

# Formelblatt zur Prüfung "Einführung in die Festigkeitslehre", Herbst 2011:

| Hookesches Gesetz                           |                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                           |
|---------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Beanspruchung                               | Dehnungen                                                                                                                                                                                                            | Spannungen                                                                                                                                                                                                                                                                                                                                |
| $\sigma_x$<br>( $\sigma_y = \sigma_z = 0$ ) | $\varepsilon_x = \frac{1}{E} \cdot \sigma_x$<br>$\varepsilon_y = -\frac{\mu}{E} \cdot \sigma_x$<br>$\varepsilon_z = -\frac{\mu}{E} \cdot \sigma_x$                                                                   | $\sigma_x = E \cdot \varepsilon_x$<br>$\sigma_y = 0$<br>$\sigma_z = 0$                                                                                                                                                                                                                                                                    |
| $\sigma_x; \sigma_y$<br>( $\sigma_z = 0$ )  | $\varepsilon_x = \frac{1}{E} (\sigma_x - \mu \sigma_y)$<br>$\varepsilon_y = \frac{1}{E} (\sigma_y - \mu \sigma_x)$<br>$\varepsilon_z = -\frac{\mu}{E} (\sigma_x + \sigma_y)$                                         | $\sigma_x = \frac{E}{1-\mu^2} (\varepsilon_x + \mu \cdot \varepsilon_y)$<br>$\sigma_y = \frac{E}{1-\mu^2} (\varepsilon_y + \mu \cdot \varepsilon_x)$<br>$\sigma_z = 0$                                                                                                                                                                    |
| $\sigma_x; \sigma_y; \sigma_z$              | $\varepsilon_x = \frac{1}{E} (\sigma_x - \mu (\sigma_y + \sigma_z))$<br>$\varepsilon_y = \frac{1}{E} (\sigma_y - \mu (\sigma_x + \sigma_z))$<br>$\varepsilon_z = \frac{1}{E} (\sigma_z - \mu (\sigma_x + \sigma_y))$ | $\sigma_x = \frac{E}{1+\mu} \left( \varepsilon_x + \frac{\mu}{1-2\mu} \cdot e \right)$<br>$\sigma_y = \frac{E}{1+\mu} \left( \varepsilon_y + \frac{\mu}{1-2\mu} \cdot e \right)$<br>$\sigma_z = \frac{E}{1+\mu} \left( \varepsilon_z + \frac{\mu}{1-2\mu} \cdot e \right)$<br>mit ( $e = \varepsilon_x + \varepsilon_y + \varepsilon_z$ ) |

Schubspannungs-,  
Schiebungs-  
beziehungen:

$$\tau_{xy} = G \cdot \gamma_{xy}$$

$$\tau_{yz} = G \cdot \gamma_{yz}$$

$$\tau_{zx} = G \cdot \gamma_{zx}$$

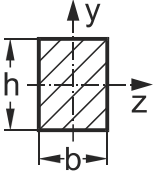
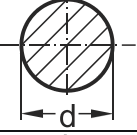
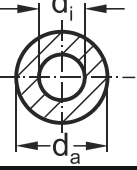
$$\left( G = \frac{E}{2(1+\mu)} \right)$$

| Vergleichsspannungen |                                                                                                         |                                                      |                                                                                             |                                                                  |
|----------------------|---------------------------------------------------------------------------------------------------------|------------------------------------------------------|---------------------------------------------------------------------------------------------|------------------------------------------------------------------|
| Spannungszustand     |                                                                                                         |                                                      |                                                                                             |                                                                  |
| Hypo-<br>these       | dreiachsig                                                                                              | zweiachsig                                           |                                                                                             |                                                                  |
|                      | $\sigma_1, \sigma_2, \sigma_3$                                                                          | $\sigma_1, \sigma_2, \sigma_3 = 0^{+)}$              | $\sigma_x, \sigma_y, \tau_{xy}$                                                             | $\sigma_b, \tau_t$                                               |
| NH                   | $\sigma_1$ (Zug) bzw.<br>$\sigma_3$ (Druck)                                                             | $\sigma_1$                                           | $\frac{\sigma_x + \sigma_y}{2} + \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$ | $\frac{\sigma_b}{2} + \frac{1}{2} \sqrt{\sigma_b^2 + 4\tau_t^2}$ |
| SH                   | $\sigma_1 - \sigma_3$                                                                                   | $\sigma_1$                                           | $\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}^{++)}$                                       | $\sqrt{\sigma_b^2 + 4\tau_t^2}$                                  |
| GEH                  | $\frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$ | $\sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2}$ | $\sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2}$                         | $\sqrt{\sigma_b^2 + 3\tau_t^2}$                                  |

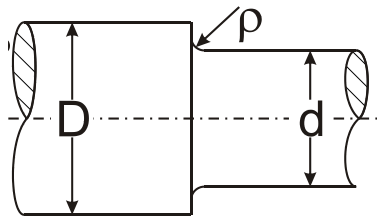
<sup>+)</sup>  Für die Fälle  $\sigma_1 > 0; \sigma_2 = 0; \sigma_3 < 0$  bzw.  $\sigma_1 = 0; \sigma_2 < 0; \sigma_3 < 0$  ergibt sich  $\sigma_v$  aus Spalte 1

<sup>++)</sup>  gilt nur für  $\sigma_x \sigma_y \leq \tau_{xy}^2$ ; für  $\sigma_x \sigma_y > \tau_{xy}^2$  ist  $\sigma_v$  (SH) =  $\sigma_v$  (NH)

## Widerstandsmomente:

| Querschnitts-<br>form                                                             | Biegung                                    |                                                        | Torsion                                    |                                                        |
|-----------------------------------------------------------------------------------|--------------------------------------------|--------------------------------------------------------|--------------------------------------------|--------------------------------------------------------|
|                                                                                   | Axiales Flächen-<br>trägheitsmoment        | Widerstandsmoment<br>gegen Biegung                     | Polares Flächen-<br>trägheitsmoment        | Widerstandsmoment<br>gegen Torsion                     |
|  | $I_z = \frac{1}{12}bh^3$                   | $W_{b,z} = \frac{1}{6}bh^2$                            | nicht elementar lösbar                     |                                                        |
|  | $I = \frac{\pi}{64}d^4$                    | $W_b = \frac{\pi}{32}d^3$                              | $I_p = \frac{\pi}{32} \cdot d^4$           | $W_t = \frac{\pi}{16} \cdot d^3$                       |
|  | $I = \frac{\pi}{64} \cdot (d_a^4 - d_i^4)$ | $W_b = \frac{\pi}{32} \cdot \frac{d_a^4 - d_i^4}{d_a}$ | $I = \frac{\pi}{32} \cdot (d_a^4 - d_i^4)$ | $W_t = \frac{\pi}{16} \cdot \frac{d_a^4 - d_i^4}{d_a}$ |

## Formzahlen bei statischer Beanspruchung:



$$\alpha_{k,zd} = 1 + \frac{1}{\sqrt{0,62 \cdot \frac{2 \cdot p}{(D-d)} + 7 \cdot \frac{p}{d} \cdot \left(1 + 2 \cdot \frac{p}{d}\right)^2}}$$

$$\alpha_{k,b} = 1 + \frac{1}{\sqrt{0,62 \cdot \frac{2 \cdot p}{(D-d)} + 116 \cdot \frac{p}{d} \cdot \left(1 + 2 \cdot \frac{p}{d}\right)^2 + 0,2 \cdot \frac{d}{D} \cdot \left(\frac{2 \cdot p}{(D-d)}\right)^3}}$$

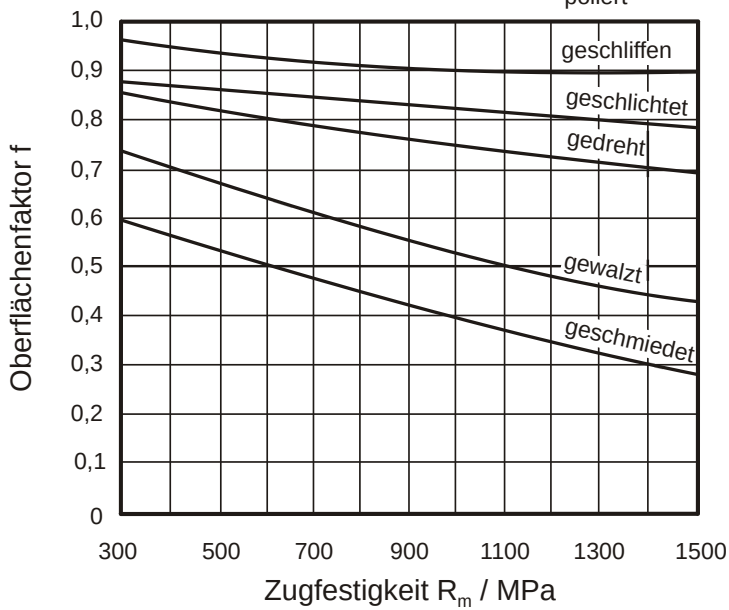
$$\alpha_{kt} = 1 + \frac{1}{\sqrt{3,4 \frac{2p}{(D-d)} + 38 \frac{p}{d} \left(1 + 2 \frac{p}{d}\right)^2 + \left(\frac{2p}{(D-d)}\right)^2 \frac{d}{D}}}$$

## Kerbwirkungszahl bei Schwingbeanspruchung

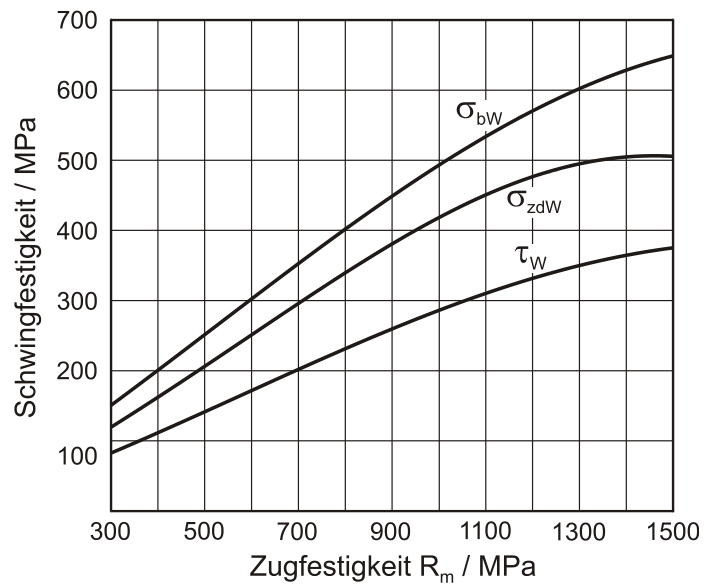
$$\beta_{kb} = \frac{\alpha_{kb}}{1 + \frac{55}{R_e} \sqrt{\frac{c}{\rho}}}$$

mit: c=2 bei Zug/Druck, Biegung; c=1 bei Torsion.

**Oberflächenfaktor f** in Abhängigkeit von der  
Zugfestigkeit ( $f = \sigma_W / \sigma_{W \text{ poliert}}$ )



**Schwingfestigkeitskennwerte** in Abhängigkeit  
von der Zugfestigkeit



**Näherungsformeln für die dauerfest ertragbare Spannungsamplitude:**

Für zähe Werkstoffe:

$$\sigma_A = \sigma_W \sqrt{1 - \frac{\sigma_m}{R_m}}$$

Für spröde Werkstoffe:

$$\sigma_A = \sigma_W \left( 1 - \frac{\sigma_m}{R_m} \right)$$