

NONPARAMETRIC ANALYSIS

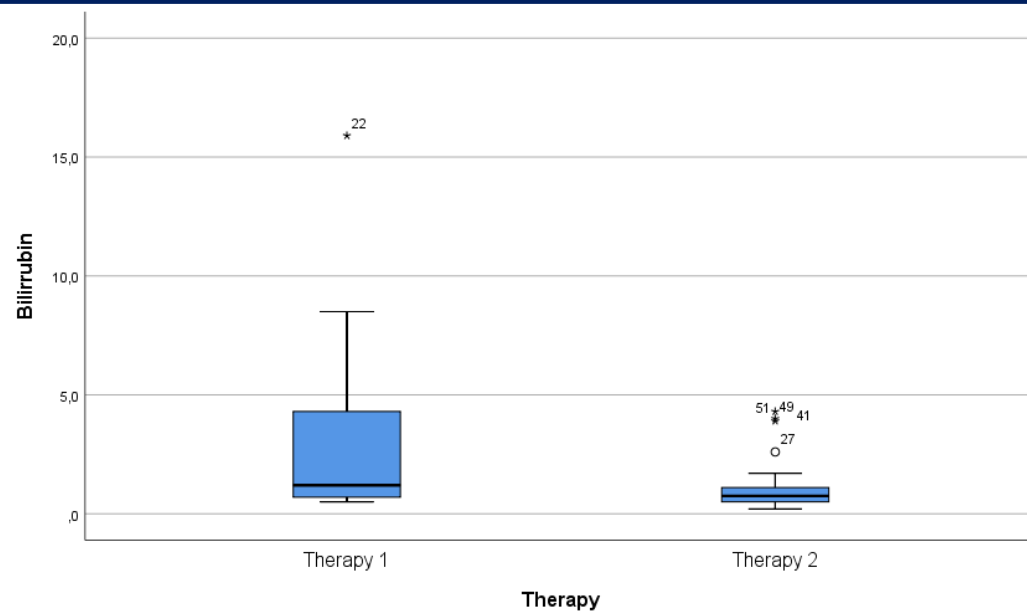
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DATASET

Verarbeitete Fälle

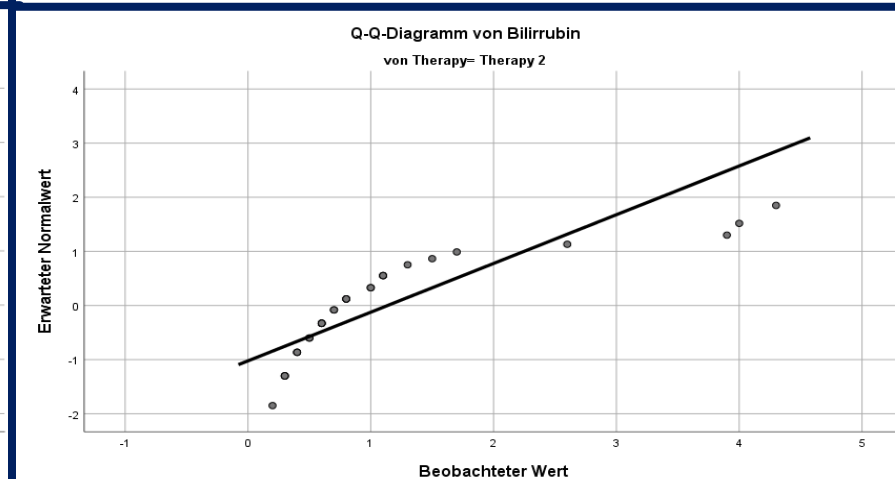
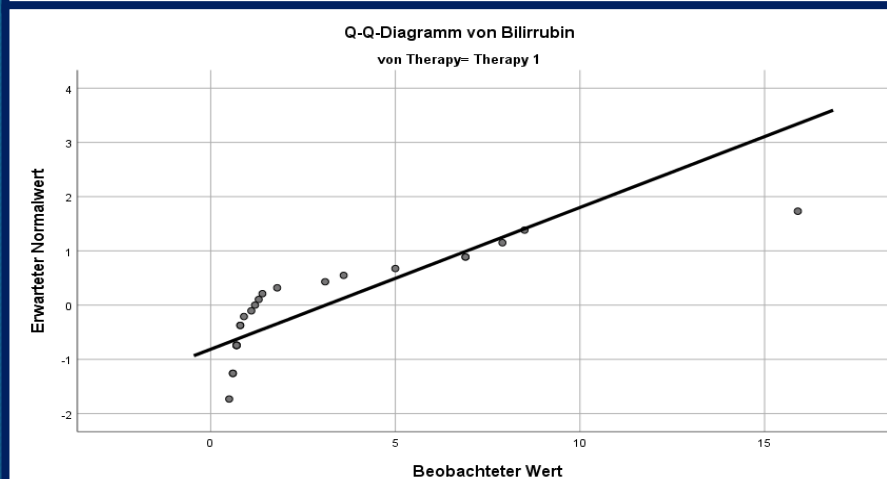
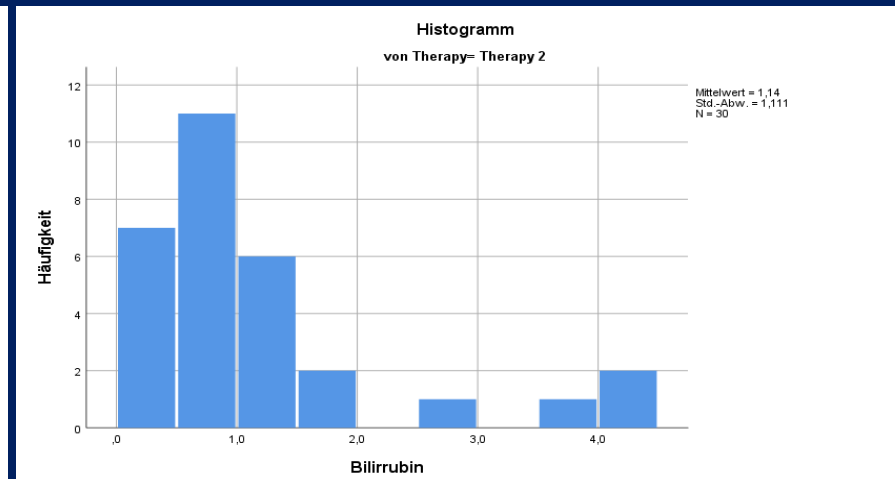
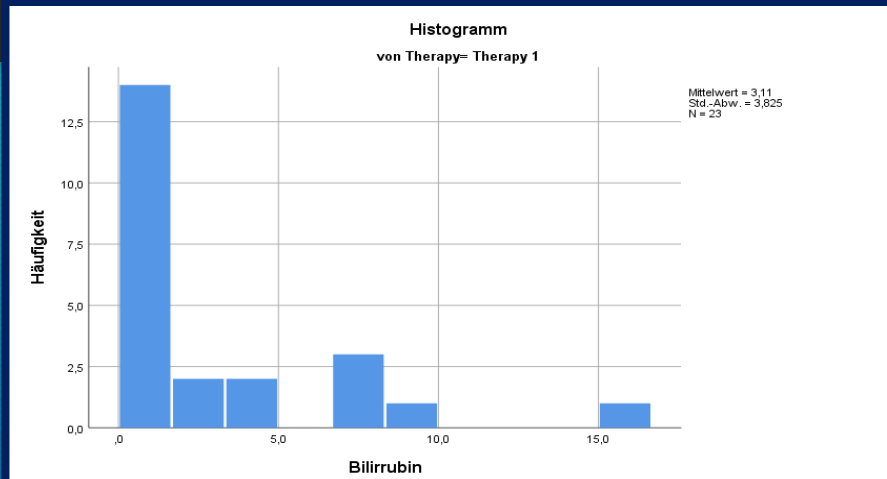
		Gültig		Fälle Fehlend		Gesamt	
	Therapy	N	Prozent	N	Prozent	N	Prozent
Bilirubin	Therapy 1	23	95,8%	1	4,2%	24	100,0%
	Therapy 2	30	100,0%	0	0,0%	30	100,0%



Deskriptive Statistik

Therapy		Statistik		Std.-Fehler
Bilirubin	Therapy 1	Mittelwert		3,113
		95% Konfidenzintervall des Mittelwerts	Untergrenze	1,459
			Obergrenze	4,767
		5% getrimmtes Mittel		2,601
		Median		1,200
		Varianz		14,633
		Std.-Abweichung		3,8253
		Minimum		,5
		Maximum		15,9
		Spannweite		15,4
		Interquartilbereich		4,3
		Schiefe		2,048
		Kurtosis		4,612
	Therapy 2	Mittelwert		1,137
		95% Konfidenzintervall des Mittelwerts	Untergrenze	,722
			Obergrenze	1,551
		5% getrimmtes Mittel		1,017
		Median		,750
		Varianz		1,234
		Std.-Abweichung		1,1109
		Minimum		,2
		Maximum		4,3
		Spannweite		4,1
		Interquartilbereich		,7
		Schiefe		2,005
		Kurtosis		3,187

What do we do in these cases?



NON-PARAMETRIC TESTS

- Assumption-free test ??
- They do make distributional assumptions but they are less restrictive ones than their parametric counterparts.
- Non-parametric tests are mostly used when assumptions aren't met for other tests such as ANOVA or T Tests (normal distribution).
- Non-parametric tests are suitable for ordinal variables too.

NON-PARAMETRIC TESTS

- They work on the principle of Ranking the data
- Variable age: 3, 1, 13, 8



- The analysis is then carried out on the ranks rather than the actual data.

NON-PARAMETRIC TESTS

- Kruskal-Wallis Test.
- Friedman's Test.
- Nonparametric Analysis of Longitudinal Data in Factorial Experiments.
- Wald-type statistic (WTS) and ANOVA-type statistic (ATS) (factorial designs).

KRUSKAL-WALLIS TEST

- The Kruskal-Wallis test is an alternative for a one-way ANOVA if the assumptions of the latter are violated.
- It is used for comparing two or more independent samples of equal or different sample sizes.
- A scientist look if that the chemical “genistein”, which is naturally occurring in soya, was linked to lowered sperm counts in males. He took 80 males and split them into four groups that varied in the number of soya meals they ate per week over a year-long period. At the end of the year, all of the participants were sent away to produce some sperm.

KRUSKAL-WALLIS TEST

No soya meals
n=20

1 soya meal per week
n=20

4 soya meals per week
n=20

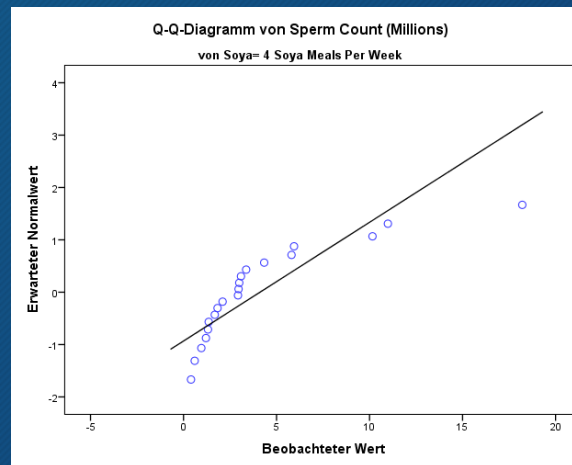
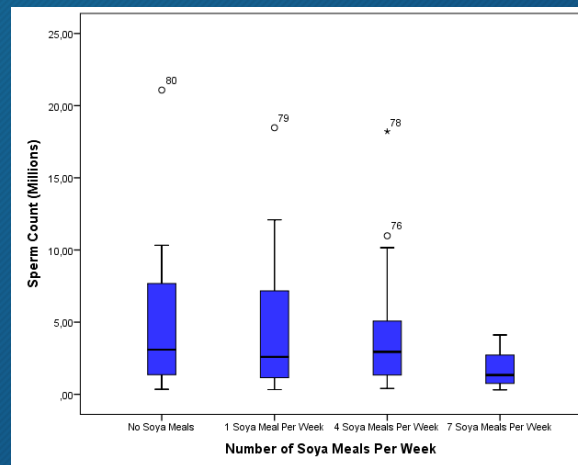
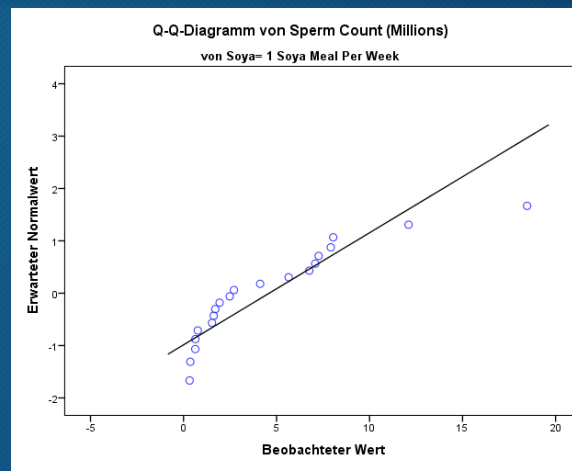
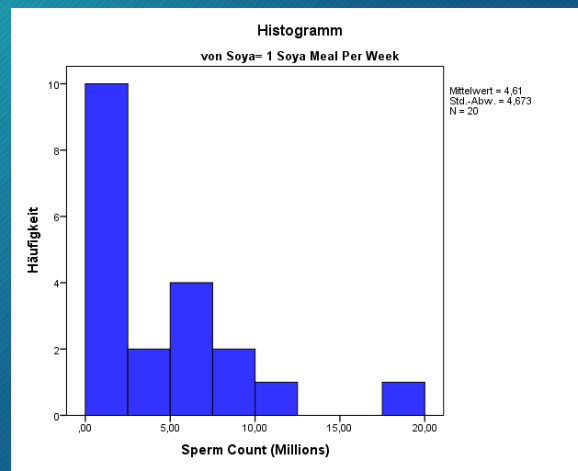
7 soya meals a week
n=20

1 year
after



sperm count

KRUSKAL-WALLIS TEST



*Soya.sav [DataSet1] - IBM SPSS Statistics Dateneditor

1: Ranks					
	ID	Soya	Sperm	Ranks	var
1	S061	7 Soya Meals Per Week	,31	1	
2	S062	7 Soya Meals Per Week	,32	2	
3	S021	1 Soya Meal Per Week	,33	3	
4	S001	No Soya Meals	,35	4	
5	S022	1 Soya Meal Per Week	,36	5	
6	S041	4 Soya Meals Per Week	,40	6	
7	S063	7 Soya Meals Per Week	,56	7	
8	S064	7 Soya Meals Per Week	,57	8	
9	S002	No Soya Meals	,58	9	
10	S042	4 Soya Meals Per Week	,60	10	
11	S023	1 Soya Meal Per Week	,63	11	
12	S024	1 Soya Meal Per Week	,64	12	
13	S065	7 Soya Meals Per Week	,71	13	
14	S025	1 Soya Meal Per Week	,77	14	
15	S066	7 Soya Meals Per Week	,81	15	
16	S067	7 Soya Meals Per Week	,87	16	
17	S003	No Soya Meals	,88	17	
18	S004	No Soya Meals	,92	18	
19	S043	4 Soya Meals Per Week	,96	19	
20	S068	7 Soya Meals Per Week	1,18	20	
21	S044	4 Soya Meals Per Week	1,20	21	
22	S005	No Soya Meals	1,22	22	
23	S069	7 Soya Meals Per Week	1,25	23	
24	S045	4 Soya Meals Per Week	1,31	24	
25	S070	7 Soya Meals Per Week	1,33	25	
26	S071	7 Soya Meals Per Week	1,34	26	
27	S046	4 Soya Meals Per Week	1,35	27	
28	S072	7 Soya Meals Per Week	1,49	28	
29	S073	7 Soya Meals Per Week	1,50	29	
30	S006	No Soya Meals	1,51	30	
31	S007	No Soya Meals	1,52	31	
32	S026	1 Soya Meal Per Week	1,53	32	
33	S008	No Soya Meals	1,57	33	
34	S027	1 Soya Meal Per Week	1,62	34	
35	S047	4 Soya Meals Per Week	1,68	35	
36	S028	1 Soya Meal Per Week	1,71	36	
37	S048	4 Soya Meals Per Week	1,83	37	
38	S029	1 Soya Meal Per Week	1,94	38	

Datenansicht Variablenansicht

	No soya		1 soya meal		4 soya meals		7 soya meals	
	Sperm	Rank	Sperm	Rank	Sperm	Rank	Sperm	Rank
	0,35	4	0,33	3	0,40	6	0,31	1
	0,58	9	0,36	5	0,60	10	0,32	2
	...							
Total (R _i)		927		833		833		547

$$H = \frac{12}{N(N-1)} \sum_{i=1}^k \frac{R_i^2}{n_i} - 3(N+1)$$

This test statistic chi-square distribution degrees of freedom: one less than the number of groups (k - 1)=3

$$\begin{aligned}
 H &= \frac{12}{80(81)} \left(\frac{972^2}{20} + \frac{883^2}{20} + \frac{833^2}{20} + \frac{547^2}{20} \right) - 3(81) \\
 &= \frac{12}{6480} (42,966.45 + 38,984.45 + 38,984.45 + 14,960.45) - 243 \\
 &= 0.0019(135,895.8) - 243 \\
 &= 251.66 - 243 \\
 &= 8.659
 \end{aligned}$$

*Soya.sav [DataSet1] - IBM SPSS Statistics Dateneditor

Datei Bearbeiten Ansicht Daten Transformieren Analysieren Direktmarketing

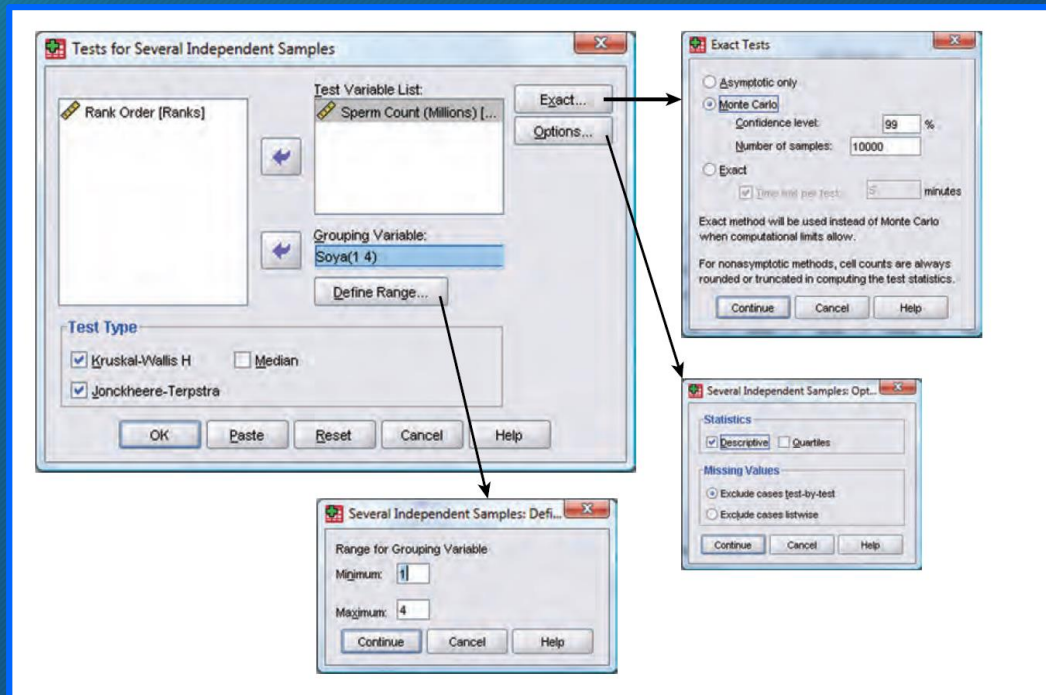
1: Ranks 1

	ID	Soya	Sperm	Ranks	var
1	S061	7 Soya Meals Per Week	,31	1	
2	S062	7 Soya Meals Per Week	,32	2	
3	S021	1 Soya Meal Per Week	,33	3	
4	S001	No Soya Meals	,35	4	
5	S022	1 Soya Meal Per Week	,36	5	
6	S041	4 Soya Meals Per Week	,40	6	
7	S063	7 Soya Meals Per Week	,56	7	
8	S064	7 Soya Meals Per Week	,57	8	
9	S002	No Soya Meals	,58	9	
10	S042	4 Soya Meals Per Week	,60	10	
11	S023	1 Soya Meal Per Week	,63	11	
12	S024	1 Soya Meal Per Week	,64	12	
13	S065	7 Soya Meals Per Week	,71	13	
14	S025	1 Soya Meal Per Week	,77	14	
15	S066	7 Soya Meals Per Week	,81	15	
16	S067	7 Soya Meals Per Week	,87	16	
17	S003	No Soya Meals	,88	17	
18	S004	No Soya Meals	,92	18	
19	S043	4 Soya Meals Per Week	,96	19	
20	S068	7 Soya Meals Per Week	1,18	20	
21	S044	4 Soya Meals Per Week	1,20	21	
22	S005	No Soya Meals	1,22	22	
23	S069	7 Soya Meals Per Week	1,25	23	
24	S045	4 Soya Meals Per Week	1,31	24	
25	S070	7 Soya Meals Per Week	1,33	25	
26	S071	7 Soya Meals Per Week	1,34	26	
27	S046	4 Soya Meals Per Week	1,35	27	
28	S072	7 Soya Meals Per Week	1,49	28	
29	S073	7 Soya Meals Per Week	1,50	29	
30	S006	No Soya Meals	1,51	30	
31	S007	No Soya Meals	1,52	31	
32	S026	1 Soya Meal Per Week	1,53	32	
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Datenansicht Variablenansicht

KRUSKAL-WALLIS TEST SPSS

- Analyze -> Nonparametric Tests -> Legacy Dialogs -> K Independent Samples



Exact test:
Large samples: Monte Carlo method.
Small samples: Exact test.

NPART TESTS

/K-W=Sperm BY Soya(1 4)
/STATISTICS DESCRIPTIVES
/MISSING ANALYSIS
/METHOD=MC CIN(99) SAMPLES(10000).

KRUSKAL-WALLIS TEST SPSS

Ränge			
	Number of Soya Meals Per Week	N	Mittlerer Rang
Sperm Count (Millions)	No Soya Meals	20	46,35
	1 Soya Meal Per Week	20	44,15
	4 Soya Meals Per Week	20	44,15
	7 Soya Meals Per Week	20	27,35
	Gesamt	80	

Statistik für Test ^{a,b}			
			Sperm Count (Millions)
Chi-Quadrat			8,659
df			3
Asymptotische Signifikanz			,034
Monte-Carlo-Signifikanz	Signifikanz		,033 ^c
	99%-Konfidenzintervall	Untergrenze	,028
		Obergrenze	,037

a. Kruskal-Wallis-Test

b. Gruppenvariable: Number of Soya Meals Per Week

c. Basiert auf 10000 Stichprobentabellen mit einem Startwert von 622500317.

Sperm counts were significantly affected by eating soya meals, $H(3) = 8.66$, $p < .05$.
 Post hoc test: Mann–Whitney tests using Bonferroni correction

KRUSKAL-WALLIS TEST R

```
newModel <- kruskal.test(Sperm ~ Soya, data = Soya)
Soya$Ranks <- rank(Soya$Sperm)
by(Soya$Ranks, Soya$Soya, mean)
newModel
```

OUTPUT

```
data: Sperm by Soya
Kruskal-Wallis chi-squared = 8.6589, df = 3, p-value =
0.03419
```

MEAN RANK FOR EACH GROUP

```
Soya$Soya: 1
[1] 46.35
```

```
-----
Soya$Soya: 2
[1] 44.15
```

```
-----
Soya$Soya: 3
[1] 44.15
```

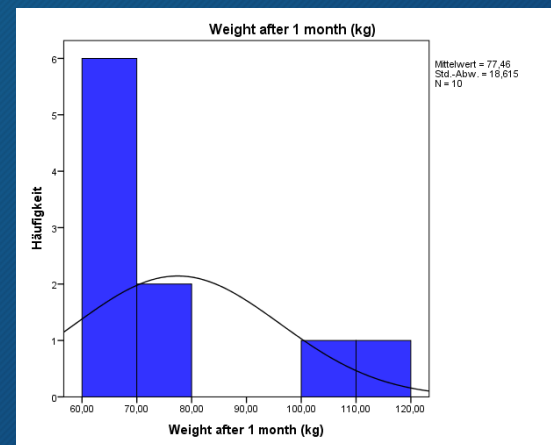
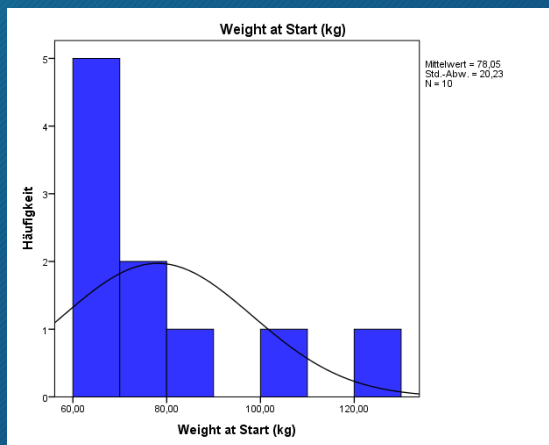
```
-----
Soya$Soya: 4
[1] 27.35
```

FRIEDMAN'S ANOVA

- Alternative to the repeated-measures case.
- It is used for testing differences between conditions when there are more than two conditions and the same participants have been used in all conditions.
- Example: A study was conducted to investigate the effects of dietary and exercise interventions on women who feel the need to lose weight. Their weight was measured in kilograms at the start of the diet and then after one month and two months.

FRIEDMAN'S ANOVA

Statistiken				
		Weight at Start (kg)	Weight after 1 month (kg)	Weight after 2 months (kg)
N	Gültig	10	10	10
	Fehlend	0	0	0
Schiefe		1,465	1,847	1,404
Standardfehler der Schiefe		,687	,687	,687
Kurtosis		1,086	2,470	2,766
Standardfehler der Kurtosis		1,334	1,334	1,334



Diet.sav [DataSet2] - IBM SPSS Statistics Dateneditor					
Datei Bearbeiten Ansicht Daten Transformieren Analysieren Direktma					
23 : Month1					
	ID	Start	Month1	Month2	var
1	D01	63,75	65,38	81,34	
2	D02	62,98	66,24	69,31	
3	D03	65,98	67,70	77,89	
4	D04	107,27	102,72	91,33	
5	D05	66,58	69,45	72,87	
6	D06	120,46	119,96	114,26	
7	D07	62,01	66,09	68,01	
8	D08	71,87	73,62	55,43	
9	D09	83,01	75,81	71,63	
10	D10	76,62	67,66	68,60	
11					
12					
13					
14					
15					
16					

FRIEDMAN'S ANOVA

	Weight			Weight Ranks		
	Start	Month 1	Month 2	Start	Month 1	Month 2
D001	63.75	65.38	81.34	1	2	3
D002	62.98	66.24	69.31	1	2	3
D003	65.98	67.7	77.89	1	2	3
D004	107.2 7	102.72	91.33	3	2	1
D005	66.58	69.45	72.87	1	2	3
						
R _i				19	20	21

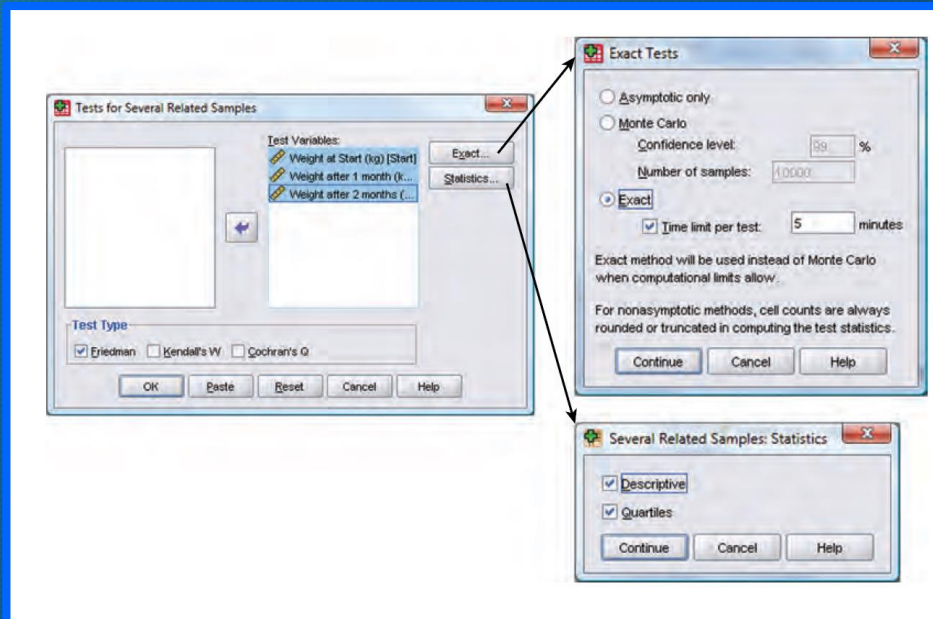
$$F_r = \left[\frac{12}{Nk(k+1)} \sum_{i=1}^k R_i^2 \right] - 3N(k+1)$$

$$\begin{aligned}
 F_r &= \left[\frac{12}{(10 \times 3)(3+1)} (19^2 + 20^2 + 21^2) \right] - (3 \times 10)(3+1) \\
 &= \frac{12}{120} (361 + 400 + 441) - 120 \\
 &= 0.1(1202) - 120 \\
 &= 120.2 - 120 \\
 &= 0.2
 \end{aligned}$$

This test statistic chi-square distribution, degrees of freedom: one less than the number of groups $(k - 1) = 2$

FRIEDMAN'S ANOVA SPSS

- Analyze -> Nonparametric Tests -> Legacy Dialogs -> K Related Samples



NPART TESTS

```
/FRIEDMAN=Start Month1 Month2  
/STATISTICS DESCRIPTIVES QUARTILES  
/MISSING LISTWISE  
/METHOD=EXACT TIMER(5).
```

FRIEDMAN'S ANOVA SPSS

Deskriptive Statistiken

	N	Mittelwert	Standardabweichung	Minimum	Maximum	Perzentile		
						25.	50. (Median)	75.
Weight at Start (kg)	10	78,0543	20,23008	62,01	120,46	63,5549	69,2288	89,0709
Weight after 1 month (kg)	10	77,4635	18,61502	65,38	119,96	66,2065	68,5728	82,5385
Weight after 2 months (kg)	10	77,0668	16,10612	55,43	114,26	68,4525	72,2493	83,8365

Statistik für Test^a

N	10
Chi-Quadrat	,200
df	2
Asymptotische Signifikanz	,905
Exakte Signifikanz	,974
Punkt-Wahrscheinlichkeit	,143

a. Friedman-Test

The weight of participants did not significantly change over the two months of the diet, $X^2(2)=0.20$, $p=0.905$.

FRIEDMAN'S ANOVA R

- Friedman's test demands that you give it a matrix rather than a dataframe.
- It wants all of the variables of interest in one data set.
- It get confused by missing data.

```
dietCompleteCases <- na.omit(Diet)
```

```
friedman.test(as.matrix(Diet))
```

OUTPUT

```
data: as.matrix(Diet)
```

```
Friedman chi-squared = 0.2, df = 2, p-value =  
0.9048
```

NP FACTORIAL DESIGNS

- ANOVA Test : valid under the assumption of normally distributed errors, and equal variances across the different treatment groups.
- The accuracy of the F test depends on the data distribution, sample size and the degree of variance heteroscedasticity.
- ANOVA-type test based on ranks has been implemented in R.
- In order to test the null hypotheses: The Wald type statistic (WTS) and the ANOVA-type statistic (ATS).

NP FACTORIAL DESIGNS (1 way)

Example: dataset on startup costs of companies. The data represent business startup costs in thousands of dollars for five different kinds of shops.

```
data("startup", package = "GFD")
```

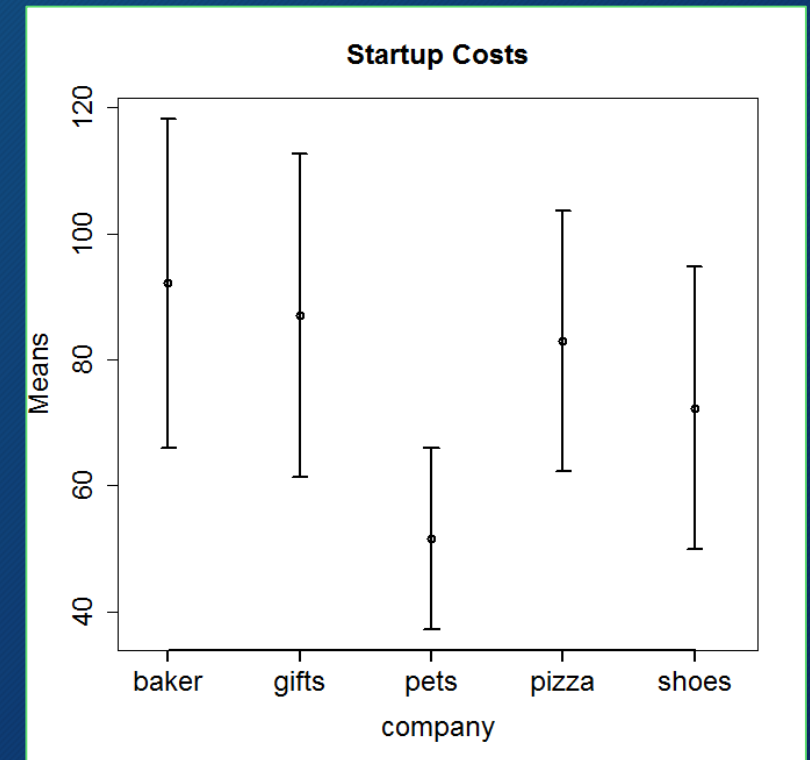
```
set.seed(456)
```

```
model1 <- GFD(Costs ~ company, data = startup, nperm = 10000,  
  alpha = 0.05)
```

NP FACTORIAL DESIGNS

Descriptive:

	company	n	Means	Variances	Lower 95 % CI	Upper 95 % CI
1	Baker	11	92.09091	1512.6909	65.96202	118.21980
2	gifts	10	87.00000	1289.1111	61.31569	112.68431
3	pets	16	51.62500	733.0500	37.19781	66.05219
4	pizza	13	83.00000	1165.1667	62.37271	103.62729
5	shoes	10	72.30000	983.7889	49.86254	94.73746



NP FACTORIAL DESIGNS

Wald-Type Statistic (WTS):

Test statistic	df	p-value	p-value WTPS
15.037830399	4.000000000	0.004623394	0.024600000

- WTS: liberal
- ATS: conservative
- WTPS: in between.

ANOVA-Type Statistic (ATS):

Test statistic	df1	df2	p-value
2.57248203	3.70623134	44.51042721	0.05456579

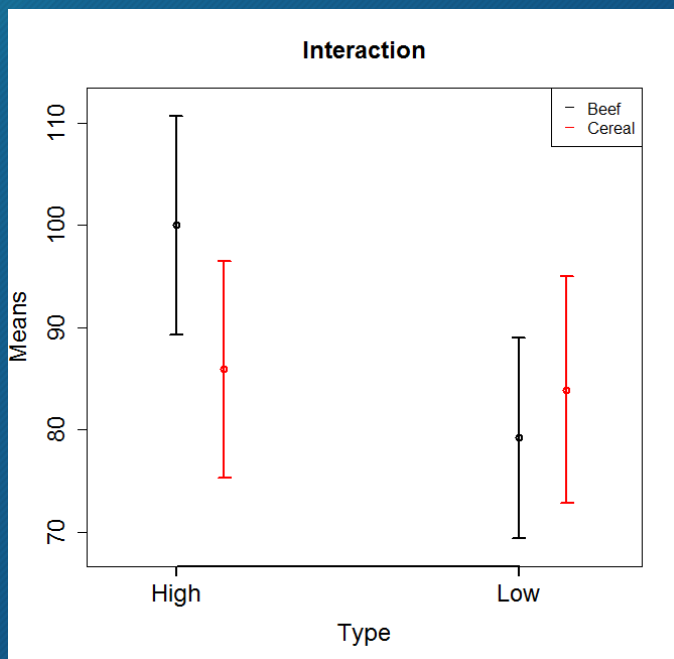
NP FACTORIAL DESIGNS (2 way)

The data contains information on the weight gain (in grams) of rats which were randomized to one of four diets, distinguished by the amount of protein (high and low) and the source of protein (beef and cereal).

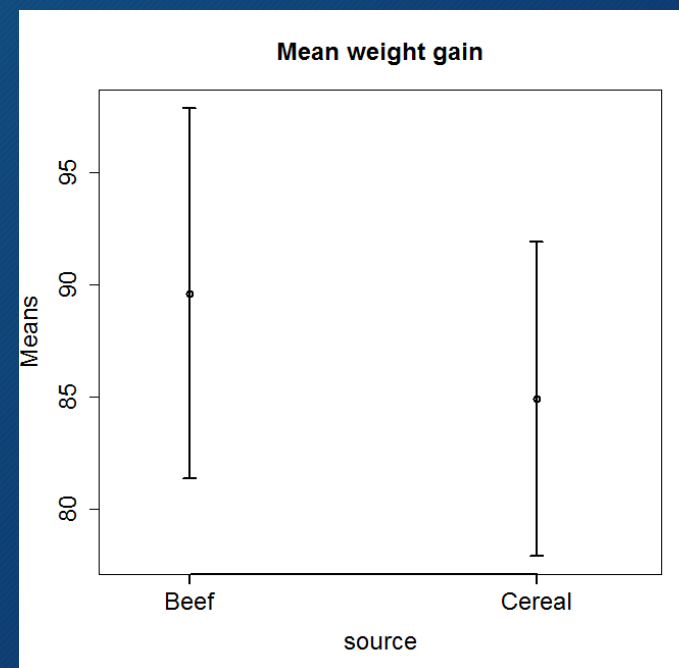
```
data("weightgain", package = "HSAUR")  
  
View(weightgain)  
set.seed(789)  
  
model2 <- GFD(weightgain ~ source * type, data = weightgain)  
  
summary(model2)
```

NP FACTORIAL DESIGNS (2 way)

```
plot(model2, factor = "source:type", main =  
"Interaction", xlab = "Type", cex.axis = 1.5, cex.lab =  
1.5, cex.main = 1.5)
```



```
plot(model2, factor = "source", main = "Mean weight gain",  
xlab = "source", cex.axis = 1.5, cex.lab = 1.5, cex.main = 1.5)
```



NP FACTORIAL DESIGNS (2 way)

Descriptive:

	source	type	n	Means	Variances	Lower 95 % CI	Upper 95 % CI
1	Beef	High	10	100.0	229.1111	89.17206	110.82794
3	Beef	Low	10	79.2	192.8444	69.26595	89.13405
2	Cereal	High	10	85.9	225.6556	75.15403	96.64597
4	Cereal	Low	10	83.9	246.7667	72.66260	95.13740

Wald-Type Statistic (WTS):

	Test statistic	df	p-value	p-value WTPS
source	0.9879494	1	0.32024407	0.3305
type	5.8123090	1	0.01591439	0.0182
source:type	3.9517976	1	0.04682133	0.0562

ANOVA-Type Statistic (ATS):

	Test statistic	df1	df2	p-value
source	0.9879494	1	35.72893	0.32692829
type	5.8123090	1	35.72893	0.02118641
source:type	3.9517976	1	35.72893	0.05452616

The factor type, i.e., high or low amount of protein in the food, has a significant impact on the weight gain at 5% level of significance using all three different tests. The source of the protein, in contrast, does not have a significant influence. The interaction has a borderline significance at 5% level.

NP ANALYSIS OF LONGITUDINAL DATA

- LD are measurements collected from the same experimental units (subject or individuals) over time.
- One homogeneous group of subjects is observed repeatedly at $s=1, \dots, t$ time points.
- One homogeneous group of subjects is observed repeatedly under $l=1, \dots, c$ conditions where each subject is observed under each condition at $s=1, \dots, t$ time points.
- More than one homogeneous group of subjects (male/female, different treatment groups) are observed repeatedly at $s=1, \dots, t$ time points.

NP ANALYSIS OF LONGITUDINAL DATA

Typical questions:

1. Do the treatments have the same effect? -> Treatment effects
2. Is the time profile flat? -> Time effects
3. Are the effects of the treatments similar over time? Or Are the treatment profiles parallel? -> Interaction effects between treatment and time

NP ANALYSIS OF LONGITUDINAL DATA

OPTIONS

- Repeated measures ANOVA
- Generalized linear models (GLM)

Requisites:

Presence of homogeneous variances.
Normal distribution of variables

NP ANALYSIS OF LONGITUDINAL DATA

- In case of skewed distributions, outliers, or small sample sizes, parametric statistical procedures may result in unreliable or even false conclusions.
- Alternative: nonparametric rank-based methods
- They offer a flexible and robust framework for the analysis of a variety of longitudinal studies.
- the rank-based methodology is not restricted to data on a continuous scale and enables to analyze ordered categorical, dichotomous, and
- heavily skewed data.

NP ANALYSIS OF LONGITUDINAL DATA

- SPSS does not provide any package so far.
- R package nparLD offers nonparametric methodology for the most frequently arising factorial designs.
- Npar: nonparametric
- LD: longitudinal data.

Warning: the asymptotic approximation deteriorates with an increase of a number of factor levels as well as for smaller sample sizes ($n < 7$).

NP ANALYSIS OF LONGITUDINAL DATA

- The factor which stratifies samples in independent groups, is called a whole-plot factor: sex is the whole-plot
- The factor, stratifying repeated measurements, is called a sub-plot factor, time is the sub-plot factor.
- Each design is denoted by $F_x\text{-LD-}F_y$, x and y are the number of whole-plot and sub-plot factors, respectively.

LD F1 EXAMPLE

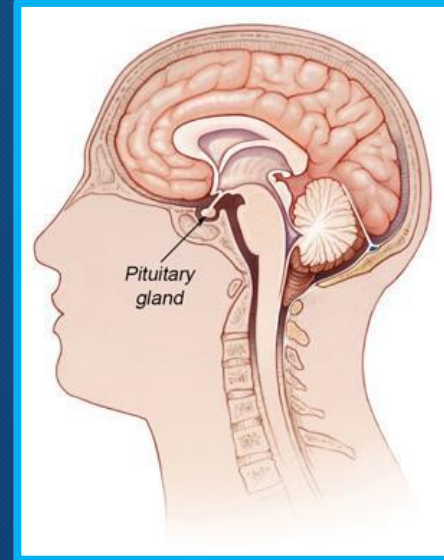
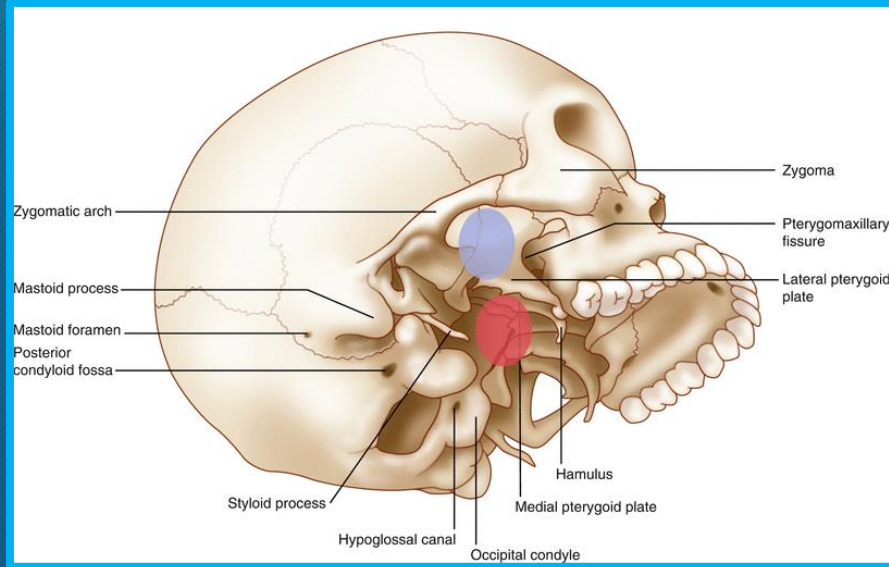
LD F1

Pottho and Roy (1964) assess distances (in millimeters) between the center of the pituitary and the pterygomaxillary fissure of 16 boys on four different occasions, (at the ages 8, 10, 12, and 14).

In particular, the researchers are interested in testing the hypothesis:

$H = \bigcap_{s=8,10,12,14} (F_s : F_8 = F_{10} = F_{12} = F_{14})$ of no time effect, where F_s denotes the marginal distribution of the distances at age s .

LD F1 EXAMPLE

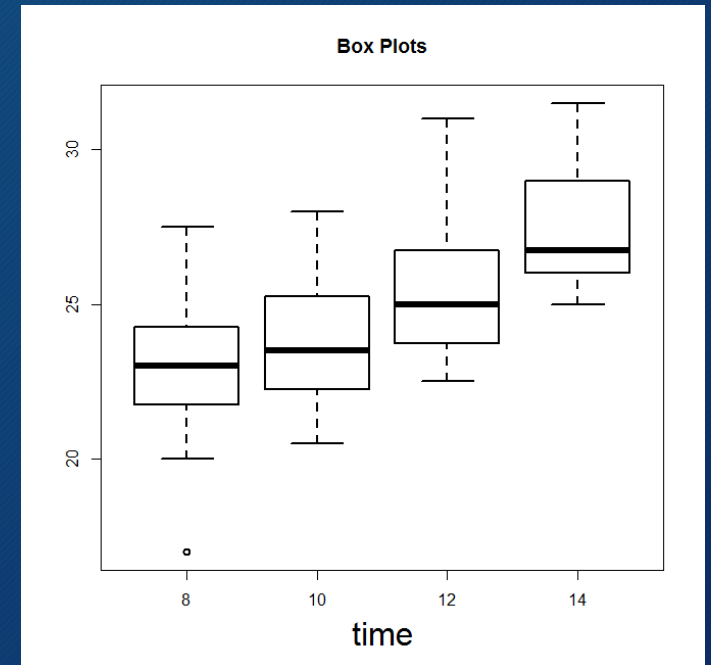


https://www.google.com/search?biw=1136&bih=831&tbm=isch&sa=1&ei=XDWAXLvADvuN1fAPnZO-6Ao&q=pituitary+&oq=pituitary+&gs_l=img.3..35i39j0i67j0i67j0i5j0i67.4260.4260..5534...0.0..0.100.100.0j1.....1....1..gws-wiz-img.0hH0XwoCA54#imgsrc=YYmz7jELfoKc7M:

LD F1 EXAMPLE

```
install.packages("nparLD")  
library("nparLD")  
data("dental")  
View(dental)
```

```
boxplot(resp ~ time, data = dental, lwd = 2, xlab = "time",  
        font.lab = 2, cex.lab = 2, main = "Box Plots")
```



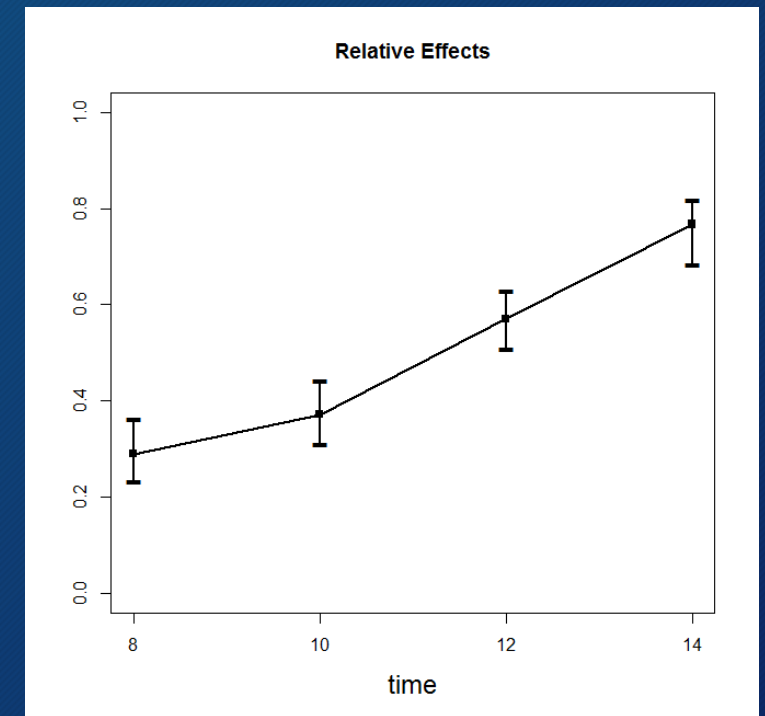
The box plots show the minimum, first quartile, median, third quartile, and the maximum distance measured for each time point separately. They indicate that the measured distances have a somewhat skewed distribution.

LD F1 EXAMPLE

```
ex.f1np <- nparLD(resp ~ time, data = dental, subject = "subject",  
  description = FALSE)
```

The Figure shows the lower bound, point estimate, and the upper bound for each time point separately (95% CI).

The point estimates increase, meaning the older the boys, the larger the observed distances between pituitary and the pterygomaxillary fissure.



LD F1 EXAMPLE

```
plot(ex.f1np)$Conf.Int
```

	Group	Time	Nobs	RankMeans	RTE	Bias	Variance	Lower	Upper
1	Group1	8	16	19.0625	0.2900	-0.0048	0.0177	0.2340	0.3642
2	Group1	10	16	24.3125	0.3721	-0.0024	0.0190	0.3097	0.4436
3	Group1	12	16	37.0312	0.5708	0.0021	0.0154	0.5087	0.6291
4	Group1	14	16	49.5938	0.7671	0.0051	0.0177	0.6846	0.8175

The obtained result of 0.29 for the age group 8 (time8) can be interpreted, for example, as follows: a randomly chosen observation from the whole dataset results in a smaller value than a randomly chosen observation from the age group 8 with an estimated probability of 29%. Since $bp_8 < bp_{10} < bp_{12} < bp_{14}$, The increase in the effect seems to indicate the increase in the measured distances.

LD F1 EXAMPLE

```
summary(ex.f1np)
```

Wald-Type Statistic (WTS):

	Statistic	df	p-value
time	94.47718	3	2.391503e-20

ANOVA-Type Statistic (ATS):

	Statistic	df	p-value
time	31.48774	2.700785	1.437729e-18

To test the hypothesis of no time effect WTS and ATS can be applied. Both WTS and ATS yield highly statistically significant p values of 2.39×10^{-20} , and 1.44×10^{-18} , respectively, indicating that the null hypothesis of no time effect can be rejected.

LD F1 EXAMPLE

Comparison	Hypothesis	p value	Adjusted p value
Time 8 vs. Time 10	$H_0^F : F_8 = F_{10}$	0.2204	0.6612
Time 8 vs. Time 12	$H_0^F : F_8 = F_{12}$	< 0.0001	< 0.0001
Time 8 vs. Time 14	$H_0^F : F_8 = F_{14}$	< 0.0001	< 0.0001

Table 4: Multiple comparisons against the control in the dental study with Bonferroni adjustment.

- the Bonferroni-adjusted p value of 0.6612, obtained for testing the age group 8 against the age group 10 (Time 8 vs. Time 10), is calculated by multiplying the original p value of 0.2204 by 3.
- We can conclude that the distance between the center of the pituitary and the pterygomaxillary fissure significantly increases over time. In addition, there are significant differences between the distributions of the measured distances for the age groups 8 and 12, and age groups 8 and 14, respectively.

F1 LD F1 EXAMPLE

Study: Each rat was randomly assigned to one of three treatments (control, thyroxin, or thiouracil) with sample sizes of 10, 7, and 10, respectively. The weight (in grams) of each rat was recorded at baseline and subsequent four weeks.

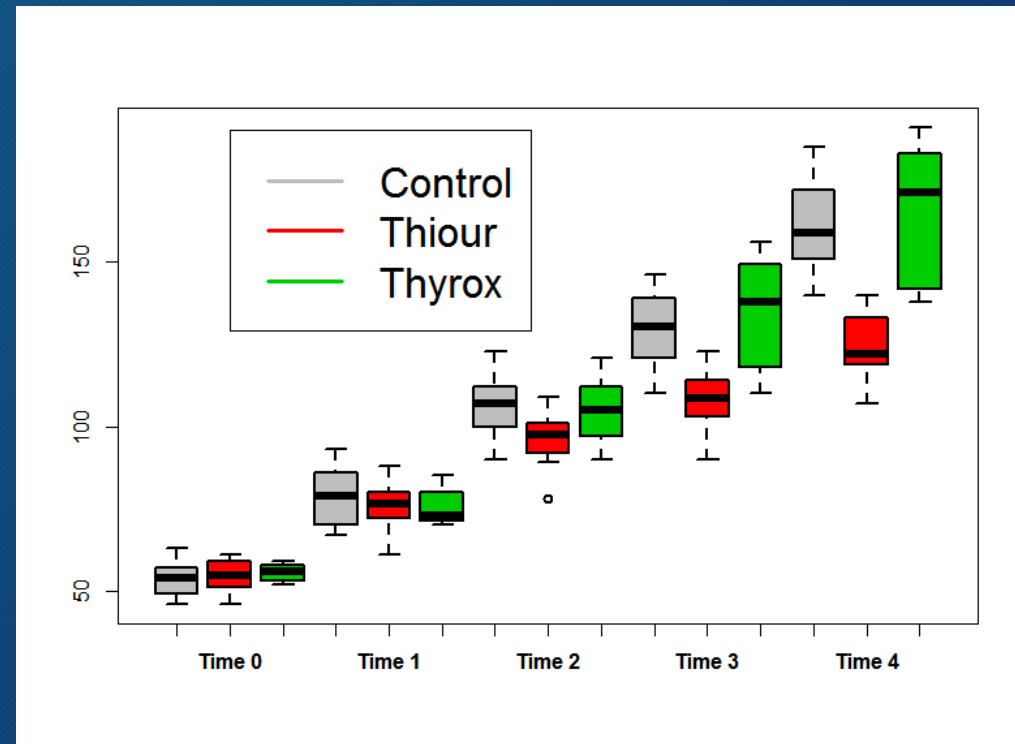
Although a standard technique of the logarithmic transformation makes the data more homoscedastic, there is a concern that such a nonlinear transformation distorts the time and treatment effects.

```
data("rat")
View(rat)

boxplot(resp ~ group * time, data = rat, names = FALSE,
        col = c("grey", 2, 3), lwd = 2)
axis(1, at = 2, labels = "Time 0", font = 2, cex = 2)
axis(1, at = 5, labels = "Time 1", font = 2, cex = 2)
axis(1, at = 8, labels = "Time 2", font = 2, cex = 2)
axis(1, at = 11, labels = "Time 3", font = 2, cex = 2)
axis(1, at = 14, labels = "Time 4", font = 2, cex = 2)
legend(2, 190, c("Control", "Thiour", "Thyrox"), lwd = c(3, 3, 3),
      col = c("grey", 2, 3), cex = 2)
```

F1 LD F1 EXAMPLE

- The Figure shows box plots of the rat growth for the three treatments and each time point separately.
- The box plots indicate that the data follow a skewed distribution.

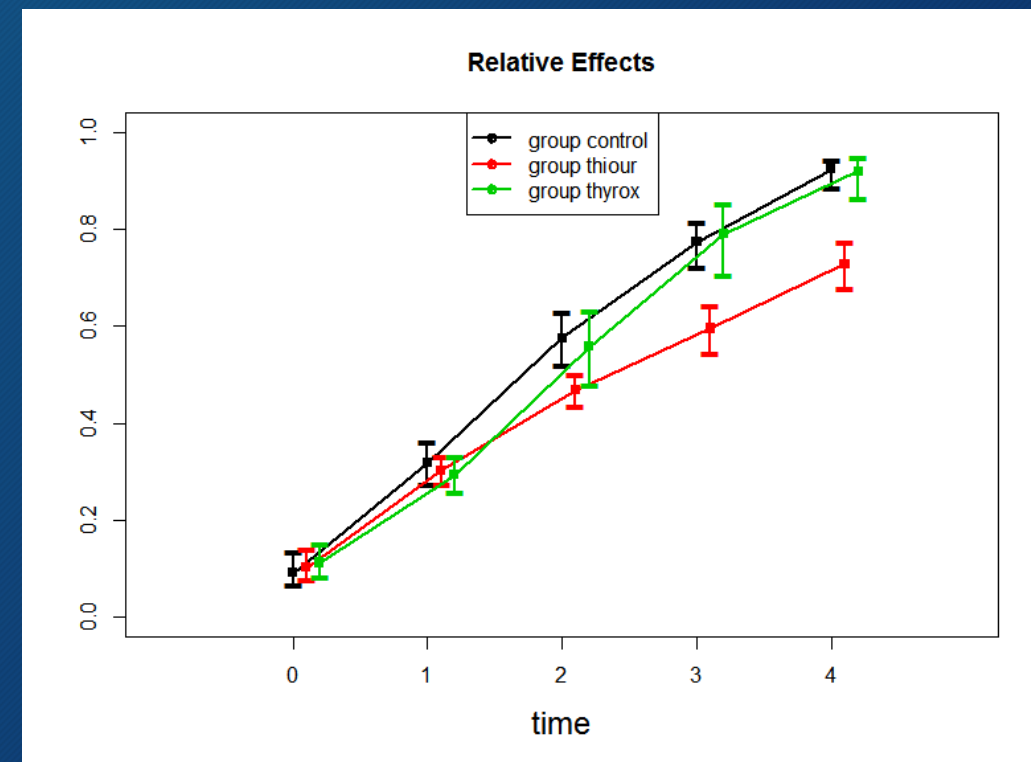


F1 LD F1 EXAMPLE

```
ex.f1f1np <- nparLD(resp ~ time * group, data = rat,  
  subject = "subject", description = FALSE)
```

```
plot(ex.f1f1np)$Conf.Int
```

	Group	Time	Nobs	RankMeans	RTE	Bias	Variance	Lower	Upper
1	groupcontrol	0	10	12.7500	0.0907	0.0000	0.0077	0.0657	0.1352
2	groupcontrol	1	10	43.3000	0.3170	-0.0002	0.0133	0.2756	0.3623
3	groupcontrol	2	10	78.2500	0.5759	-0.0005	0.0222	0.5189	0.6307
4	groupcontrol	3	10	105.0000	0.7741	0.0003	0.0153	0.7231	0.8165
5	groupcontrol	4	10	125.3000	0.9244	0.0003	0.0049	0.8874	0.9437
11	groupthiour	0	10	14.3000	0.1022	0.0000	0.0064	0.0778	0.1395
12	groupthiour	1	10	41.3000	0.3022	-0.0003	0.0061	0.2738	0.3326
13	groupthiour	2	10	63.7000	0.4681	-0.0010	0.0071	0.4365	0.5001
14	groupthiour	3	10	80.8500	0.5952	0.0000	0.0170	0.5451	0.6431
15	groupthiour	4	10	98.9000	0.7289	0.0014	0.0164	0.6774	0.7738
6	groupthyrox	0	7	15.5714	0.1116	0.0000	0.0075	0.0841	0.1505
7	groupthyrox	1	7	40.0714	0.2931	0.0000	0.0102	0.2567	0.3329
8	groupthyrox	2	7	75.7857	0.5577	-0.0008	0.0410	0.4804	0.6319
9	groupthyrox	3	7	107.2143	0.7905	-0.0009	0.0374	0.7065	0.8528
10	groupthyrox	4	7	124.7143	0.9201	0.0017	0.0109	0.8643	0.9484



F1 LD F1 EXAMPLE

Questions:

1. Are the time profiles of the three experiments are parallel? ,
2. Is there a statistical interaction between treatment and time?
3. Do the treatments affect the weight gains?

The time profile from the thyroxin group does not seem to be parallel to both the control and the thiouracil group time profiles.

```
summary(ex.f1f1np)
```

F1 LD F1 EXAMPLE

Wald-Type Statistic (WTS):

	Statistic	df	p-value	
group	12.52657	2	1.904977e-03	→ Treatment effect
time	3619.03739	4	0.000000e+00	→ Time effect
group:time	70.34311	8	4.199050e-12	→ Interaction

ANOVA-Type Statistic (ATS):

	Statistic	df	p-value
group	5.286582	1.922792	5.654723e-03
time	1008.512138	1.990411	0.000000e+00
group:time	11.093940	3.516933	3.616929e-08

In this study, the hypothesis of no interaction, i.e., parallel time profiles, is rejected at the 1% level using both WTS and ATS.

Modified ANOVA-Type Statistic for the Whole-Plot Factors:

	Statistic	df1	df2	p-value
group	5.286582	1.922792	19.23468	0.01563658

REFERENCES

1. Friedrich et al, GFD: An R Package for the Analysis of General Factorial Designs, Journal of Statistical Software. July 2017.
2. Noguchi, K et al, nparLD: An R Software Package for the Nonparametric Analysis of Longitudinal Data in Factorial Experiments. Journal of Statistical Software, Sept 2012.
3. Andi Field, Discovering Statistic using R
4. Andi Field, Discovering Statistic using SPSS