

Aufgabe 5 Taylorpolynom zweite Stufe SS07

$$\begin{pmatrix} \sin \\ -\cos \\ -\sin \end{pmatrix} \cos$$

$$f(x) = x \sin(2x)$$

$$f'(x) = \frac{d}{dx}(x) \sin(2x) + x \frac{d}{dx}(\sin(2x)) = \sin(2x) + 2x \cos(2x)$$

$$f''(x) = \frac{d}{dx}(\sin(2x)) + \frac{d}{dx}(2x) \cos(2x) + 2x \frac{d}{dx}(\cos(2x)) = \underbrace{2 \cos(2x) + 2 \cdot \cos(2x) - 4x \sin(2x)}_{4 \cos(2x)}$$

$$f'(0) = 0 + 0 = 0$$

$$f''(0) = 2 \cdot 1 + 2 \cdot 1 - 0 = 4$$

$$\boxed{\text{TS}} \quad T_2(f, x, 0) = \frac{f(0)}{0!} (x-0)^0 + \frac{f'(0)}{1!} (x-0)^1 + \frac{f''(0)}{2!} (x-0)^2$$

$$0 + 0 + \underline{2x^2} \quad \checkmark$$

Aufgabe 6 Ableiten / Taylorpolynom SS08

$$f(x) = e^{x^2+3x+1}$$

$$f'(x) = e^{x^2+3x+1} (2x+3)$$

$$f''(x) = \left(\frac{d}{dx}(e^{x^2+3x+1}) \right) (2x+3) + (e^{x^2+3x+1}) \left(\frac{d}{dx}(2x+3) \right)$$

$$f''(x) = e^{x^2+3x+1} (2x+3)^2 + e^{x^2+3x+1} (2)$$

$$e^{x^2+3x+1} ((2x+3)^2 + 2) = e^{x^2+3x+1} (4x^2 + 12x + 11)$$

$$\text{Taylor} \quad \sum_{k=0}^n \frac{f^{(k)}(x_0)}{k!} (x-x_0)^k$$

$$T_2(f, x, 0) = \frac{f(0)}{0!} (x-0)^0 + \frac{f'(0)}{1!} (x-0)^1 + \frac{f''(0)}{2!} (x-0)^2$$

$$e + e(3)x + \frac{e(11)}{2} x^2$$

Aufgabe 4 Ableitungen SS10

$$f: \mathbb{R} \rightarrow \mathbb{R}: x \mapsto x^2 e^{-\frac{x^2}{2}}$$

$$f'(x) = \frac{d}{dx}(x^2) e^{-\frac{x^2}{2}} + x^2 \frac{d}{dx}(e^{-\frac{x^2}{2}}) = 2x e^{-\frac{x^2}{2}} + x^2 \cdot (-x) e^{-\frac{x^2}{2}} = e^{-\frac{x^2}{2}} (2x - x^3)$$

$$f''(x) = \frac{d}{dx}(2x) e^{-\frac{x^2}{2}} + 2x \frac{d}{dx}(e^{-\frac{x^2}{2}}) + \frac{d}{dx}(-x^3) e^{-\frac{x^2}{2}} + (-x^3) \frac{d}{dx}(e^{-\frac{x^2}{2}})$$

$$f''(x) = 2 e^{-\frac{x^2}{2}} + 2x \cdot (-x) (e^{-\frac{x^2}{2}}) + \underline{-3x^2 e^{-\frac{x^2}{2}}} + (-x^3) (e^{-\frac{x^2}{2}}) \cdot (-x)$$

$$f''(x) = e^{-\frac{x^2}{2}} (2 - 5x^2 + x^4)$$

$$T(f, x, 2) = 4e^{-2} + e^{-2} (4 - 8) (x-2) + e^{-2} (2 - 20 + 16) (x-2)^2$$

$$4e^{-2} + e^{-2} (-4) (x-2) + e^{-2} (-2) (x-2)^2 = e^{-2} (4 - 4(x-2) - 2(x-2)^2)$$

Aufgabe 6 Taylorpolynom SS11

$-c \begin{smallmatrix} 5 \\ -3 \end{smallmatrix} c$

$$f: \mathbb{R} \rightarrow \mathbb{R}: x \mapsto \cos(e^{3x} - 1)$$

$$f'(x) = -\sin(e^{3x} - 1) e^{3x} \cdot 3$$

$$f''(x) = -\sin(e^{3x} - 1) e^{3x} \cdot 9 + -\cos(e^{3x} - 1) e^{3x} \cdot e^{3x} \cdot 9$$

$$f''(x) = -\sin(e^{3x} - 1) 9e^{3x} - \cos(e^{3x} - 1) 9e^{6x}$$

$$T_2(f, x, 0) = \cancel{f(0)} + \cancel{f'(0)(x-0)} + \frac{\overset{0-9}{f''(0)(x)^2}}{2!} = 1 - \frac{9}{2}x^2$$

Aufgabe 2 Taylorpolynom SS13

$$f: \mathbb{R} \rightarrow \mathbb{R}: x \mapsto x e^{x^2+2x+2}$$

$$f(0) = 0$$

$$f'(0) = e^2$$

$$T_1(f, x, 0) = f(0) + f'(0)x = e^2 x$$

$$f'(x) = x(2x+2)e^{x^2+2x+2} + e^{x^2+2x+2}$$

Aufgabe 4 Taylorpolynom 2. Stufe SS14

$$\exp(\dots) = e^{\dots}$$

$$f: \mathbb{R} \rightarrow \mathbb{R}: x \mapsto \exp((x-2)^2)$$

$$f'(x) = \exp((x-2)^2) \cdot \overset{(2x-4)}{2(x-2)}$$

$$f''(x) = \exp((x-2)^2) 2 + \exp((x-2)^2) (2x-4)^2 = \exp((x-2)^2) (4x^2 - 16x + 18)$$

$$T_2(f, x, 0) = f(0) + f'(0)x + \frac{f''(0)(x)^2}{2} = e^4 - e^4 4x + 9e^4 x^2$$

$$f(0) = e^4$$

$$f'(0) = e^4(-4)$$

$$f''(0) = e^4(18)$$

Aufgabe 3 Taylorpolynom SS15

$$f: (0, \frac{\pi}{2}) \rightarrow \mathbb{R}: x \mapsto \frac{\sqrt{3}x}{\tan(x)} \quad \begin{matrix} u(x) \\ v(x) \end{matrix}$$

$$\frac{u' \cdot v - u \cdot v'}{v^2} = \frac{\sqrt{3} \cdot \tan(x) - \sqrt{3}x \cdot \frac{1}{\cos^2(x)}}{(\tan(x))^2} = \sqrt{3} \left(\frac{\tan(x) - x \cdot \frac{1}{\cos^2(x)}}{(\tan(x))^2} \right)$$

$$f\left(\frac{\pi}{6}\right) = \sqrt{3} \left(\frac{\pi}{6}\right) \cdot \frac{3}{\sqrt{3}} = \frac{\pi}{2}$$

$$f'\left(\frac{\pi}{6}\right) = \frac{\sqrt{3} \cdot \frac{\sqrt{3}}{3} - \sqrt{3} \left(\frac{\pi}{6}\right) \cdot \frac{1}{\frac{4}{9}}}{\frac{3}{9}} = \left(1 - \sqrt{3} \left(\frac{\pi}{6}\right) \cdot \frac{9}{4}\right) \frac{9}{3} = 3 - \frac{\sqrt{3} \cdot \pi \cdot 2}{3}$$

$$T_1 = \frac{\pi}{2} + \left(3 - \frac{\sqrt{3} \cdot \pi \cdot 2}{3}\right) \left(x - \frac{\pi}{6}\right)$$

Aufgabe 6 Taylorpolynom SS 17

$$f: \left(-\frac{1}{3}, \frac{1}{3}\right) \rightarrow \mathbb{R} : x \mapsto f(x) := (-3x+1)^2 \ln(-3x+1) + 2$$

$$f'(x) = (18x-6) \ln(-3x+1) + (-3x+1)^2 \cdot \frac{-3}{-3x+1} = (18x-6) \ln(-3x+1) + 3x-3$$

$$f''(x) = 18 \ln(-3x+1) + \overset{(-3x+1)(-6)}{(18x-6)} \frac{-3}{-3x+1} = 18 \ln(-3x+1) + 18+9$$

$$f(0) = 2 \quad T_2(f, x, 0) = 2 - 3x + \frac{27}{2}x^2$$

$$f'(0) = -3$$

$$f''(0) = 27$$