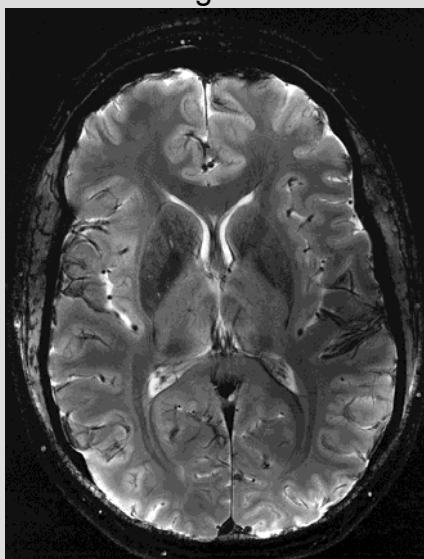


MR-Technik für Bildgebung und Spektroskopie

Prof. Klaus Scheffler, PhD
MPI for Biological Cybernetics
Dept. Biomedical Magnetic Resonance
Tübingen

Biomedical Magnetic Resonance: 1 Introduction

Biomedical Magnetic Resonance

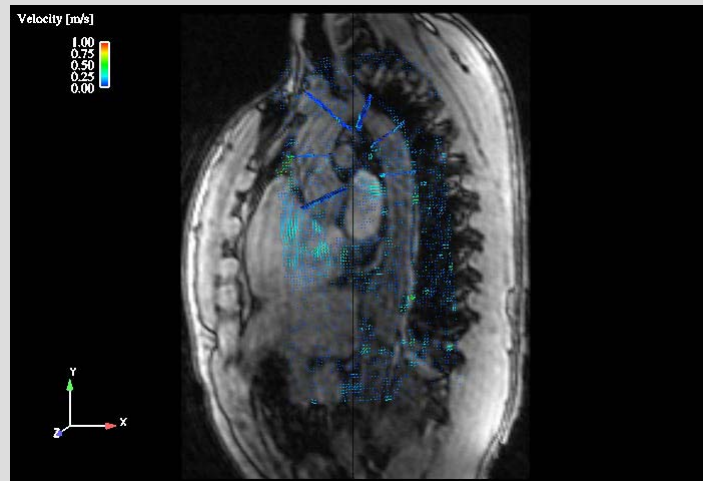


Biomedical Magnetic Resonance: 1 Introduction

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Biomedical Magnetic Resonance



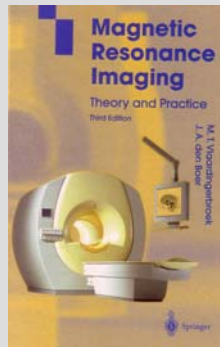
Biomedical Magnetic Resonance

Contents:

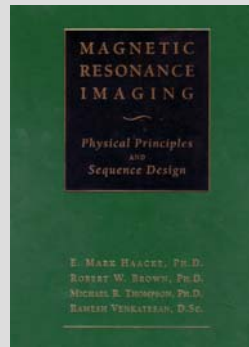
- 1 Introduction
- 2 Nuclear Magnetic Moments
- 3 Motion of Magnetization
- 4 Excitation and reception
- 5 Magnetic Resonance Imaging

Biomedical Magnetic Resonance

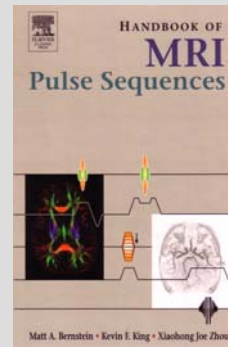
Recommended Literature:



Magnetic Resonance Imaging
M.T. Vlaardingerbroek, J.A. den Boer
Springer



Magnetic Resonance Imaging
E.M. Haacke, R.W. Brown, M.R. Thomson,
R. Venkatesan. Wiley-Liss



Handbook of MRI Pulse Sequences
M.A. Bernstein, K.F. King, X.J. Zhou
Elsevier

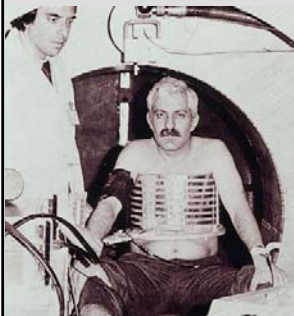
Biomedical Magnetic Resonance

Some links on the web:

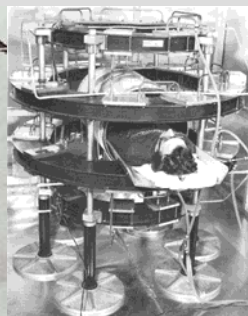
- http://sunnybrook.ca/research/?page=sri_groups_cmri_mrp_1
- <http://www.e-mri.org/>
- www.mritutor.org
- <http://www.cis.rit.edu/htbooks/mri>
- <http://mrsrl.stanford.edu/~brian/mritools.html/> (including Matlab code)
- <http://mrsrl.stanford.edu/~brian/blochsim/> (including Matlab code)
- <http://www-mrsrl.stanford.edu/~brian/mri-movies/> (including Matlab code)
- <http://www-mrsrl.stanford.edu/~brian/bloch/> (including Matlab code)
- <http://www.revisemri.com/>

Biomedical Magnetic Resonance

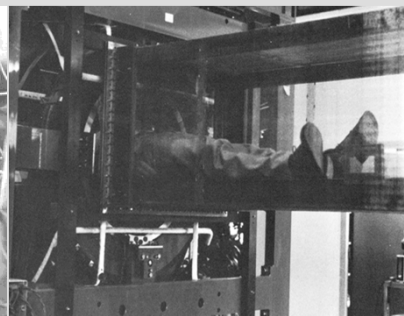
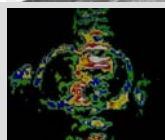
History:



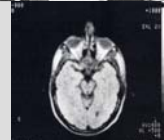
1974



1979



1981



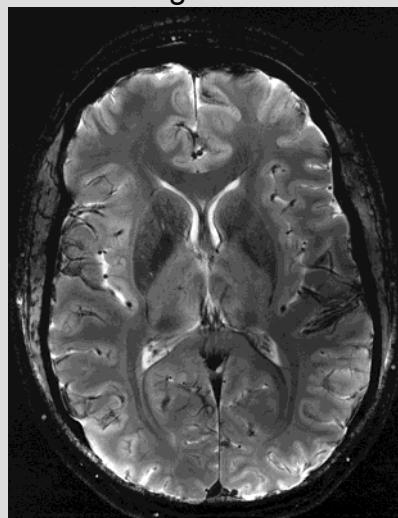
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Biomedical Magnetic Resonance

Today:



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Nuclear Magnetic Moments

Properties of a nucleus:



For example: ^1H , ^{13}C , ^{19}F , ^{31}P

Nucleus is charged (multiples of e^+)

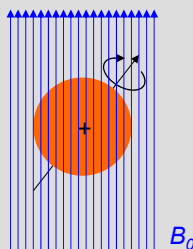
Some nuclei have a spin angular momentum J :
 $J = \hbar I$ (I multiples of $\frac{1}{2}$)

Then, they have a dipolar magnetic moment μ :
 $\mu = \gamma J = \gamma \hbar I$

$\hbar$ = Planck's quantum constant
 γ = gyromagnetic ratio

Nuclear Magnetic Moments

Nucleus in a magnetic field:



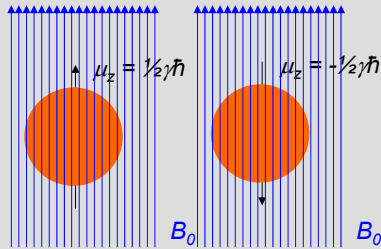
In a magnetic field,
the dipolar magnetic moment μ is
quantized along the field:

$\mu_z = \gamma \hbar m$
and m can be $m = I, I-1, \dots, -I$.

$\hbar$ = Planck's quantum constant
 γ = gyromagnetic ratio

Nuclear Magnetic Moments

Nucleus in a magnetic field:



We assume $I = 1/2$ (^1H , ^{13}C , ^{19}F , ^{31}P)

Then, the magnetic moment along the field is:

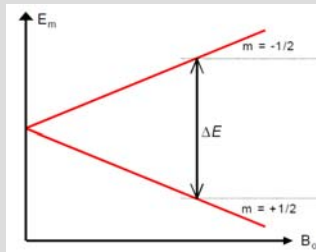
$$\mu_z = \gamma \hbar m = \pm \frac{1}{2} \gamma \hbar$$

The magnetic moment μ_z is parallel or antiparallel to the field B_0

$\hbar$ = Planck's quantum constant
 γ = gyromagnetic ration

Nuclear Magnetic Moments

Nucleus in a magnetic field:



In an external magnetic field, spin up or spin down are associated to different energy levels:

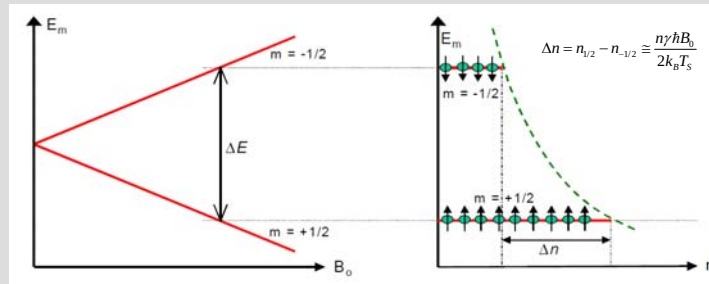
$$E_m = -\mu_z B_0 = -\gamma \hbar B_0 m, \quad m = \pm \frac{1}{2}$$

$$E_{\downarrow} = +\frac{1}{2} \gamma \hbar B_0, \quad E_{\uparrow} = -\frac{1}{2} \gamma \hbar B_0$$

$$\Delta E = \gamma \hbar B_0$$

$\hbar$ = Planck's quantum constant
 γ = gyromagnetic ration

Nuclear Magnetic Moments



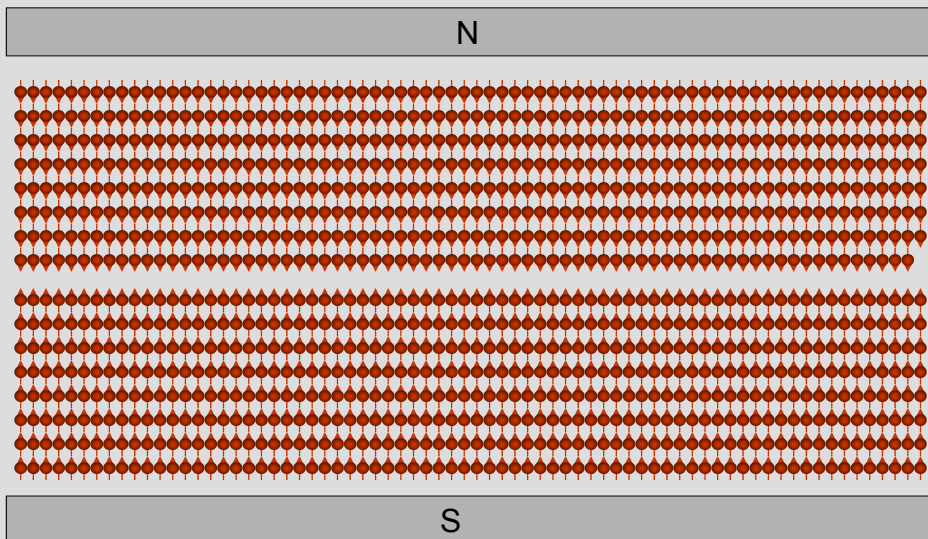
In thermal equilibrium, there are different probabilities for spin up and down occupation (Boltzmann statistics).

The difference in occupation Δn is given by:

$$\Delta n = n_{1/2} - n_{-1/2} \cong \frac{n\gamma\hbar B_0}{2k_B T_S}$$

k_B : Boltzmann constant

T_S : Temperature



Nuclear Magnetic Moments

Macroscopic magnetization:

Visible magnetization: $M_0 = \sum_i \mu_{iz}$

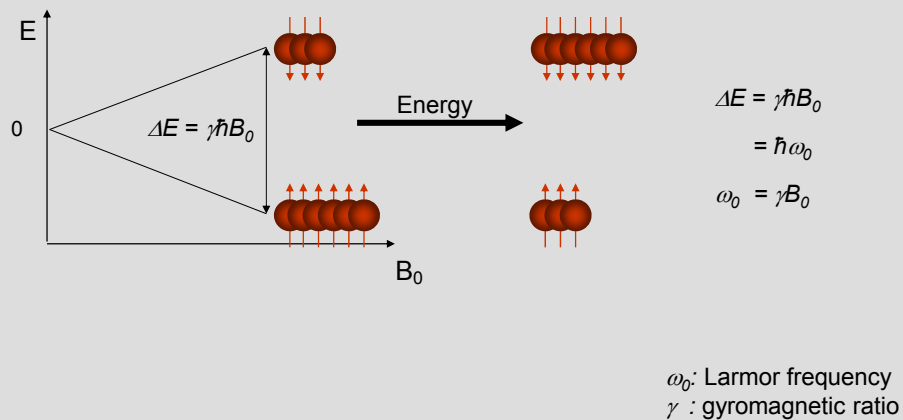
Sum (macroscopic Magnetization along B_0):

$$M_0 = \Delta n \mu_z = \frac{n \gamma \hbar B_0}{2 k_B T_S} \mu_z = \frac{n \gamma^2 \hbar^2}{4 k_B T_S} B_0 = \chi_0(T_S) B_0$$

$\chi_0(T_S)$: magnetic susceptibility
 k_B : Boltzmann constant

Nuclear Magnetic Moments

Spin excitation:



Nuclear Magnetic Moments

Some MR-relevant nuclei:

Isotop	Spin Quanten Zahl	Gyromagn. Verhältnis [MHz/Tesla]	Natürliche Häufigkeit [%]	Relative Sensitivität für gleiche Zahl Spins ^a bei konst. Feld	Relative Empfindlichkeit korr. mit natürlicher Häufigkeit
¹ H	1/2	42.58	99.98	1.0	1.0
¹³ C	1/2	10.71	1.11	1.59 · 10 ⁻²	1.8 · 10 ⁻⁴
¹⁴ N	1	3.08	99.64	1.01 · 10 ⁻³	1.0 · 10 ⁻³
¹⁷ O	5/2	5.77	0.04	2.91 · 10 ⁻²	1.1 · 10 ⁻⁵
¹⁹ F	1/2	40.06	100.	8.30 · 10 ⁻¹	8.3 · 10 ⁻¹
²³ Na	3/2	11.26	100.	9.27 · 10 ⁻²	9.3 · 10 ⁻²
³¹ P	1/2	17.24	100.	6.64 · 10 ⁻²	6.6 · 10 ⁻⁷
³⁹ K	3/2	1.99	93.08	5.08 · 10 ⁻⁴	4.7 · 10 ⁻⁴
⁴³ Ca	7/2	2.87	0.14	6.40 · 10 ⁻³	9.3 · 10 ⁻⁶

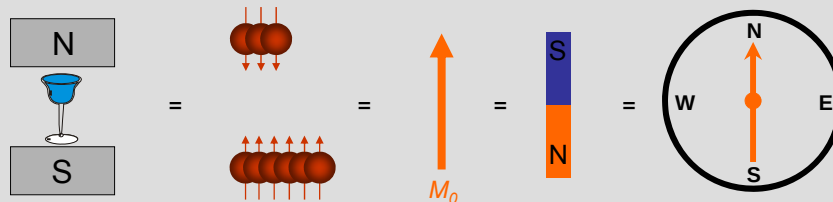
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Motion of Magnetization

Magnetization and magnetic field:



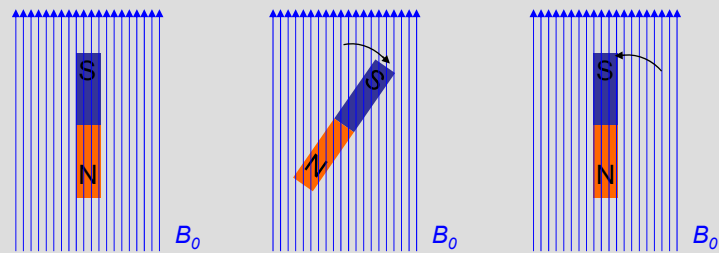
Biomedical Magnetic Resonance: 3 Motion of Magnetization

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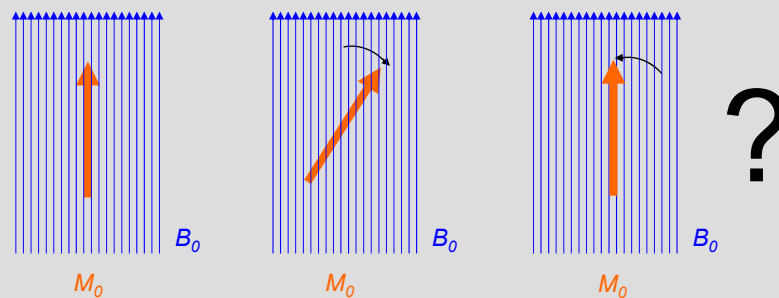
Motion of Magnetization

Magnetization and magnetic field:



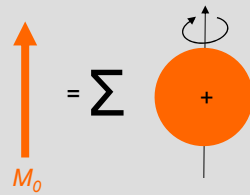
Motion of Magnetization

Magnetization and magnetic field:



Motion of Magnetization

Magnetization and magnetic field:



Nucleus is charged (multiples of e^+)

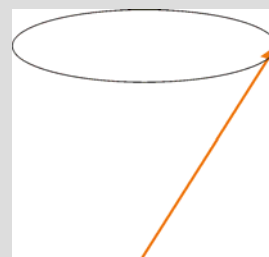
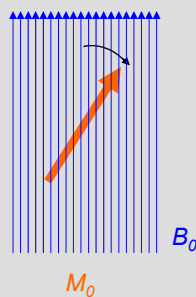
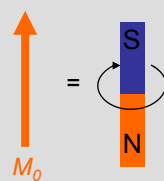
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 $J = \hbar I$ (I multiples of $\frac{1}{2}$)

Then, they have a dipolar magnetic moment μ :
 $\mu = \gamma J = \gamma \hbar I$

$\hbar$ = Planck's quantum constant
 γ = gyromagnetic ratio

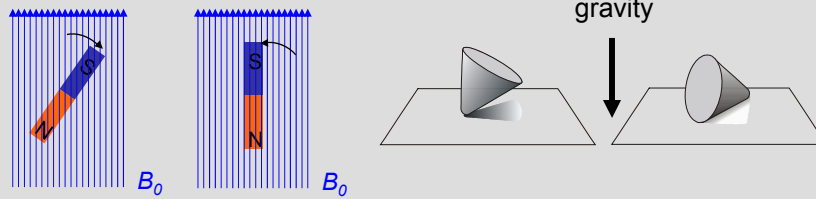
Motion of Magnetization

Magnetization and magnetic field:



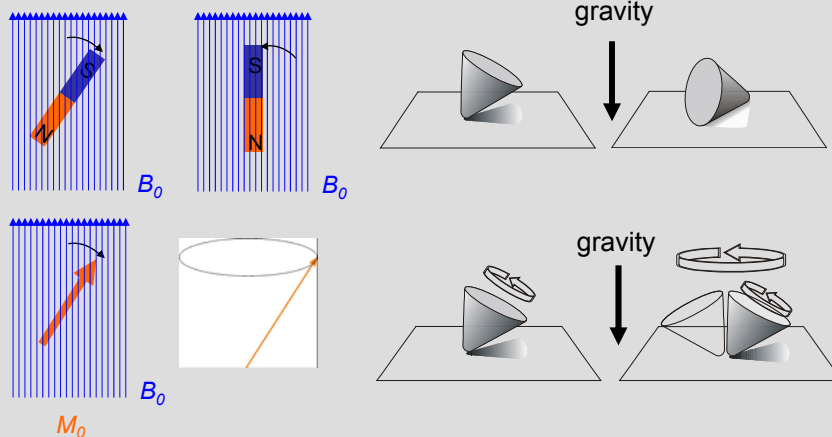
Motion of Magnetization

Magnetization and magnetic field:



Motion of Magnetization

Magnetization and magnetic field:



Motion of Magnetization

Motion of magnetization:

The temporal change of spin angular momentum J , and thus of the dipolar magnetic moment $\mu = \gamma J$ is equal to the torque that acts on μ .

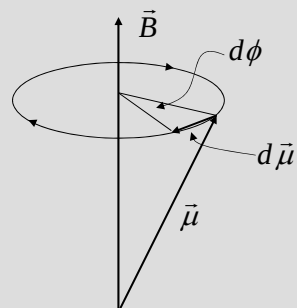
The torque that acts on a dipolar magnetic moment μ in a magnetic field B is given by $\mu \times B$.

Thus, the equation of motion becomes:

$$\frac{d\vec{\mu}}{dt} = \gamma(\vec{\mu} \times \vec{B})$$

Motion of Magnetization

Motion of magnetization: precession



$$\frac{d\vec{\mu}}{dt} = \gamma(\vec{\mu} \times \vec{B})$$

$$\frac{d\phi}{dt} = -\omega = \gamma B$$

Motion of Magnetization

Motion of magnetization:

The same equation is valid for the volume magnetization M :

$$\frac{d\vec{M}}{dt} = \gamma(\vec{M} \times \vec{B})$$

in Cartesian coordinates:

$$\frac{dM_x}{dt} = \gamma(M_y B_z - M_z B_y)$$

$$\frac{dM_y}{dt} = \gamma(M_z B_x - M_x B_z)$$

$$\frac{dM_z}{dt} = \gamma(M_x B_y - M_y B_x)$$

Motion of Magnetization

Motion of magnetization:

Example: $\vec{B} = (0, 0, B_0)$

$$\frac{dM_x}{dt} = \gamma M_y B_0$$

$$\frac{dM_y}{dt} = -\gamma M_x B_0$$

$$\frac{dM_z}{dt} = 0$$

Motion of Magnetization

Motion of magnetization:

Example: $\vec{B} = (0, 0, B_0)$

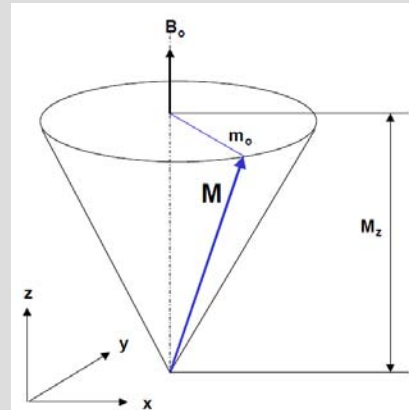
Solution

$$M_x = m_0 \cos(\omega_0 t + \delta)$$

$$M_y = -m_0 \sin(\omega_0 t + \delta)$$

$$M_z = \text{const}$$

with $\omega_0 = \gamma B_0$



Biomedical Magnetic Resonance: 3 Motion of Magnetization

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Motion of Magnetization

Motion of magnetization:

Back to equilibrium:

$$\frac{dM_x}{dt} = \gamma(M_y B_z - M_z B_y) - \frac{1}{T_2} M_x$$

$$\frac{dM_y}{dt} = \gamma(M_z B_x - M_x B_z) - \frac{1}{T_2} M_y$$

$$\frac{dM_z}{dt} = \gamma(M_x B_y - M_y B_x) - \frac{1}{T_1} (M_z - M_0)$$

Bloch Equation

T_1 : longitudinal Relaxation
 T_2 : transverse Relaxation

Biomedical Magnetic Resonance: 3 Motion of Magnetization

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Motion of Magnetization

Motion of magnetization:

Example: $B=0$

$$\frac{dM_x}{dt} = -\frac{1}{T_2} M_x$$

$$\frac{dM_y}{dt} = -\frac{1}{T_2} M_y$$

$$\frac{dM_z}{dt} = -\frac{1}{T_1} (M_z - M_0)$$

Motion of Magnetization

Motion of magnetization:

Solution:

$$M(t)_x = M_{xi} e^{-t/T_2}$$

$$M(t)_y = M_{yi} e^{-t/T_2}$$

$$M(t)_z = (M_{zi} - M_0) e^{-t/T_1} + M_0$$

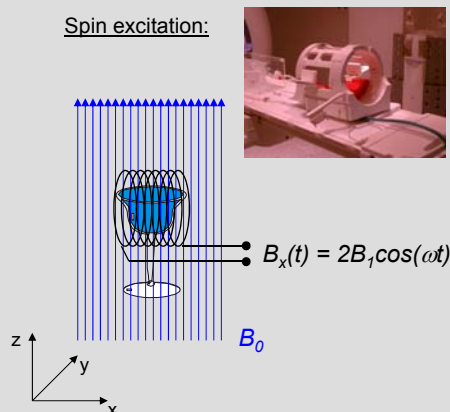
Initial condition $M_i = (M_{xi}, M_{yi}, M_{zi})$

Table of relaxation times at 1.5T and 3T

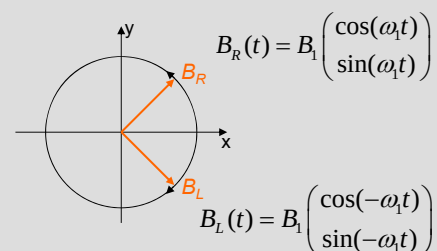
	1.5T		3T	
	T ₁ [ms]	T ₂ [ms]	T ₁ [ms]	T ₂ [ms]
Liver ¹	576	46/54	812	42
Skeletal muscle ¹	1008	44/35	1412	50/32
Heart ¹	1030	40	1471	47
Kidney ²	690	44	1194	56
Cartilage 0° ³	1024	30/42	1168	27/37
Cartilage 55° ³	1038	44	1156	43
White matter ³	884/778	72	1084	69/56
Gray matter ³	1124	95	1820/1470	99/71
Optic nerve ³	815	77	1083	78
Spinal cord ²	745	74	993	78
Blood ⁴	1441/1200	290/327	1932/1550	275

Excitation and Reception

Spin excitation:



Decomposition of $B_x(t)$ into two counterrotating fields:
 $B_x(t) = B_R(t) + B_L(t)$

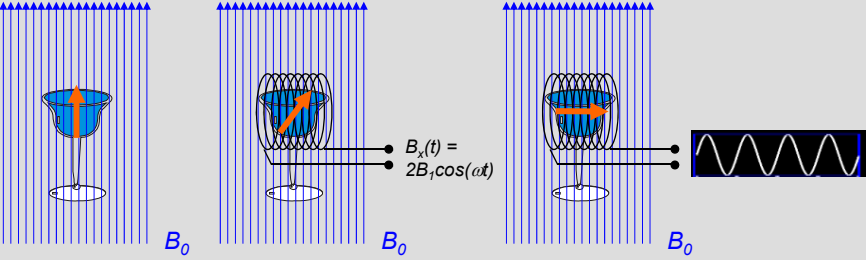




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Excitation and Reception



B_0 B_0 B_0

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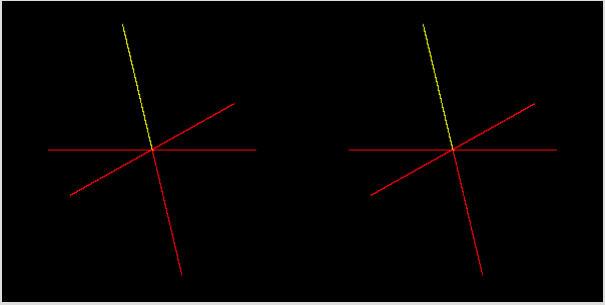


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Excitation and Reception

Rotating frame: $\left[\frac{d\vec{M}}{dt} \right]_{rot} = \gamma \vec{M} \times \vec{B}_1$ for $\omega_1 = \omega_0$ (on resonance)



lab frame
rotating frame

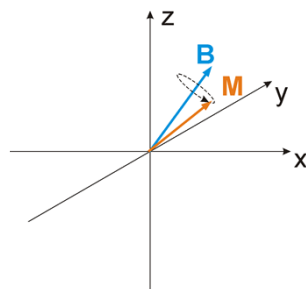
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Bloch equation in the rotating frame



Prof. Bloch

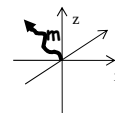
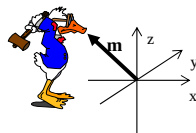
$$\frac{d}{dt} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \gamma \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} - \frac{1}{T_2} \begin{pmatrix} M_x \\ M_y \\ 0 \end{pmatrix} + \frac{1}{T_1} \begin{pmatrix} 0 \\ 0 \\ M_0 - M_z \end{pmatrix}$$



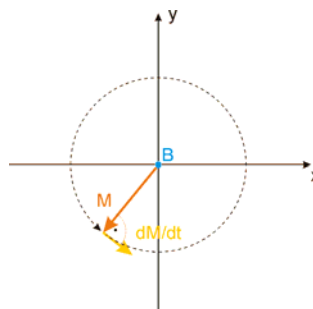
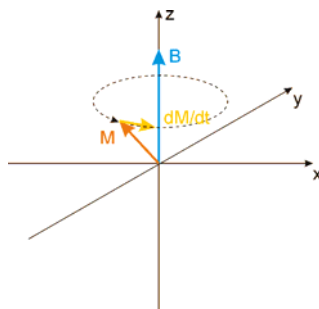
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Precession



$$\frac{d}{dt} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \gamma \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ B_z \end{pmatrix} \longrightarrow \mathbf{R}_z(\phi) = \begin{pmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

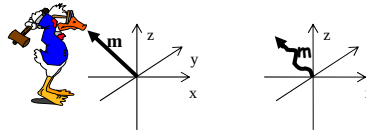


$$\phi = \gamma B_z t$$

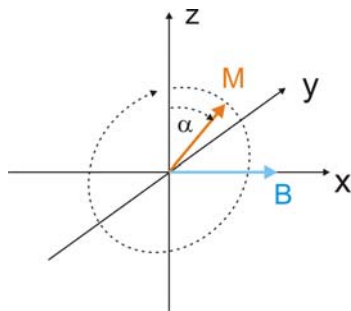
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RF-Puls



$$\frac{d}{dt} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \gamma \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} \times \begin{pmatrix} B_x \\ 0 \\ 0 \end{pmatrix} \longrightarrow \mathbf{R}_x(\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{pmatrix}$$

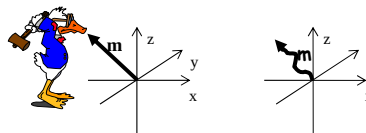


$$\alpha = \gamma B_x t$$

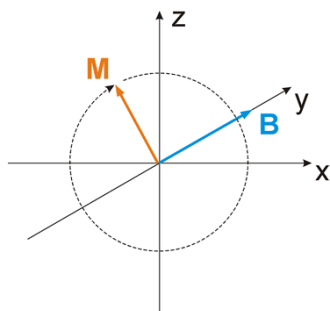
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RF-Puls



$$\frac{d}{dt} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \gamma \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} \times \begin{pmatrix} 0 \\ B_y \\ 0 \end{pmatrix} \longrightarrow \mathbf{R}_y(\alpha) = \begin{pmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{pmatrix}$$

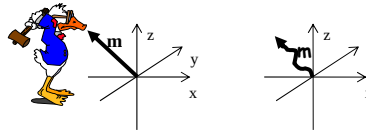


$$\alpha = \gamma B_y t$$

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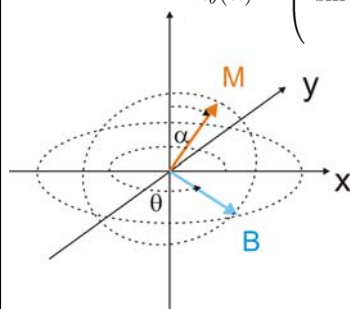
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RF-Puls



$$\frac{d}{dt} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \gamma \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} \longrightarrow$$

$$\mathbf{R}_\theta(\alpha) = \begin{pmatrix} \cos^2 \theta + \sin^2 \theta \cos \alpha & \sin \theta \cos \theta (1 - \cos \alpha) & \sin \theta \sin \alpha \\ \sin \theta \cos \theta (1 - \cos \alpha) & \sin^2 \theta + \cos^2 \theta \cos \alpha & -\cos \theta \sin \alpha \\ -\sin \theta \sin \alpha & \cos \theta \sin \alpha & \cos \alpha \end{pmatrix}$$



$$\alpha = \gamma |B| t$$

$$\theta = \angle(B_x, B_y)$$

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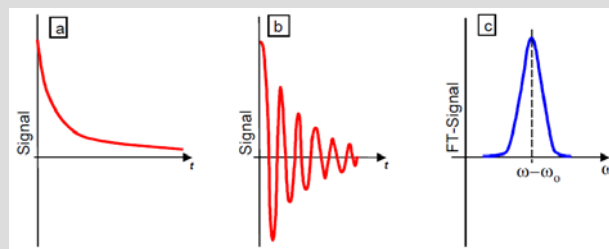
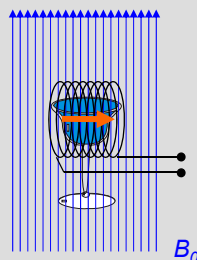
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Excitation and Reception

FID: free induction decay

precession of magnetization



a) on-resonance, b) off-resonance, c) spectrum

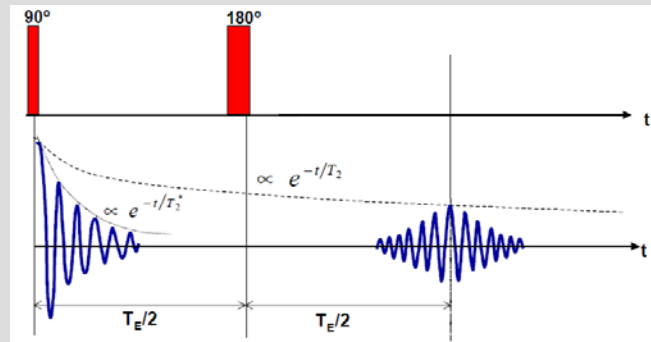
Biomedical Magnetic Resonance: 4 Excitation and Reception

K. Scheffler, 2017

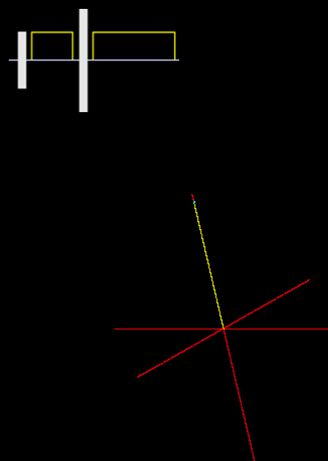
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Excitation and Reception

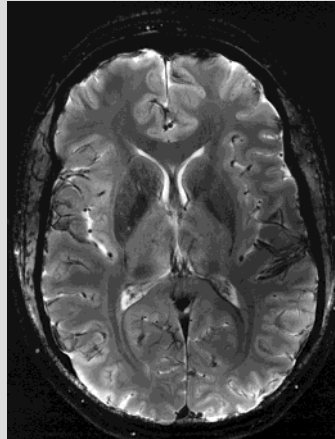
Spinecho



Spin Echoes



Magnetic Resonance Imaging

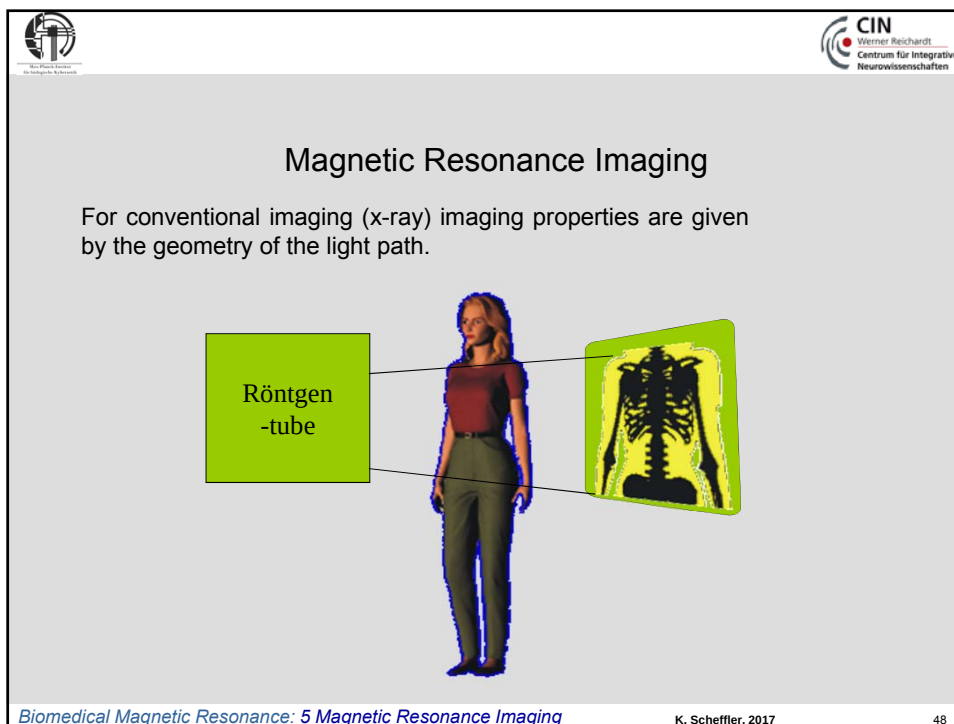
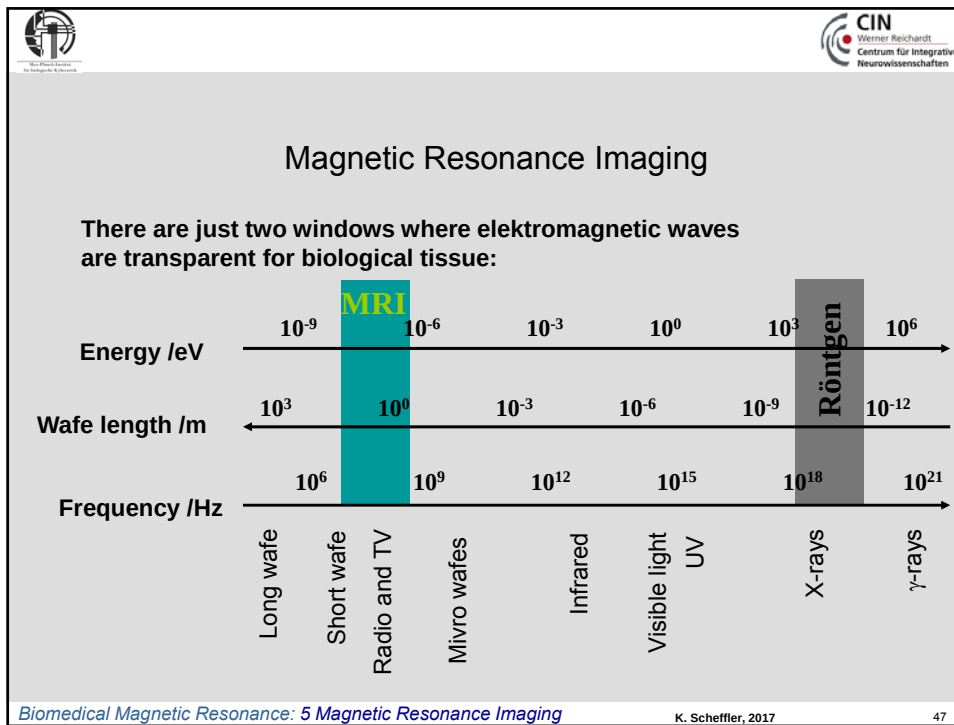


Imaging = spatial discrimination

Magnetic Resonance Imaging

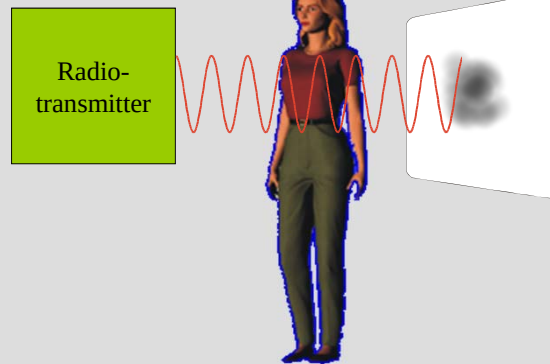
Contents:

- Magnetic Field Gradients
- Selective Excitation
- Hardware

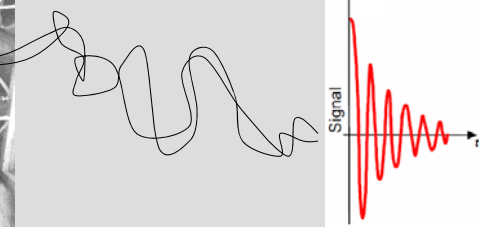
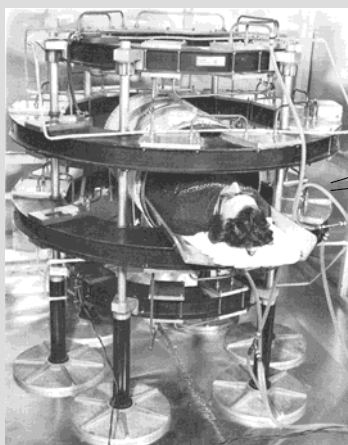


Magnetic Resonance Imaging

Objects that are smaller than the wave length cannot be imaged (geometric imaging not possible!)



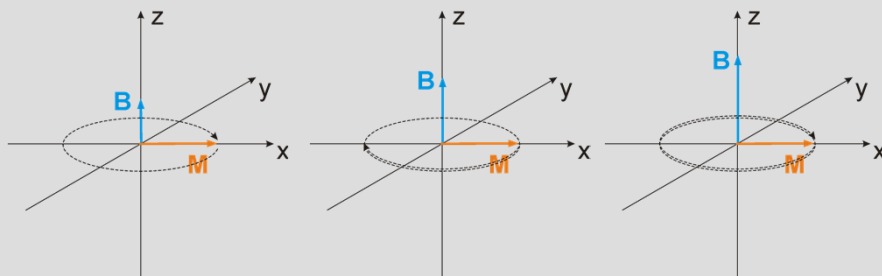
Magnetic Resonance Imaging



Magnetic Resonance Imaging

Bloch equation

$$\frac{d\vec{M}}{dt} = \gamma (\vec{M} \times \vec{B})$$

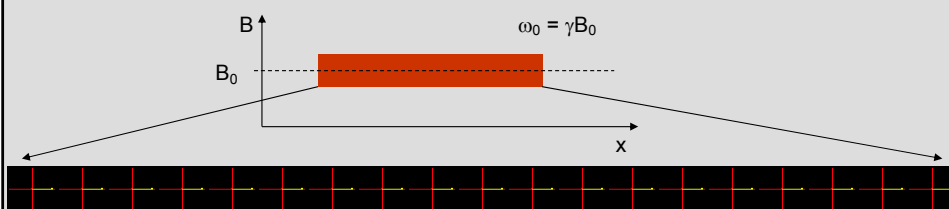


$$\omega = \gamma B \quad \text{Larmor equation}$$

Magnetic Resonance Imaging

Magnetic Field Gradients

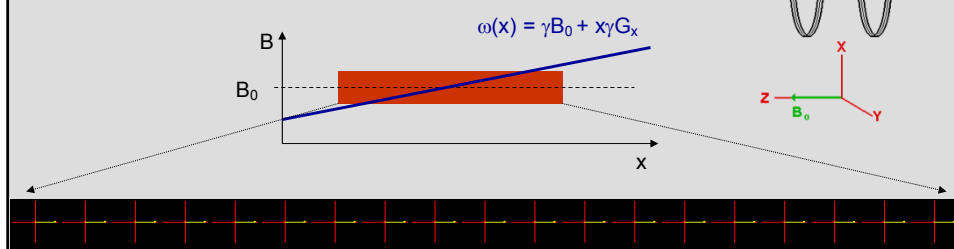
Sample in a homogenous magnetic field B_0



Magnetic Resonance Imaging

Magnetic Field Gradients

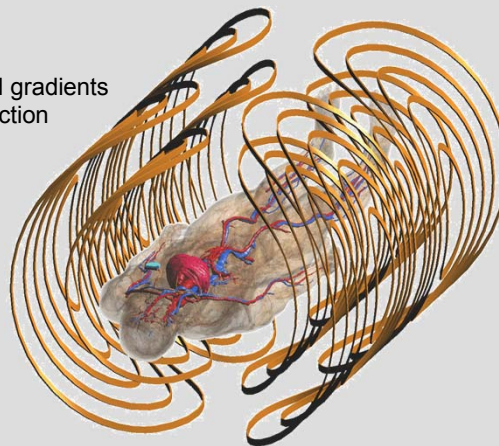
Sample in a magnetic gradient field $B_0 + \Delta B$



Magnetic Resonance Imaging

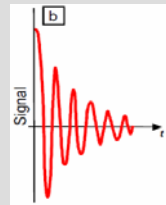
Magnetic Field Gradients

Switchable, linear magnetic field gradients independently in x, y and z direction

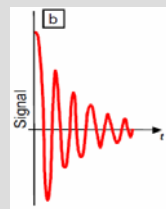
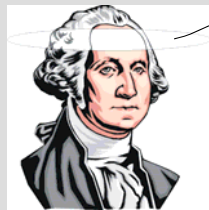


Magnetic Resonance Imaging

Selective Excitation



Signal from all magnetization inside coil

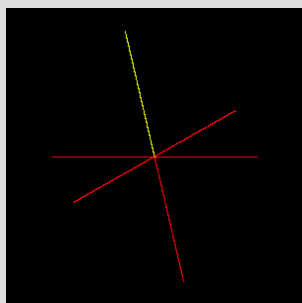


Signal only from a selected volume/slice

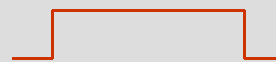
Magnetic Resonance Imaging

Selective Excitation

Frequency of B_1 field is $\omega = \gamma B_0$ (resonance condition)



B_1 :

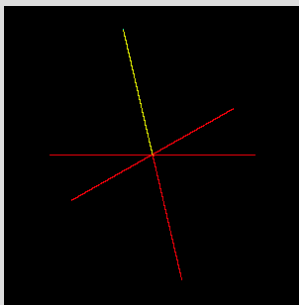


Result: 90 degrees pulse!

Magnetic Resonance Imaging

Selective Excitation

Frequency of B_1 field is $\omega > \gamma B_0$ (off resonance condition)



B_1 :

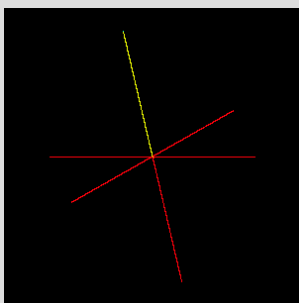


Result: 32 degrees pulse!

Magnetic Resonance Imaging

Selective Excitation

Frequency of B_1 field is $\omega \gg \gamma B_0$ (off resonance condition)



B_1 :



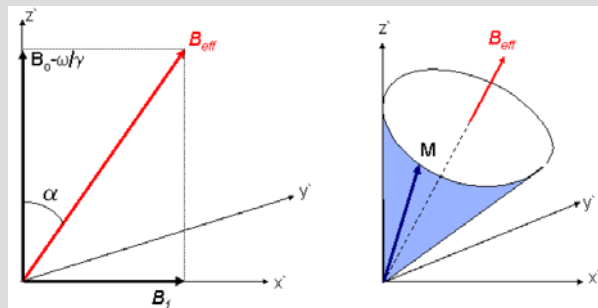
Result: 1 degrees pulse!

Magnetic Resonance Imaging

Selective Excitation

$$\left[\frac{d\vec{M}}{dt} \right]_{rot} = \gamma \vec{M} \times \vec{B}_{eff}$$

$$\vec{B}_{eff} = \begin{pmatrix} B_1 \\ 0 \\ B_0 - \frac{\omega_1}{\gamma} \end{pmatrix}$$



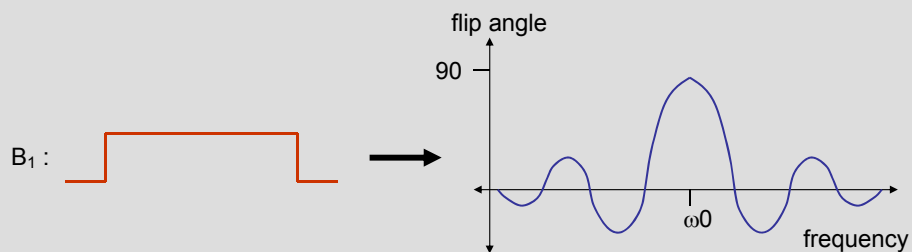
Biomedical Magnetic Resonance: 5 Magnetic Resonance Imaging

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Magnetic Resonance Imaging

Selective Excitation

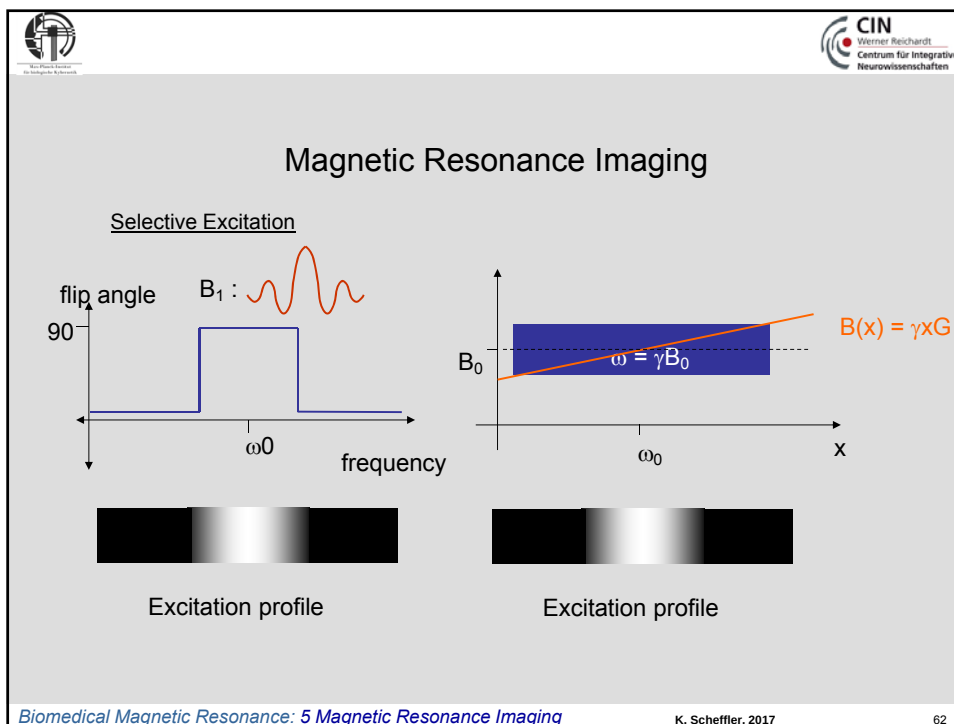
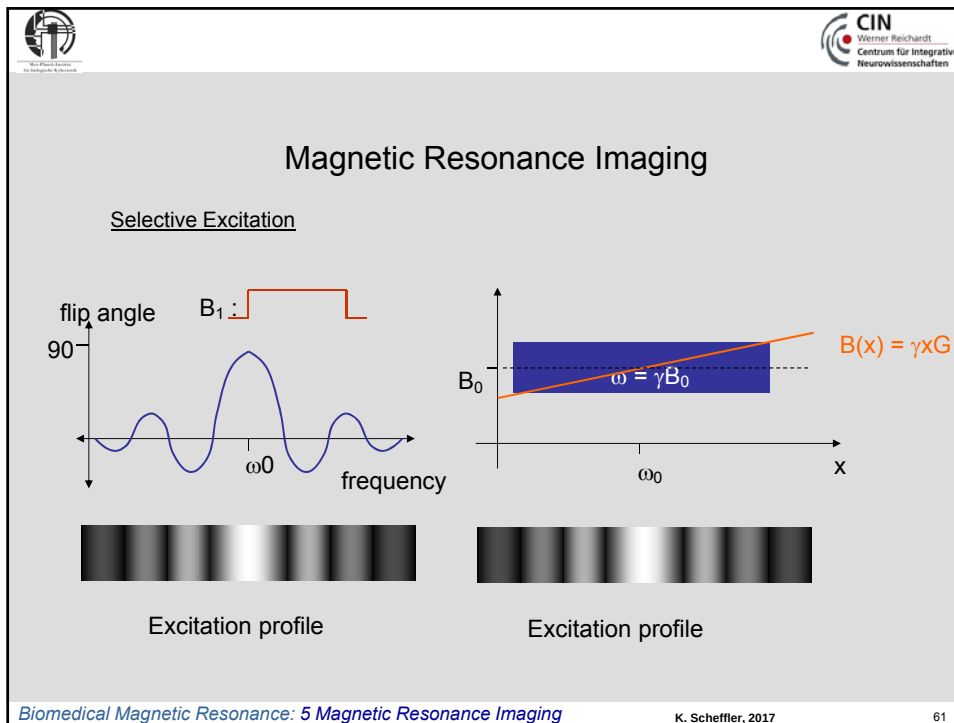


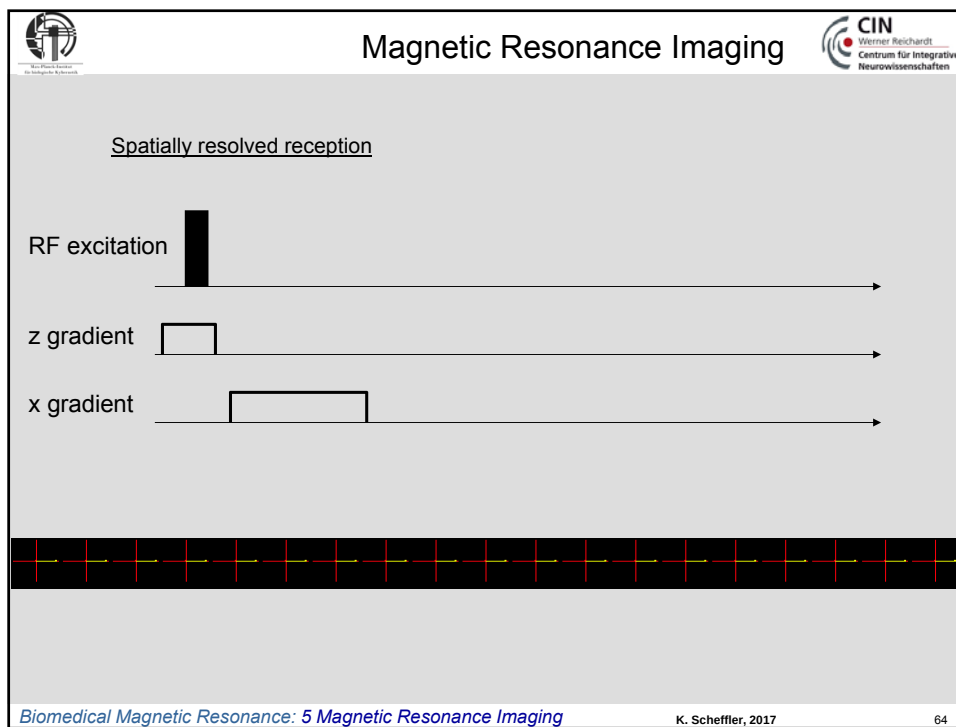
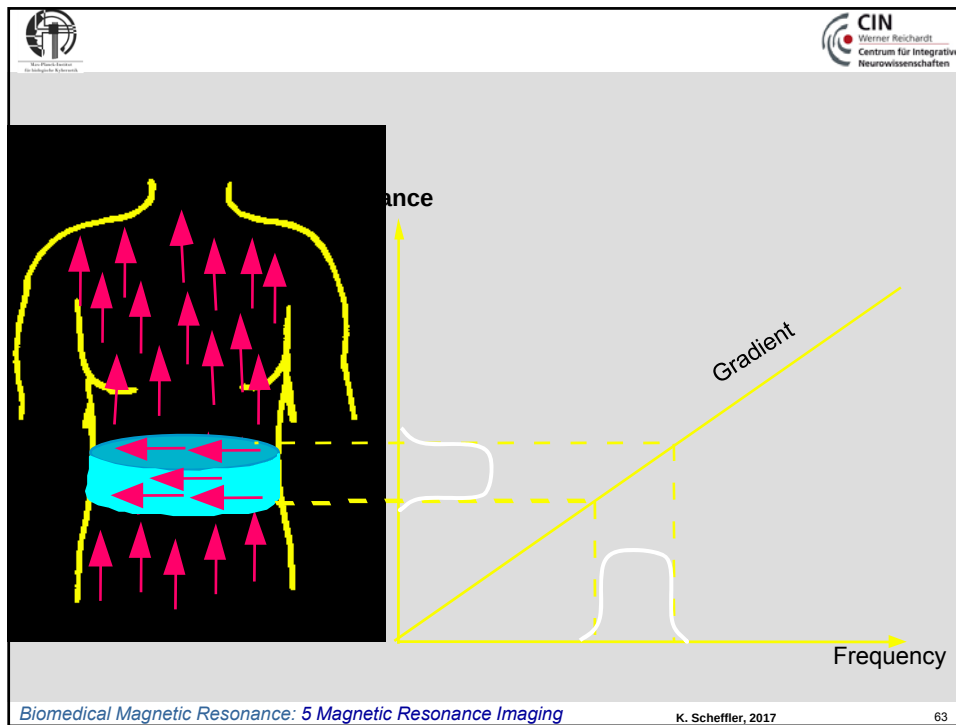
How can this be used to achieve a spatially selective excitation ?

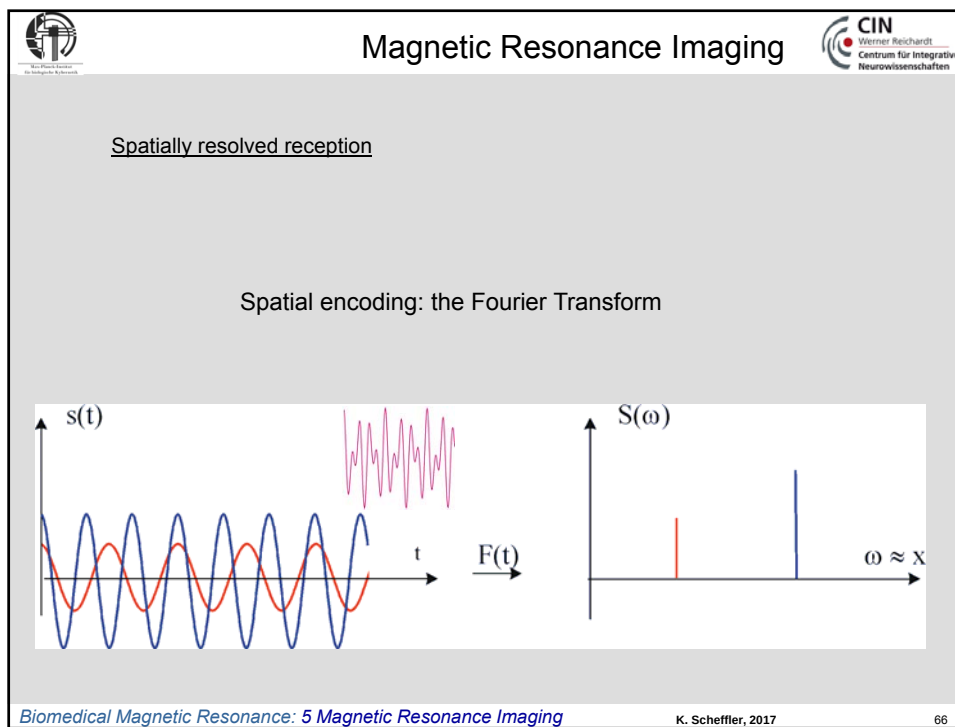
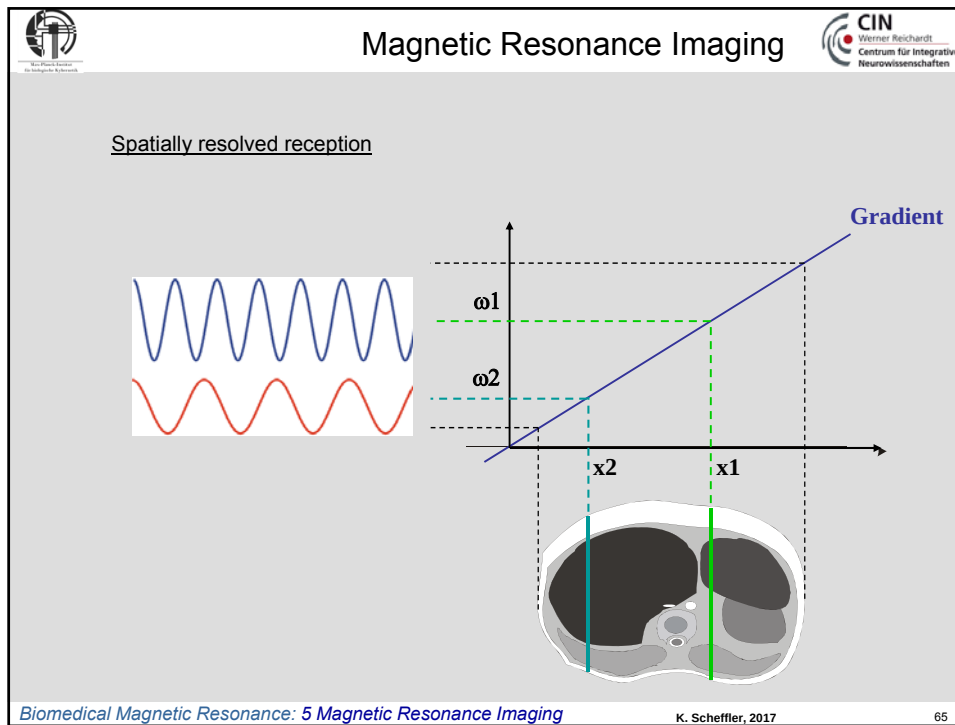
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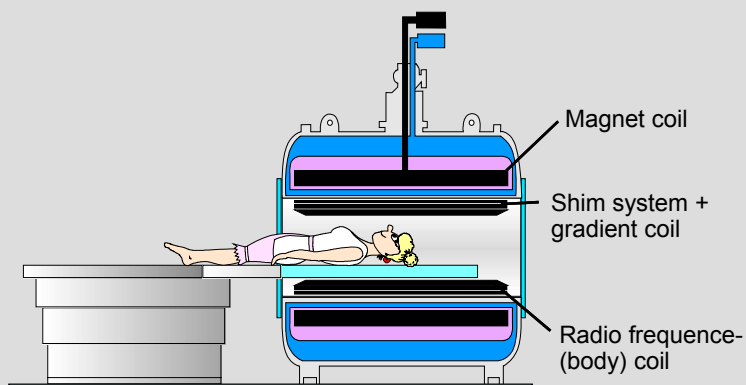


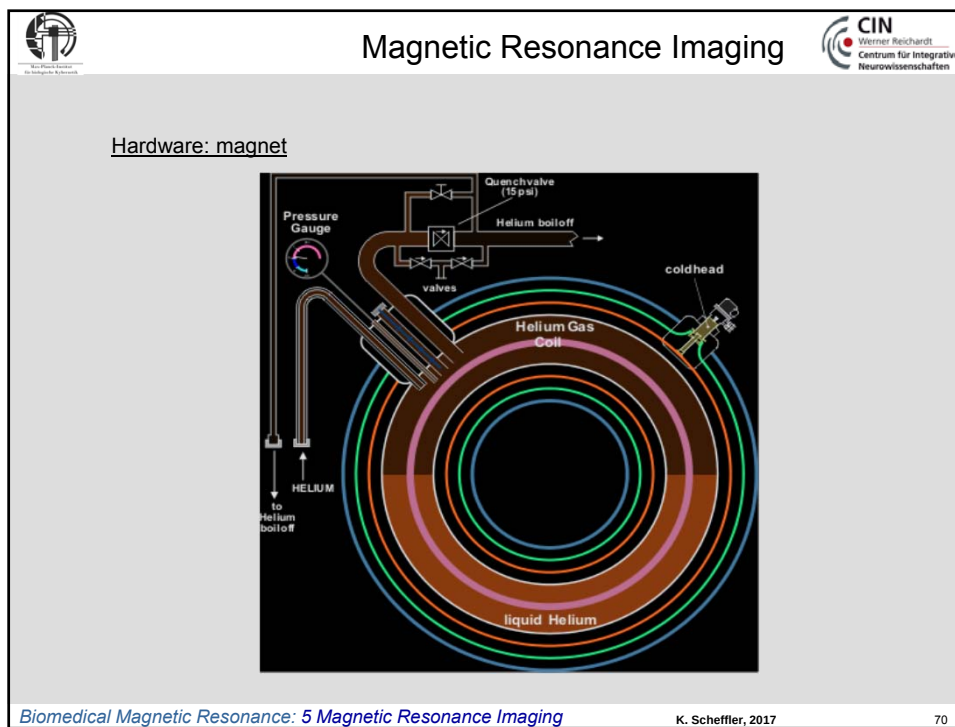
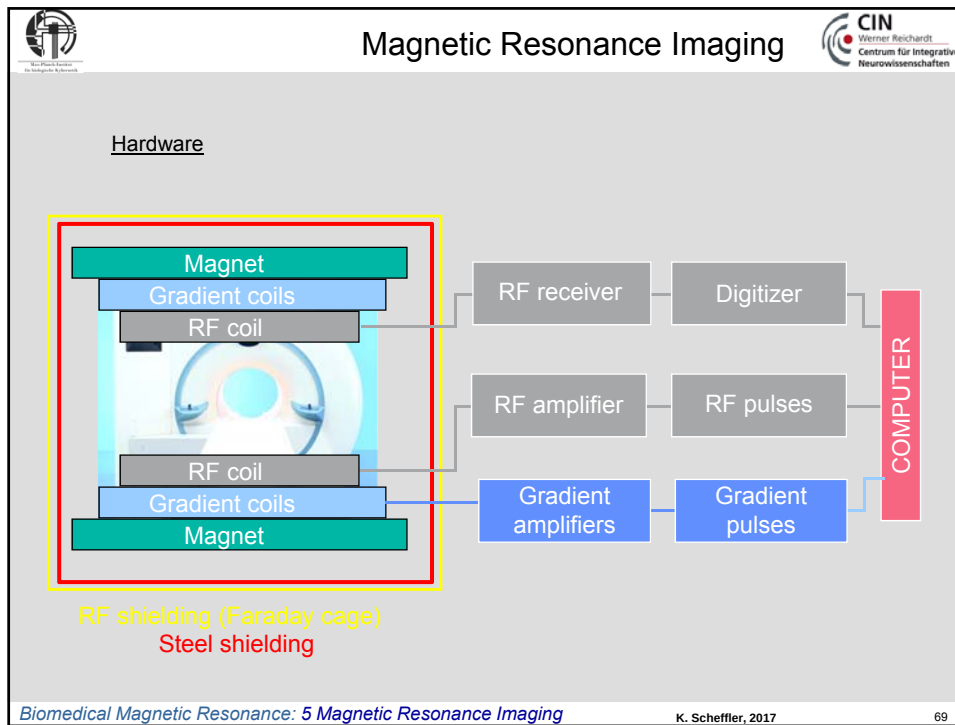


Hardware



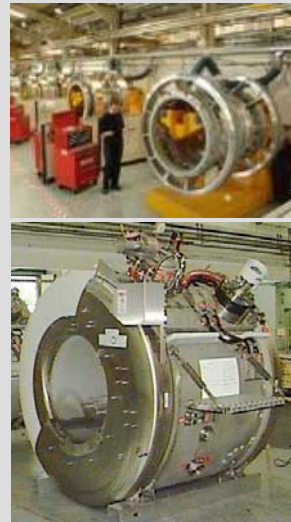
Hardware





Hardware: magnet

- The material of the superconductive wire is Niob-Titan (NbTi).
- At -269° the wire offers no resistance to the current ($-269^{\circ} = 4^{\circ}$ Kelvin).
- Liquid helium keeps the system cold at such low temperature.
- With no resistance the current can flow in the magnet „forever“ while keeping the magnetic field stable



Hardware: magnet



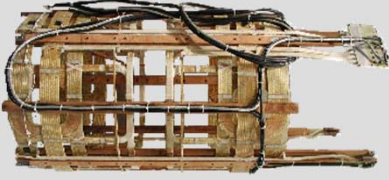


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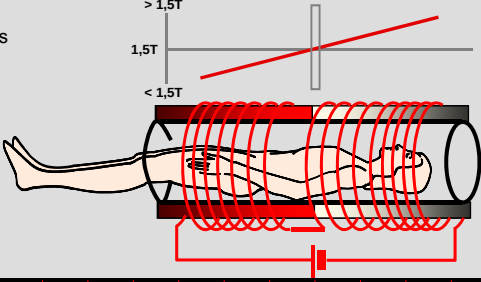
Magnetic Resonance Imaging



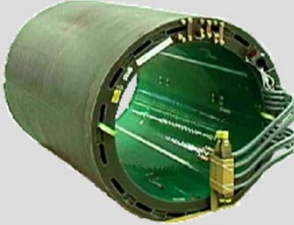
Hardware: gradient coil



1986
3 mT/m in 1.5ms



> 1.5T
1.5T
< 1.5T



2006
45 mT/m in 0.2ms



Biomedical Magnetic Resonance: 5 Magnetic Resonance Imaging

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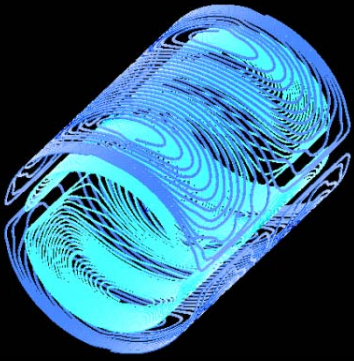


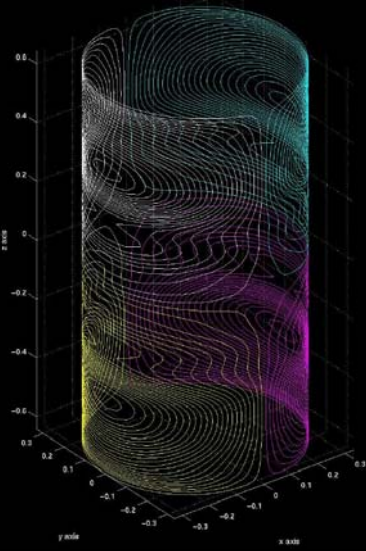
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Magnetic Resonance Imaging



Hardware: gradient coil





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Hardware: gradient amplifier

- Voltage:
max. 2 kV
 - Current:
max. 600 A
per axis
- (1.2 MW / axis)



Hardware: rf amplifier

For 1.5 T system:

- 15 kW Pulse power
- 600 W average power
- System control compensates for non-linearities and phase errors



Hardware: rf coils

