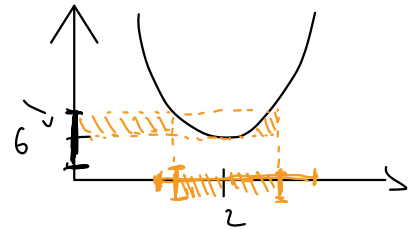


## Aufgabe 7 $\delta$ - $\epsilon$ - SS13

$$u_\alpha(z) = (z - \alpha, z + \alpha)$$

$$u_\epsilon(f(z)) = 6$$



$$f: \mathbb{R} \rightarrow \mathbb{R} : x \mapsto 5(x-2)^2 + 6$$

Berechne für die Stelle  $x_0=2$

$$|f(x) - f(x_0)| = 5(x-2)^2 + 6 - 6$$

$$\begin{aligned} |f(x) - f(x_0)| &> \epsilon \\ 5(x-2)^2 &> \epsilon \\ (x-2)^2 &> \frac{\epsilon}{5} \\ |x-2| &> \sqrt{\frac{\epsilon}{5}} \\ &\triangleq x - x_0 \\ &\text{also } \delta \end{aligned}$$

## Aufgabe 8 Stetigkeit (unstetigkeit) SS13

$$f: \mathbb{R} \rightarrow \mathbb{R} : x \mapsto \begin{cases} \frac{7}{100}x^2 + \frac{3}{5} & \text{für } x \leq 1 \\ \frac{100x^2 - 137x + 37}{100x - 100} & \text{für } x > 1 \end{cases}$$

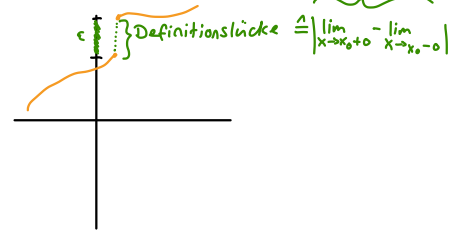
$$\lim_{x \rightarrow 1-0} = \lim_{x \rightarrow 1} \frac{7}{100}x^2 + \frac{3}{5} = \frac{7}{100} + \frac{3}{5} = \frac{7}{100} + \frac{60}{100} = \frac{67}{100}$$

$$\begin{aligned} \lim_{x \rightarrow 1+0} &= \lim_{x \rightarrow \infty} \frac{100(1+\frac{1}{n})^2 - 137(1+\frac{1}{n}) + 37}{100(1+\frac{1}{n}) - 100} = \frac{100 + \frac{200}{n} + \frac{100}{n^2} - 137 - \frac{137}{n} + 37}{100 + \frac{100}{n} - 100} = \left( \frac{200}{n} + \frac{100}{n^2} - \frac{137}{n} \right) \frac{n}{100} = \left( \frac{100}{n} + \frac{63}{n} \right) \frac{n}{100} \\ &= \left( \frac{100}{n} + \frac{63}{n} \right) \frac{n}{100} = \frac{63}{100} \end{aligned}$$

Stetig im Punkt  $x_0$  wenn gilt  $\lim_{x \rightarrow x_0+0} = \lim_{x \rightarrow x_0-0}$  (Rechts und Linksseitig!)

b)  $|f(x) - f(x_0)| \triangleq \text{Sprunghöhe} < \epsilon$  also  $\frac{4}{100} < \epsilon$   $\frac{3}{100} \leq \epsilon$   $\rightarrow$  eine Lösung die die Bedingung erfüllt

c) nicht stetig in  $x_0=1$  (siehe a), weil  $\lim_{x \rightarrow x_0+0} \neq \lim_{x \rightarrow x_0-0}$ )  
 $x \in \mathbb{R} \setminus \{1\}$



## Aufgabe 4 Stetigkeit SS15

$$f: \mathbb{R} \rightarrow \mathbb{R} : x \mapsto x|x+1| + 2x \quad x_0 = -1$$

$$\text{Berechne } \lim_{x \rightarrow x_0+0} \frac{f(x) - f(x_0)}{x - x_0} = \frac{x|x+1| + 2x + 2}{x + 1} \Rightarrow \lim_{n \rightarrow \infty} \frac{(-1 + \frac{1}{n})|(-1 + \frac{1}{n} + 1)| + 2(-1 + \frac{1}{n}) + 2}{-1 + \frac{1}{n} + 1} \quad \text{Kehrwert}$$

$$\lim_{n \rightarrow \infty} \left( (-1 + \frac{1}{n}) \left| \frac{1}{n} \right| - 2 + \frac{2}{n} + 2 \right) \cdot \frac{n}{1} = \frac{1}{n} \left( -1 + \frac{1}{n} + \frac{2}{n} - \frac{2}{n} \right) \frac{n}{1} = 1$$

$$\lim_{x \rightarrow x_0-0} \frac{f(x) - f(x_0)}{x - x_0} = \frac{x|x+1| + 2x + 2}{x + 1} \Rightarrow \lim_{n \rightarrow \infty} \frac{(-1 - \frac{1}{n})|(-1 - \frac{1}{n} + 1)| + 2(-1 - \frac{1}{n}) + 2}{-1 - \frac{1}{n} + 1} \quad \text{Kehrwert}$$

$$\lim_{n \rightarrow \infty} \left( (-1 - \frac{1}{n}) \left| \frac{1}{n} \right| - 2 - \frac{2}{n} + 2 \right) \cdot \left( -\frac{1}{1} \right) = \frac{1}{n} \left( -1 - \frac{1}{n} - \frac{2}{n} - \frac{2}{n} \right) \left( -\frac{n}{1} \right) = +3$$

Stetig in  $x_0$  differenzierbar wenn  $x \in \mathbb{R} \setminus \{-1\}$

### Aufgabe 3 Stetigkeit SS17

$$X_0 = \frac{1}{2} \quad f(x) = \begin{cases} 7x^2 - \frac{7}{4} & \text{falls } x < \frac{1}{2} \\ \alpha \ln(2x) + \frac{1}{2} & \text{falls } x \geq \frac{1}{2} \end{cases}$$

$$\lim_{x \rightarrow \frac{1}{2}^-} = \frac{f(x) - f(x_0)}{x - x_0} = \frac{7x^2 - \frac{7}{4}}{x - \frac{1}{2}} = \lim_{n \rightarrow \infty} \frac{7(\frac{1}{2} - \frac{1}{n})^2 - \frac{7}{4}}{\frac{1}{2} - \frac{1}{n} - \frac{1}{2}} = \frac{7(\frac{1}{4} - \frac{1}{n} + \frac{1}{n^2}) - \frac{7}{4}}{-\frac{1}{n}} = \frac{\frac{7}{4} - \frac{7}{n} + \frac{7}{n^2} - \frac{7}{4}}{-\frac{1}{n}} = \frac{-\frac{7}{n} + \frac{7}{n^2}}{-\frac{1}{n}} = \frac{1}{n} \left( -\frac{7}{n} + \frac{7}{n^2} \right) = \left( -\frac{7}{n} + \frac{7}{n^2} \right) \xrightarrow{n \rightarrow \infty} 0$$

$$\lim_{x \rightarrow \frac{1}{2}^+} = \frac{f(x) - f(x_0)}{x - x_0} = \frac{\alpha \ln(2x) + \frac{1}{2} - \frac{1}{2}}{x - \frac{1}{2}} = \frac{\alpha \ln(2x)}{x - \frac{1}{2}} = \lim_{n \rightarrow \infty} \frac{\alpha \ln(2(\frac{1}{2} + \frac{1}{n}))}{\frac{1}{2} + \frac{1}{n} - \frac{1}{2}} = \frac{0}{0} \stackrel{\text{L'H}}{=} \frac{\alpha \left( \frac{-2}{n^2 + 2n} \right)}{-\frac{1}{n^2}} = \alpha \left( \frac{-2}{n^2 + 2n} \right) \cdot (-n^2) = \alpha \left( \frac{2n^2}{n^2 + 2n} \right) = \alpha \left( \frac{2n^2}{n^2(1 + \frac{2}{n})} \right) = \alpha \left( \frac{2}{1 + \frac{2}{n}} \right) \xrightarrow{n \rightarrow \infty} 2\alpha$$

### Aufgabe 8 Stetigkeit SS18

$$f(x) = (\alpha + 5)|x^2 - 9|(x - 1) \quad x_0 = 3$$

$$\lim_{x \rightarrow 3^-} \frac{f(x) - f(3)}{x - 3} = \frac{(\alpha + 5)|x^2 - 9|(x - 1)}{x - 3} = \frac{(\alpha + 5)|\left(3 - \frac{1}{n}\right)^2 - 9|(3 - \frac{1}{n} - 1)}{-\frac{1}{n}} = \frac{(\alpha + 5)\left|9 - \frac{6}{n} + \frac{1}{n^2} - 9\right|\left(2 - \frac{1}{n}\right)}{-\frac{1}{n}} = \frac{(\alpha + 5)\left(-\frac{6}{n} + \frac{1}{n^2}\right)\left(2 - \frac{1}{n}\right)}{-\frac{1}{n}} = \frac{(\alpha + 5)(+6)(2)}{-1} = -12(\alpha + 5)$$

$$\lim_{x \rightarrow 3^+} \frac{f(x) - f(3)}{x - 3} = \lim_{n \rightarrow \infty} \frac{(\alpha + 5)\left|\left(3 + \frac{1}{n}\right)^2 - 9\right|\left(3 + \frac{1}{n} - 1\right)}{\frac{1}{n}} = \frac{(\alpha + 5)\left|9 + \frac{6}{n} + \frac{1}{n^2} - 9\right|\left(2 + \frac{1}{n}\right)}{\frac{1}{n}} = \frac{(\alpha + 5)\left(\frac{6}{n} + \frac{1}{n^2}\right)\left(2 + \frac{1}{n}\right)}{\frac{1}{n}} = (\alpha + 5)(6)(2) = 12(\alpha + 5)$$

Differenzierbar wenn stetig im Punkt  $x_0$  (also  $\lim_{x \rightarrow x_0^+} = \lim_{x \rightarrow x_0^-}$ )

$$-12(\alpha + 5) = 12(\alpha + 5)$$

$$-(\alpha + 5) = \alpha + 5$$

$$-\alpha - 5 = \alpha + 5$$

$$2\alpha = -10$$

$$\alpha = -5$$