \rightarrow \infty} \sqrt[k]{\frac{(3 \cdot \sqrt{2})^k}{k^2+1}} = \frac{3 \cdot \sqrt{2}}{\sqrt[k]{k^2+1}} = 3 \cdot \sqrt{2} \quad \rho = \frac{1}{3 \cdot \sqrt{2}}$$

$$\sum_{k=3}^{\infty} \frac{(z-3)^{2k}}{k!} = \lim_{k \rightarrow \infty} \sqrt[k]{\frac{(z-3)^{2k}}{k!}} = \frac{(z-3)^2}{\sqrt[k]{k!}} = 0 \quad \rho = +\infty$$