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Title: The dose response functions of ionization chambers in photon dosimetry - Gaussian or non-Gaussian?

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Abstract: This study is concerned with the spatial resolution of air-filled ionization chambers in photon-beam dosimetry, i.e. with their dose response functions. These act as convolution kernels $K(x,y)$, transforming true dose profiles $D(x,y)$ into the measured signal profiles $M(x,y)$. One-dimensional dose response functions have been experimentally determined for nine types of cylindrical ionization chambers both in their lateral and longitudinal directions, as well as across two plane-parallel chambers and for the single chambers of two 2D arrays. All these 1D dose response functions are closely described by Gaussian functions. The associated energy-dependent values of the standard deviations σ have been measured for 6 and 15 MV photons with an uncertainty of 0.02 mm. At depths beyond secondary electron fluence build-up, there was no detectable depth dependence of the σ values. The general occurrence of Gaussian dose response functions, their extension beyond the geometrical boundaries of the chambers, and the energy dependence of their standard deviations can be understood by considering the underlying system of convolutions, which is the origin of the influences of secondary electron transport. Monte-Carlo simulations of the convolution kernels for a cylindrical, a square, and a flat ionization chamber and their Fourier analysis have been employed to show that the Gaussian convolution kernels are approximations to the true dose response functions, valid in the clinically relevant domain of the spatial frequency. This paper is conceived as the starting point for the deconvolution methods to be described in a further publication.

The dose response functions of ionization chambers in photon dosimetry - Gaussian or non-Gaussian?

Die dosisbezogenen Ansprechfunktionen von Ionisationskammern in der Photonendosimetrie – Gauß-förmig oder nicht Gauß-förmig?

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Abstract

This study is concerned with the spatial resolution of air-filled ionization chambers in photon-beam dosimetry, i.e. with their dose response functions. These act as convolution kernels $K(x,y)$, transforming true dose profiles $D(x,y)$ into the measured signal profiles $M(x,y)$. One-dimensional dose response functions have been experimentally determined for nine types of cylindrical ionization chambers both in their lateral and longitudinal directions, as well as across two plane-parallel chambers and for the single chambers of two 2D arrays. All these 1D dose response functions are closely described by Gaussian functions. The associated energy-dependent values of the standard deviations σ have been measured for 6 and 15 MV photons with an uncertainty of 0.02 mm. At depths beyond secondary electron fluence build-up, there was no detectable depth dependence of the σ values. The general occurrence of Gaussian dose response functions, their extension beyond the geometrical boundaries of the chambers, and the energy dependence of their standard deviations can be understood by considering the underlying system of convolutions, which is the origin of the influences of secondary electron transport. Monte-Carlo simulations of the convolution kernels for a cylindrical, a square, and a flat ionization chamber and their Fourier analysis have been employed to show that the Gaussian convolution kernels are *approximations* to the true dose response functions, valid *in the clinically relevant domain of the spatial frequency*. This paper is conceived as the starting point for the deconvolution methods to be described in a further publication.

Zusammenfassung

Diese Arbeit befasst sich mit dem räumlichen Auflösungsvermögen der in der Photonendosimetrie verwendeten luftgefüllten Ionisationskammern, dargestellt durch ihre Dosis-Ansprechfunktionen. Diese wirken als Faltungskerne $K(x,y)$, welche die wahren Dosisprofile $D(x,y)$ in Messwertprofile $M(x,y)$ überführen. Eindimensionale Dosis-Ansprechfunktionen wurden für neun zylindrische Ionisationskammern, sowohl in der lateralen als auch in der longitudinalen Richtung, sowie für zwei Flachkammern und für die Einzelkammern von zwei zwei-dimensionalen Arrays experimentell ermittelt. Alle Dosis-Ansprechfunktionen lassen sich gut durch Gaußverteilungen darstellen. Die zugehörigen energieabhängigen Standardabweichungen σ wurden für Photonen von 6 und 15 MV mit einer Unsicherheit von 0.02 mm bestimmt. In Tiefen jenseits des Aufbaubereiches der Sekundärelektronenfluenz ist keine Tiefenabhängigkeit der σ -Werte festzustellen. Das allgemeine Auftreten von Gauß-förmigen Dosis-Ansprechfunktionen mit nicht verschwindenden Werten außerhalb der Kammerdimensionen und mit energieabhängigen Standardabweichungen lässt sich durch das zugrundeliegende System von Faltungsprozessen erklären, welches einen Einfluss des Sekundärelektronen-Transports mit sich bringt. Eine Fourieranalyse lässt erkennen, dass die Gaußschen Faltungskerne *im klinisch relevanten Bereich der Ortsfrequenzen* gut geeignete Näherungen für die wahren Dosis-Ansprechfunktionen sind, wie am Beispiel der in dieser Arbeit durch Monte-Carlo Simulation berechneten Ansprechfunktionen einer zylindrischen Kammer, einer quadratischen Kammer und einer Flachkammer gezeigt wird. Diese Arbeit ist als Ausgangspunkt einer weiteren Publikation vorgesehen, in der die entsprechenden Entfaltungsverfahren beschrieben werden.

Keywords: Photon-beam dosimetry, ionization chambers, dose response function, Gaussian distribution, standard deviation, Monte-Carlo simulation, Fourier analysis

Schlüsselwörter: Dosimetrie von Photonenstrahlung, Ionisationskammern, Dosis-Ansprechfunktion, Gaußverteilung, Standardabweichung, Monte-Carlo Simulation, Fourieranalyse

Short title: The dose response functions of ionization chambers

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1. Introduction

Frequently, the cross-beam signal profiles and field-size dependent output ratios of photon beams measured with ionization chambers are inconsistent with the results of measurements made by other detectors with better spatial resolution such as radiochromic films, Si diodes, diamond detectors and thermoluminescence detectors or by ionization chambers of other shapes and sizes. This loss of spatial resolution is correlated with the spatial extension of the air volumes of the ionization chambers. In wide photon fields this effect influences the slopes of the measured dose profiles, and in narrow fields it also affects the dose values measured in the field center. One possible solution to this problem is to use Si diodes, diamond detectors or radiochromic films whenever high spatial resolution is required [6, 10, 11, 13, 24, 29, 30, 36, 37, 38, 44, 45]. However, some of these detectors have energy-dependent responses [13, 15, 21, 24, 26,] and thus need to be calibrated by ionization chambers under small-field conditions [1, 2, 8, 43]. Precise knowledge of the spatial resolution of the reference ionization chamber is a precondition for such cross-calibration. Other possibilities are to continue the measurement of photon-beam dose profiles with ionization chambers, correcting however for their limited spatial resolution, or to use them without corrections as long as these remain negligible.

These considerations basically necessitate the quantification and understanding of the spatial resolution of extended detectors such as ionization chambers. For the quantification of the spatial resolution, the *measured signal profile* $M(x)$ obtained by scanning the detector in the direction of a coordinate x in the presence of a *true dose profile* $D(x)$ is usually described as resulting from the convolution

$$M(x) = K(x) * D(x) \quad (1)$$

where the convolution kernel $K(x)$ is called the *dose response function of the detector* in the x direction [20]. In the short-hand notation used in eq. (1), the star symbolizes the convolution integral. This convolution may be generalized to the two-dimensional case

$$M(x,y) = K(x,y) * D(x,y) \quad (2)$$

where $K(x,y)$ is the two-dimensional dose response function [20]. In experimental studies of the dose response function, the *normalized one-dimensional Gaussian distribution*

$$K(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right) \quad (3)$$

has often been observed [3, 16, 18, 19, 26, 27, 31, 42, 46], and for a number of detectors, the values of the standard deviation σ have been published [3, 16, 18, 31, 46].

Most of these studies were focused on the dose response functions of cylindrical ionization chambers in their *lateral* direction - the direction perpendicular to their symmetry axis - thereby providing the data needed to deconvolve one-dimensional signal profiles measured *with the chamber axis at right angles with the scan direction* [18, 19, 23, 25, 26, 42, 46]. However in small fields the dose values measured in the field center are simultaneously subjected to the effect of limited spatial resolution in the *longitudinal* direction of the chamber. Thus there is a need to measure the dose response function also for the case that a dose profile is scanned by a cylindrical ionization chamber *with its cylinder axis oriented in the scan direction* [16]. Therefore, the first task of this study is devoted to measure the σ_{lat} and σ_{long} values for a set of cylindrical ionization chambers with different sizes and from different manufacturers. Furthermore, the growing interest to use flat ionization chambers with their good depth resolution even in photon-beam dosimetry and to apply 2D ionization chamber arrays for the quick measurement of dose profiles have raised concerns about their spatial resolution and have shown the necessity to measure their dose response functions.

The second task to be accomplished in the present investigation is to provide the answer to the question: Are the dose response functions of ionization chambers really Gaussian or non-Gaussian? On one hand, it is at first sight surprising that the tails of the Gaussian dose response functions determined experimentally are reaching *beyond the boundaries of the sensitive volume*. In fact, not only some earlier theoretical models [7, 23, 40] and even modern Monte-Carlo studies [9, 39] have modeled perturbation factors *confined to the geometrical boundaries of the detectors*. However, Pappas *et al* 2006 [31] and van't Veld *et al* [43] have interpreted the dose response function of a detector as *resulting from both volume averaging and secondary electron transport effects*, and this consideration will be further studied. We will show that the dose response function is part of a system of convolution kernels, linking the true dose profile in the phantom as well as the signal profile of the detector to the photon fluence profile and therefore incorporating the effects of secondary electron transport. This will also be the place to clarify the relationship between the dose response functions of ionization chambers used in photon-beam dosimetry and the signal profiles frequently measured by means of slit-irradiation experiments [17, 22, 32-35] or obtained by Monte-Carlo calculations simulating such experiments [4, 5, 20].

The study of this system of convolution kernels, including our own Monte-Carlo simulations, however suggests that the true dose response functions of ionization chambers should be *non-Gaussian* in shape. The key to reconcile this finding with the experimentally observed, *clearly Gaussian* dose response functions will be the Fourier analysis of this

problem, accounting for the spectral bandwidth of the dose profiles occurring in radiotherapy with high-energy photons.

The present investigation of the dose response functions of ionization chambers, aiming at the avoidance of measurement errors due to the limited spatial resolution of detectors, complements our 3-dimensional study of detector resolution problems, started by a study of the shift of the effective point of measurement [28]. The algorithms developed to compensate the systematic errors arising from the limited spatial resolution, particularly a method of numerical deconvolution, will be dealt with in a forthcoming publication.

2. Measurement of ionization chamber dose response functions

2.1 Experimental arrangement

Measurements of signal profiles $M(x)$ were performed at a Siemens Primus accelerator (Siemens, Erlangen, Germany) equipped with a MLC consisting of 58 leaves, each projecting to 10 mm width at the isocenter plane (except the outer leaf pairs with a projection width of 6.5 cm at isocenter plane). The investigation was carried out for nine cylindrical ionization chambers with diameters of the sensitive volume ranging from 2 to 6 mm, two parallel plate chambers and two 2D ionization chamber arrays. The geometrical details of the investigated chambers are summarized in Table 1. In order to obtain signal profiles with almost negligible modification by limited spatial resolution of the detector, measurements were also carried out using an unshielded Si diode detector with a 1 mm x 1 mm sensitive area (type 60012, PTW-Freiburg).

Signal profiles were obtained by scanning the detectors across photon beams in a water phantom tank (MP3 Phantom Tank, PTW-Freiburg) equipped with stepper motors having positioning accuracy of ± 0.1 mm, see Figure 1. The measurement sequence was programmed and controlled with the MEPHYSTO mc² software (vers. 1.8, PTW-Freiburg). The scanning step width was chosen as 0.5 mm with an integration time of 0.5 s per measurement point. This scanning resolution is sufficient as the penumbra widths of the profiles investigated lie in the range of a few millimeters. The cylindrical ionization chambers were aligned with their symmetry axes either perpendicular or parallel to the scan direction in order to obtain their lateral and longitudinal dose response functions, and these functions were characterized by the associated standard deviations σ_{lat} and σ_{long} . The flat chambers and the 2D arrays were oriented with their entrance planes normal to the beam axis.

For the determination of σ_{lat} with the necessary experimental sensitivity, a narrow photon beam of 1 cm width and 5 cm length was collimated using a combination of a 2×40 cm² MLC opening with tertiary lead blocks whose lower plane was arranged only 3 cm above

the water surface, and the chamber or diode was scanned along the short axis of the slit beam (Figure 1a). For chambers no. 1 to 9 (Table 1), the measurements were performed at a source-to-surface distance (SSD) of 67.5 cm, and the detectors were positioned at the depth of maximum dose, 1.5 cm at 6 MV and 2.5 cm at 15 MV, to ensure full secondary electron build-up. Due to the commonly larger dimension of the investigated ionization chambers in the longitudinal direction, the determinations of σ_{long} for the same chambers were carried out at SSD 100 cm by scanning a lengthy rectangular beam of field size $4 \times 40 \text{ cm}^2$ along its short axis at 5 cm and 15 cm depth using 6 and 15 MV photon beams (Figure 1b). For chambers no. 10 to 13 (Table 1), the determination of their σ values was performed at SSD 100 cm and at field size $4 \times 40 \text{ cm}^2$. The *lengthy* field shapes were chosen in order to merely measure the detector signal profiles along the short axes of these fields, avoiding interference with the spatial resolution of the detectors in the direction perpendicular to the scanning direction.

2.2 Determination of parameters σ_{lat} and σ_{long} for a set of ionization chambers

Due to the constructions of the cylindrical ionization chambers investigated in this study, their dose response functions $K(x)$, at least in the lateral direction, should have the form of symmetrical, bell-shaped functions. After testing various types of functions, such as the Lorentz function, we finally followed the proposals by other authors [3, 16, 18, 31, 46] that the lateral and longitudinal dose response functions should be well represented by Gaussian functions. In order to test their suitability in a first approximation - to be improved in a final step - it was assumed that the signal profiles obtained by scanning the Si diode with its 1 mm x 1 mm wide sensitive element over the test beam would yield the true dose profiles $D(x)$ [13, 21]. These profiles were numerically convolved with normalized one-dimensional Gaussian kernels $K(x)$ of the form of eq. (2) using the convolution function of MATLAB (The Mathworks Inc, USA). The profiles $K(x) * D(x)$ resulting from this operation were compared with the measured signal profiles $M(x)$. The value of σ was then varied stepwise and systematically, and the goodness of fit between the convolved dose profile $K(x) * D(x)$ and the measured function $M(x)$ was determined graphically for each value of σ . From the three best fitting σ values, the central one was taken as the result of the measurement. The uncertainty on σ was estimated to be 0.02 mm, since changing σ by that amount had negligible impact on the goodness of fit.

In the final step of the measurement process, the resolution of the Si diode with its 1 mm x 1 mm wide sensitive element was accounted for as a correction. The dose response

function of the Si diode was obtained by comparing the dose profiles of a 6 MV slit beam (FWHM =2.5 mm) at SSD 73 cm and 15 mm depth in water when measured with EBT3 radiochromic film and with the Si diode detector. The EBT3 measurement film was scanned together with the calibration film using an EPSON 10000 XL scanner in the transmission mode at a pixel width of 0.127 mm and a resolution of 200 dpi. The pixel values in the red and blue channels were extracted from the scanned images; the former were used for the dose evaluation and the latter to correct for the difference in coating thickness [25]. For the calibration films, the mean pixel values within an area of 30 pixels x 30 pixels were used, and the calibration curve was then fitted by a fifth degree polynomial. The dose profile measured with the film was averaged over 5 pixels on both sides of the central axis to reduce the statistical noise. The EBT3 dose profile was convolved with a Gaussian function, of which the standard deviation was varied systematically until the best fit with the diode measured signal profile was achieved. The optimal σ_{Diode} value was found to be (0.30 ± 0.02) mm. This value is not noticeably changed by the resolution of the film-scanner system ($\sigma = 0.054$ mm). In all following determinations of the dose response functions of ionization chambers, the resolution of the diode was accounted for by adding in quadrature the σ values of the examined ionization chambers obtained in the first approximation and the value σ_{Diode} of the diode. This method utilizes the advantages of the Si diode such as its easy handling in a water phantom, hence allowing fast profile acquisitions on different days of measurements, while a film measurement with all the necessary circumstances such as calibration and scanning was only necessary to study the resolution of the diode.

2.3 *Resulting standard deviation values of the chambers' dose response functions*

The examples displayed in Figures 2 to 6 illustrate the performance of the determination of the detectors' dose response functions by comparison between the signal profiles measured with the Si diode (thick lines) and the signal profiles broadened by the dose response functions of the ionization chambers (measured points). Figure 2a and b show the signal profiles of a 1 cm wide slit beam at 6 MV and 15 MV, measured with the Si diode and with the PTW 31013 Semiflex chamber scanned in its lateral direction. The results of the convolution of the Si diode profile with a Gaussian kernel after searching for the best fit are presented as thin lines. The σ_{lat} values of the Gaussian kernel giving the best fit between the result of the convolution and the profile measured by the Semiflex chamber are marked in the figures. A Gaussian kernel was also satisfactorily transforming a 4 cm wide, diode measured dose profile into the signal profile of the PTW 31013 Semiflex chamber scanned along its long axis, although the length of its air volume is 16.25 mm (Figure 2c and d). The local

differences between the convolved diode and the semiflex chamber measurements are plotted below each panel in percent of the profile's maximum value. These deviations are in most cases less than 2% of the maximum. Figure 3 shows the results of the measurements performed by scanning the IBA CC13 ionization chamber in its longitudinal direction across the 4 cm wide dose profiles at 6 MV and 15 MV and at two different depths (5 cm and 15 cm). For visual comparison, all dose profiles are normalized at their maxima.

In all of the 36 scanning experiments done with the chambers no 1 to 9, the agreement between the measured signal profiles of the cylindrical ionization chambers and the convolution products of the Si diode profiles with Gaussian kernels was of the same quality as shown in Figures 2 and 3. Hence, the initial assumption that the detector's dose response functions can be closely described by Gaussian functions was confirmed experimentally.

Table 1 shows the values of σ_{lat} and σ_{long} determined for all cylindrical ionization chambers investigated in this work and corrected by adding σ_{Diode} in quadrature: $\sigma_{\text{lat}}^2 = \sigma_{\text{lat}}^{*2} + \sigma_{\text{Diode}}^2$, where σ_{lat}^* is the value obtained in the first approximation through comparison with the diode profile. From the chambers with the smallest diameters (PTW 31014 PinPoint and IBA CC01) up to the longest ionization chamber (PTW 31013 Semiflex), the 2σ values are close to but almost always slightly lower than the cavity dimensions in the scanning direction, which means that the "tails" of the Gaussian dose response functions are reaching into the exterior of the chambers. Furthermore, the dose response functions at the higher photon energy (15 MV) are generally slightly wider than at 6 MV. These phenomena indicate the influence of the secondary electron ranges on the chambers' dose response functions (see the discussion).

Furthermore for the two flat chambers and the single chambers of the two 2D ionization chamber arrays, the signal profiles $M(x)$ obtained by scanning them across the 4 cm wide beams could be accurately fitted by convolving the $D(x)$ curves with Gaussian kernels as shown, e.g., for the Roos chamber and for the 729 chamber array (PTW 10024) in Figures 4 and 5. The resulting σ values of the flat and array chambers, corrected with σ_{diode} by addition in quadrature, have also been entered in Table 1.

2.4 The depth dependence of the detector's dose response function

We also investigated whether there were detectable depth dependent changes of the convolution kernels. Figure 6 illustrates that no changes of the σ values with depth have been detected for 6 MV (upper panels) as well as 15 MV (lower panels), even though the signal profiles were subjected to their normal depth-dependent geometrical broadening. This

invariance of the σ values was to be expected since all examined depths were larger than the depth of the full secondary electron fluence build-up, so that the spectral and directional distributions of the secondary electrons would not change with depth.

2.5 2D dose response function

When an ionization chamber is applied at a point of measurement in a narrow photon beam, e.g. at a point upon the beam axis, its signal will be affected by its limited spatial resolution in *both the lateral and the longitudinal directions*, and one may ask for its 2D dose response function. The experimental layout in which each examined cylindrical ionization chamber was scanned across a lengthy photon field (see section 2.1) had been chosen so that the lateral and longitudinal dose response functions were measured *separately*. The 1D dose response functions thereby obtained,

$$K(x) = \frac{1}{\sqrt{2\pi}\sigma_{lat}} \exp\left(-\frac{x^2}{2\sigma_{lat}^2}\right) \quad (4)$$

and

$$K(y) = \frac{1}{\sqrt{2\pi}\sigma_{long}} \exp\left(-\frac{y^2}{2\sigma_{long}^2}\right), \quad (5)$$

are therefore to be regarded as the 1D projections of a 2D dose response function

$$K(x, y) = \frac{1}{2\pi\sigma_{lat}\sigma_{long}} \exp\left[-\left(\frac{x^2}{2\sigma_{lat}^2} + \frac{y^2}{2\sigma_{long}^2}\right)\right]. \quad (6)$$

In the practical use of the ionization chambers, when photon fields may obtain any cross section, the 2D signal profile $M(x, y)$ will then be the result of the 2D convolution

$$M(x, y) = D(x, y) * K(x, y) \quad (7)$$

where $K(x, y)$ is the 2D convolution kernel, eq. (6). In the case of the flat chambers, which have circular collection electrodes, the 2D point spread function is the rotation-symmetrical special case of eq. (6),

$$K(r) = \frac{1}{\pi\sigma_r^2} \exp\left(-\frac{r^2}{\sigma_r^2}\right) \quad (8)$$

with $\sigma_r^2 = 2\sigma_x^2 = 2\sigma_y^2$. Table 1 shows the σ_x values obtained by scanning the flat chambers across lengthy fields of 4 cm width.

3. Discussion: Gaussian convolution kernels and their origin

3.1 Gaussian convolution kernels as a general result

In summary, our experiments have shown that the lateral and the longitudinal dose response functions of all investigated *cylindrical ionization chambers* can be characterized by Gaussian functions. This confirms the results of Garcia-Vicente *et al* 1998 [18], who investigated the convolution kernel of an ionization chamber of 4 mm diameter comparing its measured signal profiles with the true dose profiles obtained with film, corrected for the densitometer resolution. They compared the suitability of parabolic and Gaussian convolution kernels and found that the latter would better describe the detector's dose response function. Bednarz *et al* [3], Ulmer and Kaissl [41], Pappas *et al* [31], Yan *et al* [46] and Fox *et al* 2010 [16] also concluded that the best fitting function to describe the detectors' convolution kernels is the Gaussian distribution. Beyond that, it has now been shown that the convolution kernels characteristic for *flat chambers* and for the *single chambers of 2D arrays* also have Gaussian shapes.

Our method of systematic parameter variation to measure the values of σ_{lat} and σ_{long} , i.e. to convolve the diode-measured $D(x)$ with a Gaussian kernel under systematic variation of its standard deviation σ , and to choose the σ value providing the best fit with the chamber-measured $M(x)$, can be used to obtain the dose response functions of any ionization chamber. Yan *et al* 2008 [46] used a similar approach to obtain the σ value for the IBA CC13 chamber by minimizing the differences between ionization chambers' signal profiles and the convolution of diode-measured dose profiles with Gaussian kernels at different field sizes and depths. They obtained the σ_{lat} values 2.71 and 2.79 mm, and no significant depth or field size dependence of σ_{lat} was detected. Fox *et al* 2010 [16] investigated the σ_{lat} and σ_{long} values for the IBA CC01, CC04 and CC13 chambers using analytical fits to both $D(x)$ and $M(x)$. They scanned the signal profiles of a 10 x 10 cm² field in both the cross-plane and in-plane directions at 6 MV. The average σ_{lat} and σ_{long} values for the CC01, CC04 and CC13 chambers were 1.52, 1.78 and 2.57 mm and 1.72, 1.99 and 2.64 mm respectively. The 2σ values were comparable to the chambers' dimensions, as also shown in this study, but the σ values were significantly higher than the corresponding values obtained in this work (Table 1). This is probably due to the fact that we used lengthy photon fields in order to measure the 1D dose response function of a detector under exclusion of any effects of its dose response function at right angles to the direction of the examination. We also avoided fitting analytical functions such as the error function to the measured $D(x)$ and $M(x)$ profiles, knowing that the convolution kernel determining $D(x)$ is closer to a Lorentz function than to a Gaussian function (Djouguela *et al* [14]). De Vlamynck *et al* [12] showed that considerable dose profile broadening was caused by scanning narrow beam profiles with a Markus chamber.

This is consistent with our result $\sigma \approx 2.4$ mm (see Table 1), and we expect a similar penumbra broadening as shown in this paper in Figure 2a and b for the chamber PTW 31013 in the lateral direction ($\sigma_{\text{lat}} \approx 2.4$ mm).

3.2 The lateral tails of the Gaussian dose response function

The fact that the lateral and longitudinal dose response functions of air-filled ionization chambers have the shape of Gaussian functions has a somewhat astounding aspect insofar as these functions have non-zero valued "tails" *reaching beyond the chambers' geometrical boundaries*, although the signal is exclusively produced by ionizations *within* the chamber's air volume. In order to understand this phenomenon, we have to consider that normally the sensitive volume of a detector is *not* the same as just a geometrically circumscribed region of the surrounding medium, simply collecting the elements of the total signal from all over its detection volume, an effect termed "volume averaging" [4,9] or "voxel volume effect" [39]. Rather, the dose response function $K(x)$ of a detector [20] is correlating the detector signal profile $M(x)$ with the true dose profile $D(x)$ which would be generated in the surrounding medium *in the absence of the detector*. This means that the relationship between $M(x)$ and $D(x)$ is an intricate one, presumably involving effects of secondary electron transport in the surrounding medium and the perturbation of the secondary electron field by the detector [31, 43]. An explanation involving the transport of secondary electrons would also be in line with the slight energy dependence of the σ values measured in this work and with their observed invariability with depth as long as the σ measurements were done at depths with full secondary electron build-up.

The relationship between $M(x)$ and $D(x)$ can in principle be described by the diagram shown in Fig. 7. The true dose profile $D(x)$ in a plane at a given depth of the phantom can be regarded as resulting from the photon fluence profile $\Phi(x)$ in the same plane by convolution with a kernel $K_D(x)$, which involves the effects of the downstream, lateral and backward transport of secondary electrons and is known to be closely described by Lorentz-shaped convolution kernels [14]. On the other hand, the measured profile $M(x)$ results from $\Phi(x)$ by convolution with a kernel $K_M(x)$. Such kernels, measurable by scanning narrow slit photon beams over the investigated detectors [17, 22, 32-35] or obtained by simulating such scans by Monte Carlo calculation [4, 17, 20, 43], are coined by details of the detector geometry in combination with secondary electron transport into the sensitive volume from the detector surrounding, from its walls and its central electrode. Therefore the relationship between $D(x)$ and $M(x)$ is an *indirect* one, mediated by $\Phi(x)$. By application of Fourier's convolution theorem to the three convolutions indicated in Figure 7, one can calculate that

$$\text{FT}[K(x)] = \frac{1}{\sqrt{2\pi}} \frac{\text{FT}[K_M(x)]}{\text{FT}[K_D(x)]} \quad (9)$$

where FT represents the Fourier transform of a kernel, see also [4, 17]. If the widths of kernels $K_M(x)$ and $K_D(x)$ are taken into account, eq. (9) clarifies that there is no condition requiring $K(x)$ to vanish at the boundary of the chamber's air volume, and that the effects of secondary electron transport affecting $K_M(x)$ and $K_D(x)$ will influence $K(x)$ as well.

3.3 Explanation of the general occurrence of Gaussian convolution kernels using Fourier analysis

In order to understand the general occurrence of Gaussian convolution kernels for all ionization chambers investigated in this study, in the lateral as well as in the longitudinal direction and independent of their shapes, Monte Carlo simulations of $K_M(x)$ and $K_D(x)$ were performed by stepwise shifting a 0.25 mm wide 6 MV slit beam across (i) a cylindrical chamber (PTW type 31010), (ii) a single chamber of the 2D-ARRAY (PTW type 10024) and (iii) a parallel plate chamber, embedded in water at depth $z = 5$ cm (Fig. 8).

The resulting functions $K_M(x)$ and $K_D(x)$ for the PTW Semiflex 31010 exposed at right angles to its axis are shown in Figure 9 (upper left). $K(x)$ was obtained by deconvolving iteratively $K_M(x)$ with $K_D(x)$ (upper right). Both $K_M(x)$ and $K(x)$ exhibit increased response at the positions of the inner electrode and the chamber wall due to the release of secondary electrons into the sensitive volume. This clearly indicates that the true $K(x)$ is *not a Gaussian distribution*, and the upper right panel illustrates the difference against the Gaussian kernel $G(x)$ found experimentally ($\sigma_{\text{lat}} = 2.20$ mm). Similar results have been obtained in earlier studies [17, 20, 43]. But nevertheless, as shown in the lower left panel, $\text{FT}[G(x)]$ *closely resembles* $\text{FT}[K(x)]$ at spatial frequencies below about 0.15 mm^{-1} . Thus the Gaussian kernel $G(x)$ is a *well-suited approximation* to the true kernel $K(x)$ since clinical dose profiles have only negligible components at frequencies higher than 0.1 mm^{-1} [34], a feature attributable to physical and geometrical reasons such as the size of the focus, scattered radiations from the flattening filter and collimators as well as the transport of secondary electrons and small-angle scattered photons in a phantom. This fact is illustrated in the lower right panel for a 2 cm wide dose profile: the Fourier transforms of a 2 cm wide dose profile at 5 cm depth in water $D(x)$ convolved with $G(x)$ or $K(x)$ are not showing noticeable differences.

In an analogous manner, the results of the Monte-Carlo simulation for a single chamber of the PTW 2D-ARRAY are shown in Figure 10. Both $K_M(x)$ and $K(x)$ (upper

panels) of the quadratic chamber show increased response at the chamber's borders when the slit beam is impinging on the ridges adjacent to the chamber's volume, releasing secondary electrons into the sensitive air volume [4, 17, 22, 32-35]. In frequency space, the Gaussian kernel $G(x)$ found experimentally in this study ($\sigma = 2.27$ mm) for the single chamber of the PTW 2D-ARRAY shows again close agreement to the true $K(x)$ at spatial frequencies below about 0.15 mm^{-1} , and the convolution products of $D(x)$ with $G(x)$ and $K(x)$ do not exhibit noticeable differences (lower right panels).

Figure 11 shows the results for a simulated parallel plate chamber. Functions $K_D(x)$ and $K_M(x)$ are similar to earlier results for a flat, air-filled cavity [4]. For the parallel plate chamber, a Gaussian kernel with $\sigma = 4.90$ mm has been shown to closely represent the true $K(x)$ in the frequency domain. For the parallel plate chamber, the convolution products of $D(x)$ with $G(x)$ and $K(x)$ again do not exhibit noticeable differences (lower right panels).

These observations illustrate that Gaussian convolution kernels are *approximations to the true non-Gaussian convolution kernels* for all detector geometries, but not in the usual sense of approximations in normal space, as the Gaussians $G(x)$ are in some details of their shape quite dissimilar to the true kernels $K(x)$ (upper right of Figs. 9, 10 and 11). Rather, the Fourier transforms of the Gaussian kernels approximate the Fourier transforms of the true kernels *in the low-frequency part of the frequency space* (lower left of Figs. 9, 10 and 11), which is the decisive fact since the Gaussians act as convolution kernels to the dose profiles occurring under the conditions of high-energy photon-beam radiotherapy. Thus the Gaussian dose response kernels experimentally observed in a multitude of studies [3, 16, 18, 19, 26, 27, 31, 42, 46] *are approximations adequately characterizing the spatial resolution of ionization chambers* as long as the latter are used to measure the dose profiles of clinical photon beams. Furthermore, the use of Gaussian dose response functions can be regarded as a substantial advantage for practical applications. The Gaussian shape is on one hand a simple rule, easy to recollect, and each chamber can be associated with its own σ values. On the other hand, the development of deconvolution methods and correction factors can be uniformly based on the Gaussian shapes of the convolution kernels. These methods and correction factors, based on Gaussian convolution kernels, will be dealt with in a further publication.

4. Conclusions

The present study of the limited spatial resolution of ionization chambers, occurring when they are used in the dosimetry of narrow photon beams, has shown that the dose response functions of these chambers, determined experimentally from measured dose

profiles, can be generally described by Gaussian distributions, confirming earlier observations. The lateral and longitudinal dose response functions of cylindrical chambers have therefore been characterized by their standard deviations σ_{lat} and σ_{long} . The σ values for nine types of cylindrical chambers, two types of parallel plate chambers and the single chambers of two types of 2D ionization chamber arrays were measured and tabulated. The phenomenon that the lateral tails of the Gaussian dose response functions are extended beyond the geometrical dimensions of the chambers can be explained by the ranges of the secondary electrons and the underlying convolutions. Fourier analysis and consideration of the low-frequency domain occupied by clinical photon dose profiles has revealed the reason for the general occurrence of Gaussian kernels, namely their property as *adequate approximations*. This insight reconciles the existing conflict with the true, non-Gaussian convolution kernels obtained in scanning experiments or by their Monte Carlo simulation and supports the practical use of Gaussian kernels in clinical photon-beam dosimetry.

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List of figure captions

Figure 1: Measurement setup: (a) A 1 cm wide slit beam collimated by tertiary lead blocks. For the determination of σ_{lat} , the ionization chambers were positioned with their symmetry axes perpendicular to the scan direction at the depth of measurement $d_m = 1.5$ cm for 6 MV beam and $d_m = 2.5$ cm for the 15 MV beam; (b) Field size 4×40 cm², achieved using the beam-head collimators. For the determination of σ_{long} , the ionization chambers were positioned with their symmetry axes parallel to the scan direction at $d_m = 5$ cm and 15 cm for both 6 and 15 MV photon beams. The entrance planes of the flat chambers and the 2D arrays were arranged normal to the beam axis.

Figure 2: Determination of the dose response functions of a cylindrical ionization chamber PTW 31013 semiflex. Measurements of the lateral dose response functions were carried out with a 1 cm wide slit beam profile at depths of full secondary electron equilibrium build-up: (a) 6 MV, depth = 1.5 cm; (b) 15 MV, depth = 2.5 cm. Measurements of the longitudinal dose response functions were carried out with a 4 cm wide beam at 5 cm depth: (c) 6 MV; (d) 15 MV. The noted σ_{lat} values were determined by varying the σ values until optimal fit of the thin lines with the measured values was achieved. The σ values corrected for σ_{Diode} have been entered in Table 1. The local differences between the convolved diode and the Semiflex chamber profile values are plotted below each panel.

Figure 3: Determination of the dose response function in the longitudinal direction of the cylindrical CC13 ionization chamber. Beam profiles were obtained by scanning a 4×40 cm² field at (a) 6 MV and 5 cm depth; (b) 6 MV and 15 cm depth; (c) 15 MV and 5 cm depth and (d) 15 MV and 15 cm depth. The σ values corrected for σ_{Diode} have been entered in Table 1.

Figure 4: Determination of the dose response function of the Roos chamber (PTW 34001). Beam profiles were obtained by scanning a 4×40 cm² field at (a) 6 MV and 5 cm depth; (b) 15 MV and 5 cm depth. The σ values corrected for σ_{Diode} have been entered in Table 1.

Figure 5: Determination of the dose response function of a single ionization chamber of the 2D array 729 (PTW 10024). Beam profiles were obtained by scanning a 4×40 cm² field at (a) 6 MV and 5 cm depth; (b) 15 MV and 5 cm depth. The σ values corrected for σ_{Diode} have been entered in Table 1.

Figure 6: Determination of the lateral dose response function of the 31010 cylindrical ionization chamber at three different depths using a 1 cm wide slit beam at energy 6 MV (upper panels) and 15 MV (lower panels).

Figure 7: Diagram illustrating the relationship between the photon fluence profile $\Phi(x)$, the true dose profile $D(x)$, the measured signal profile $M(x)$ and the associated convolution kernels.

Figure 8: Geometry of the ionization chambers investigated using Monte-Carlo simulations (i) PTW 31010 cylindrical chamber; (ii) a single quadratic chamber of the PTW 2D-ARRAY type 10024; (iii) a parallel plate chamber. The 0.25 mm wide slit beam is impinging from above as depicted in the figure.

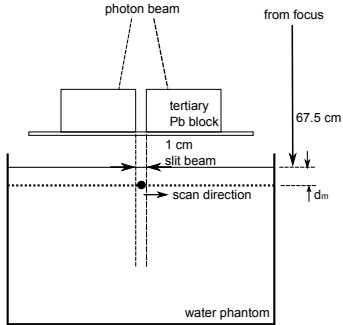
Figure 9: Upper left: Maximum-normalized values of $K_M(x)$ of PTW 31010 cylindrical chamber and $K_D(x)$ according to a Monte Carlo simulation. Upper right: $K(x)$ obtained through iterative deconvolution is compared to its Gaussian approximation $G(x)$ with $\sigma = 2.20$ mm obtained in this study; Lower left: Plot of the maximum-normalized magnitudes of $\text{FT}[K_M(x)]$, $\text{FT}[K_D(x)]$, $\text{FT}[K(x)]$. Lower right: Fourier spectra of $D(x)$ of a 2 cm wide dose profile and of its convolution with $G(x)$ and $K(x)$.

Figure 10: upper left: Maximum-normalized values of $K_M(x)$ of a chamber of the PTW 2D-ARRAY type 10024 and $K_D(x)$ according to a Monte Carlo simulation; upper right: $K(x)$ obtained through iterative deconvolution is compared to its Gaussian approximation $G(x)$ with $\sigma = 2.70$ mm obtained in

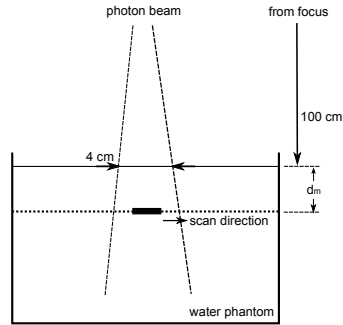
this study; lower left: Plot of the maximum-normalized magnitudes of $FT[K_M(x)]$, $FT[K_D(x)]$, $FT[K(x)]$; lower right: Fourier spectra of $D(x)$ of a 2 cm wide dose profile and its convolution with $G(x)$ and $K(x)$.

Figure 11: upper left: Maximum-normalized values of $K_M(x)$ of a parallel plate chamber with 7.5 mm radius and 2 mm height and $K_D(x)$ according to a Monte Carlo simulation; upper right: $K(x)$ obtained through iterative deconvolution is compared to its Gaussian approximation $G(x)$ with $\sigma = 4.90$ mm; lower left: Plot of the maximum-normalized magnitudes of $FT[K_M(x)]$, $FT[K_D(x)]$, $FT[K(x)]$; lower right: Fourier spectra of $D(x)$ of a 2 cm wide dose profile and its convolution with $G(x)$ and $K(x)$.

Figure1



(a)



(b)

Figure2

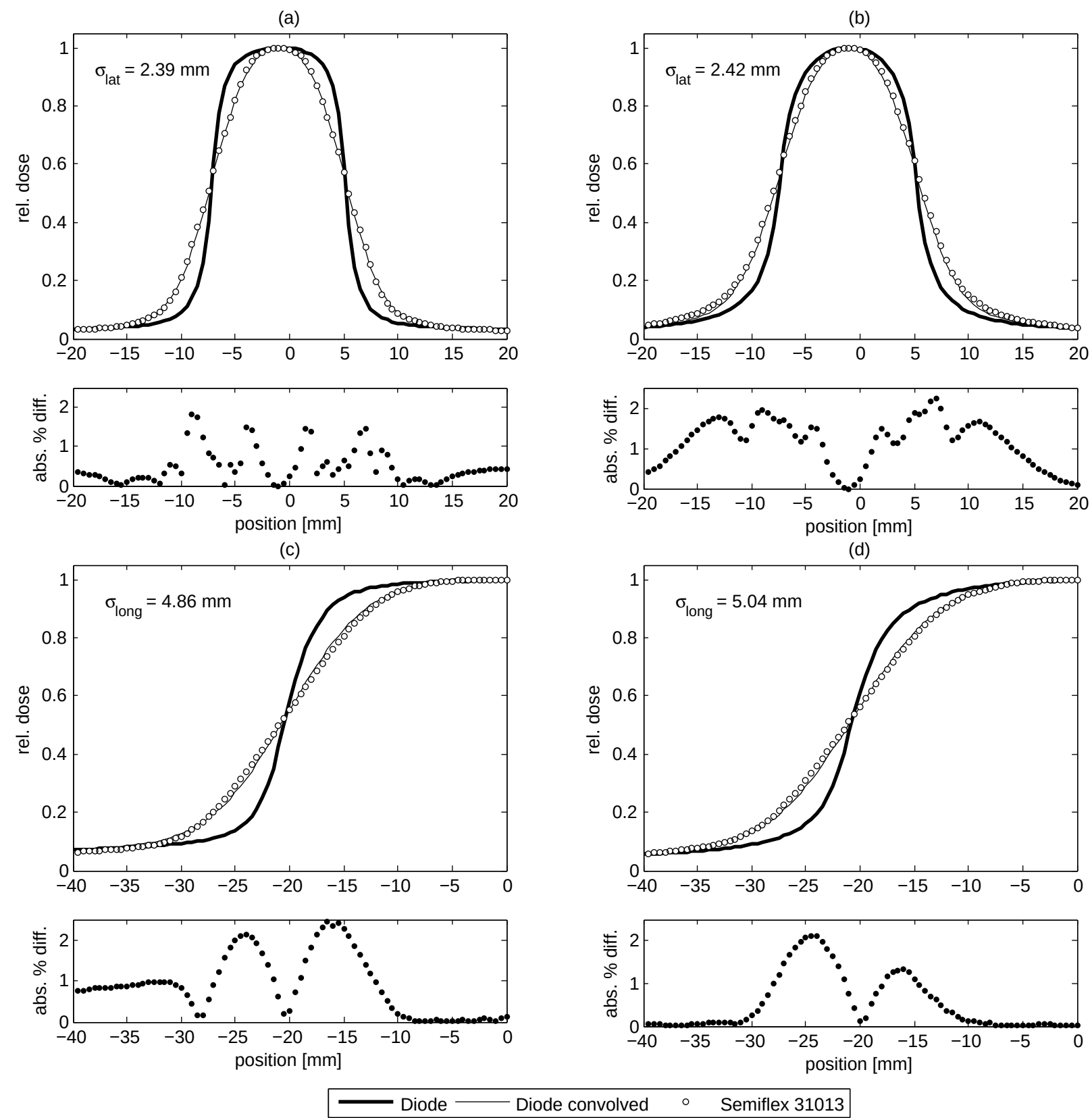


Figure3

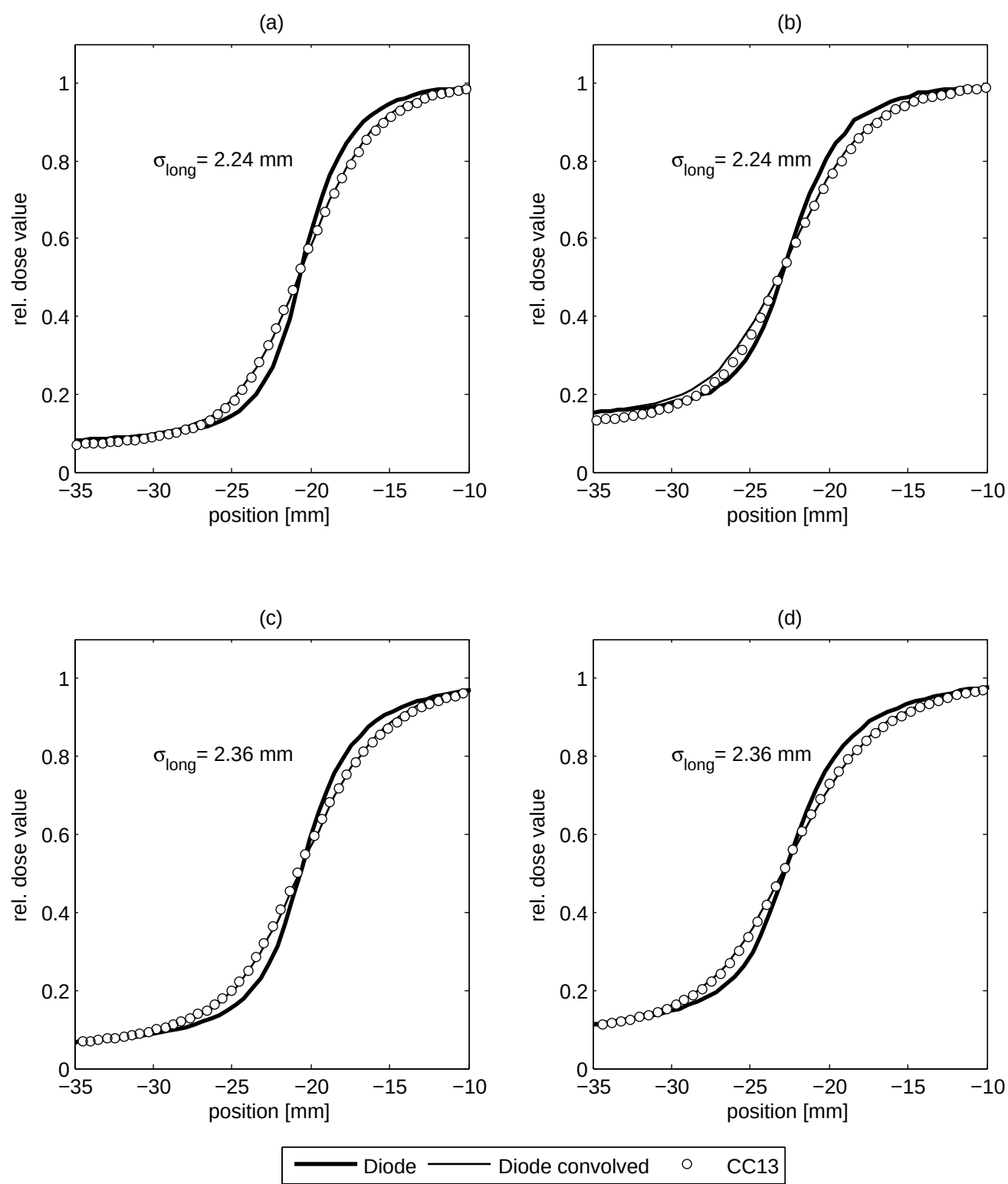


Figure4

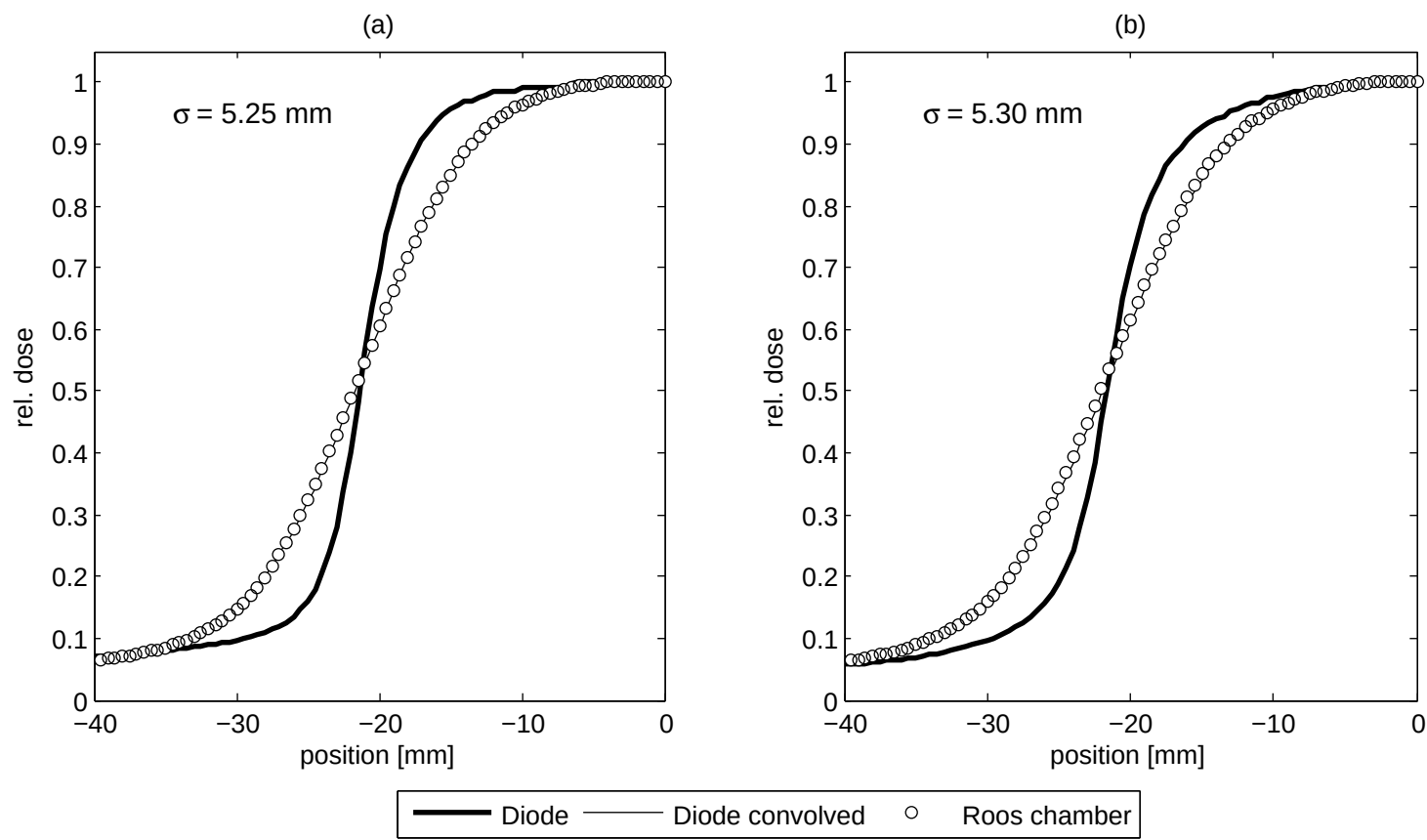


Figure5

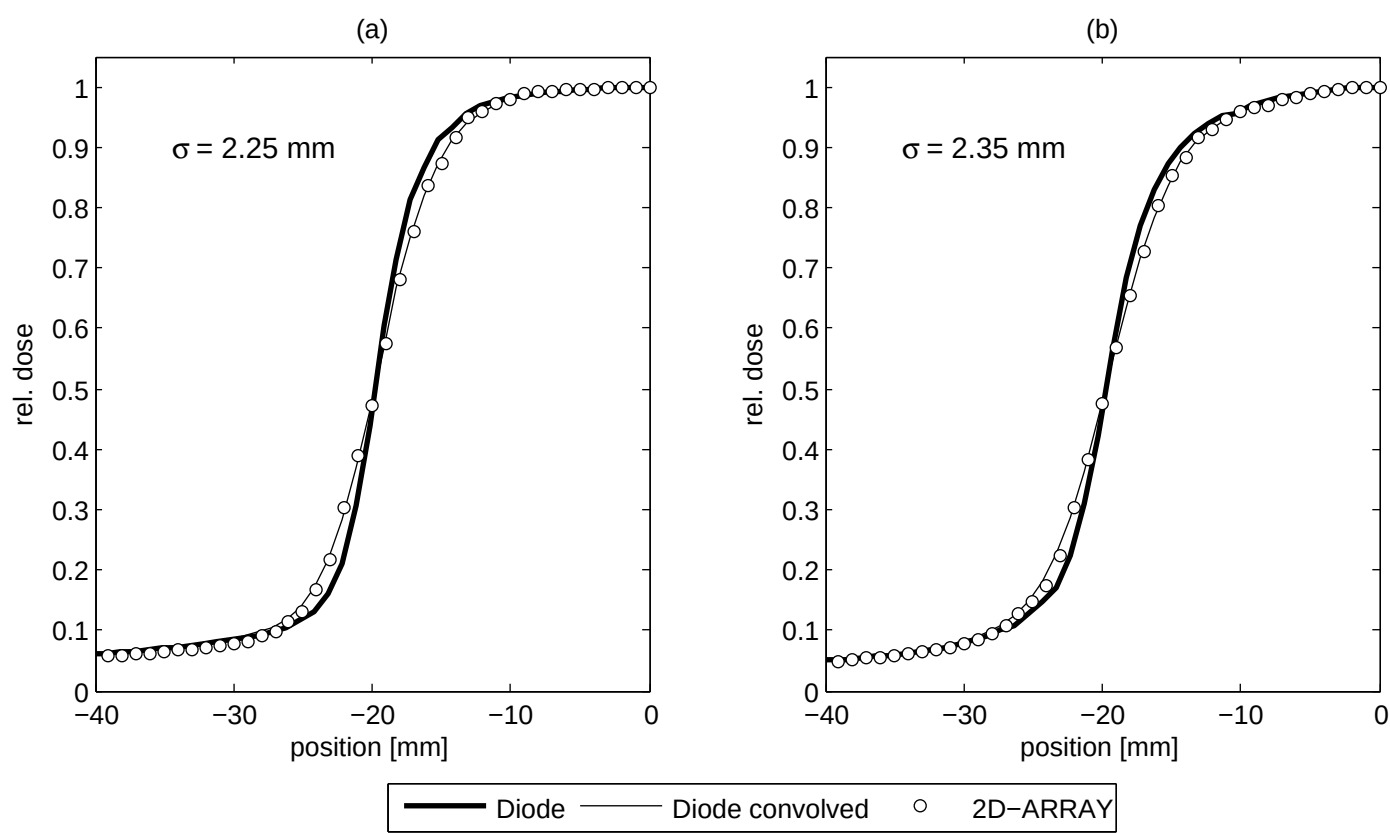


Figure6

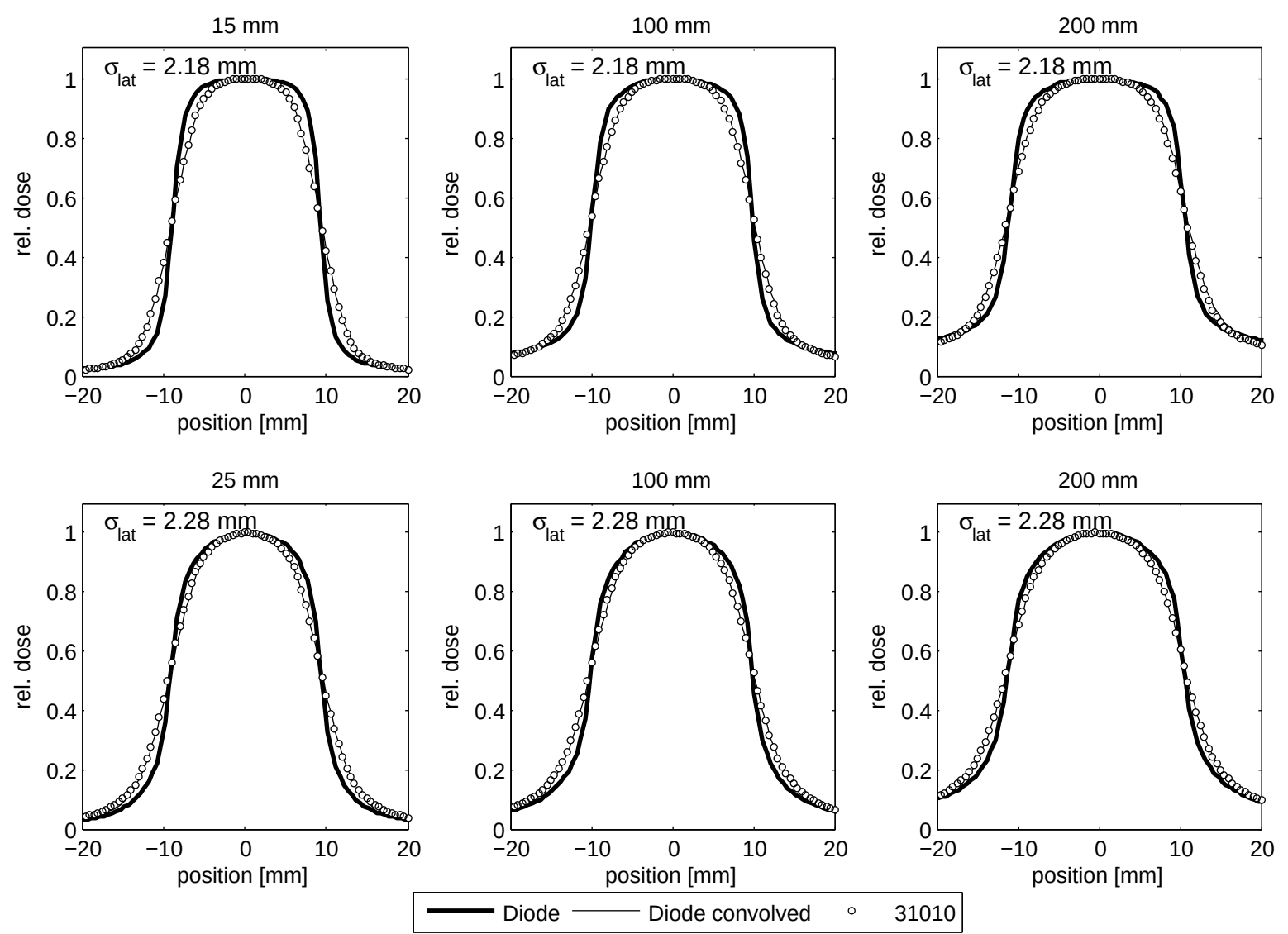


Figure7

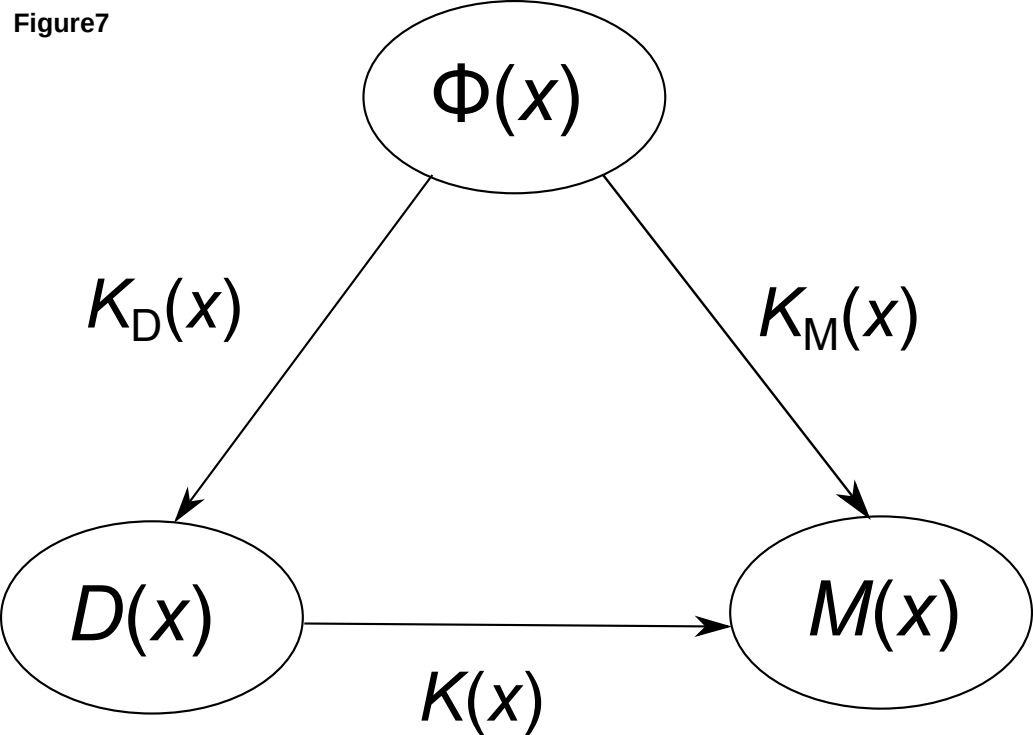


Figure 8

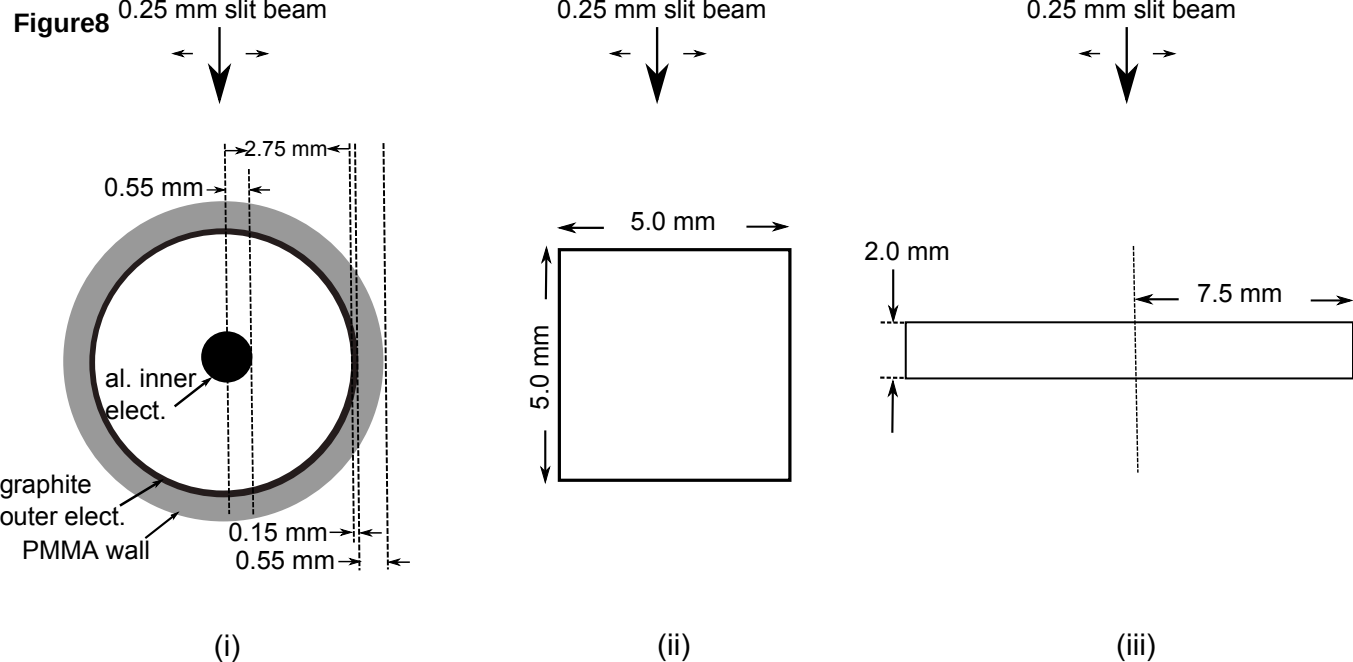


Figure9

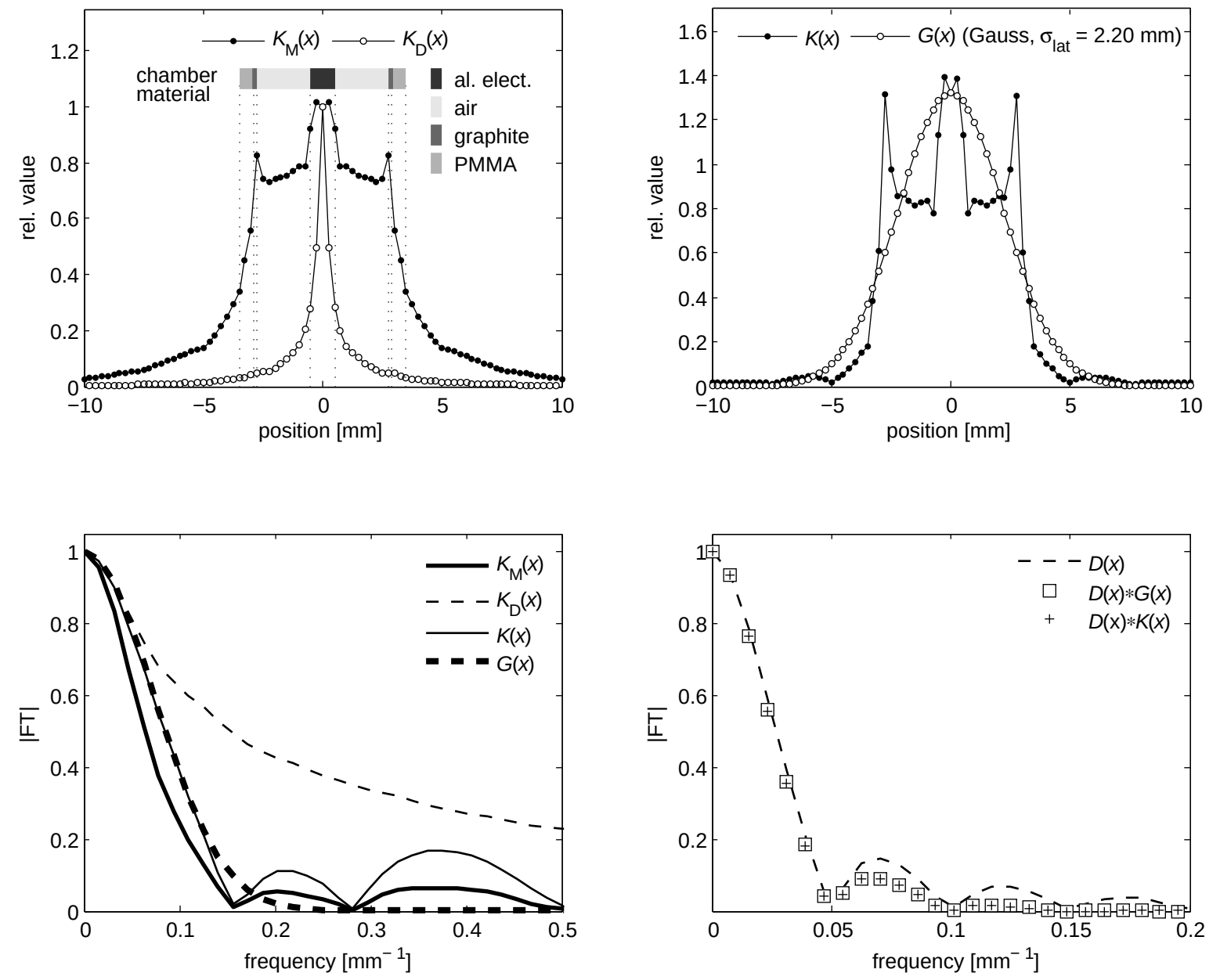


Figure10

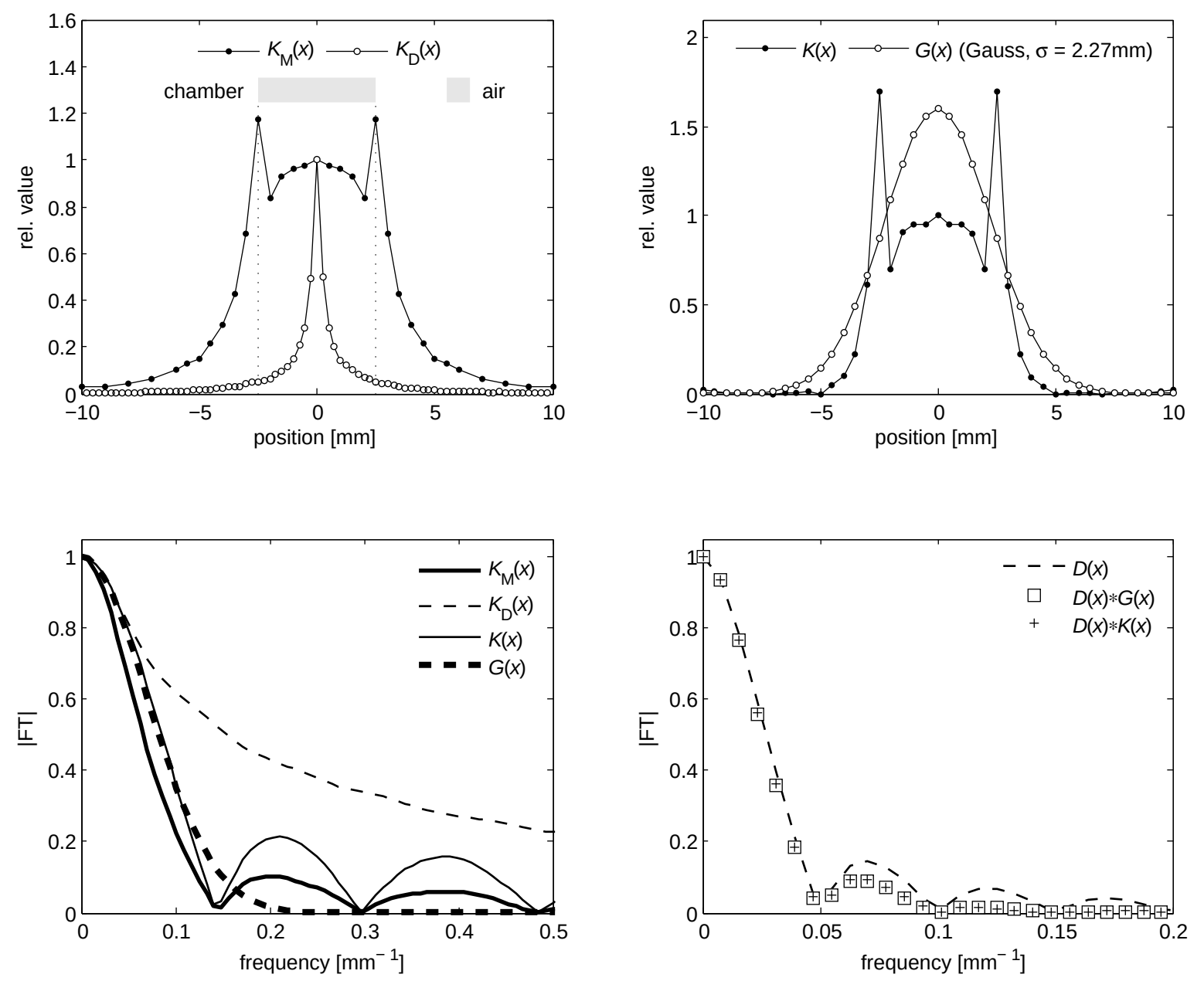


Figure11

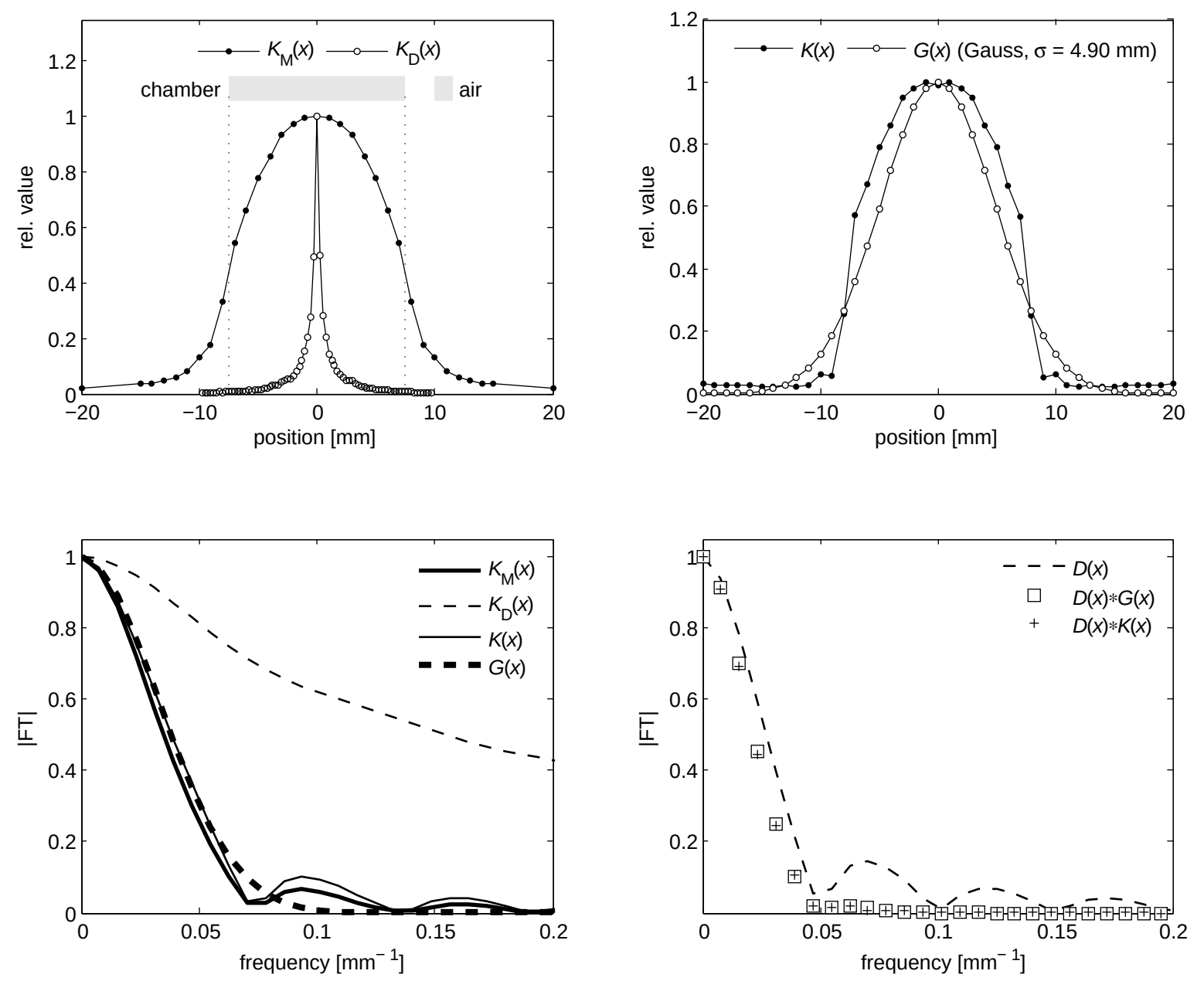


Table 1. The dimensions of the ionization chambers investigated in this work and the associated standard deviations σ_{lat} and σ_{long} of their lateral respectively longitudinal dose response functions.

#	Detector	Sensitive volume [cm ³]	Diameter of sensitive volume [mm]	Length of sensitive volume [mm]	σ_{lat} [mm] 6/15 MV (± 0.02 mm)	σ_{long} [mm] 6/15 MV (± 0.02 mm)
1	PTW 31014 PinPoint	0.015	2.0	5.0	0.99/0.99	1.98/2.02
2	PTW 31015 PinPoint	0.03	2.9	5.0	1.40/1.49	2.23/2.30
3	PTW 31016 PinPoint 3D	0.016	2.9	2.9	1.38/1.38	1.78/1.90
4	PTW 31010 SemiFlex	0.125	5.5	6.5	2.20/2.30	2.28/2.50
5	PTW 31013 SemiFlex	0.3	5.5	16.25	2.41/2.44	4.87/5.05
6	IBA CC01	0.01	2.0	3.6	0.69/0.71	1.18/1.24
7	IBA CC04	0.08	4.0	3.6	1.49/1.51	1.49/1.51
8	IBA CC08	0.08	6.0	4.0	2.04/2.22	1.73/2.00
9	IBA CC13	0.13	6.0	5.8	2.26/2.38	2.26/2.38
10	PTW 23343 Markus	0.055	5.3 ^a	-	2.37/2.45	-
11	PTW 34001 Roos	0.35	15.0 ^a	-	5.26/5.31	-
12	PTW 10024 729 2D-ARRAY	0.125	5.0 ^b	5.0 ^c	2.27/2.37	2.27/2.37
13	PTW 10032 STARCHECK	0.05	2.0 ^b	5.0 ^c	1.63/1.73	-

^a parallel plate chamber; ^b width of a single array chamber; ^c length of a single array chamber