

Fluid dynamics and nano and micro fluidics

by Quim Martí Baena

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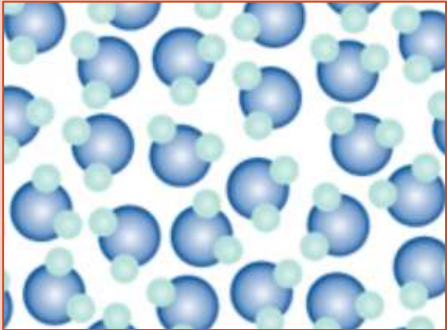
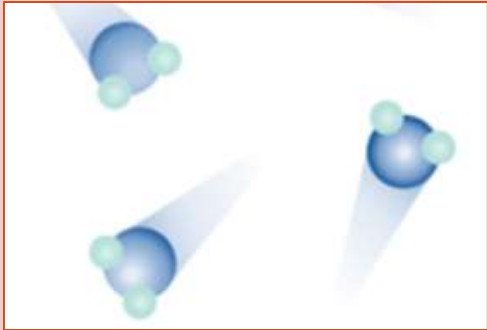


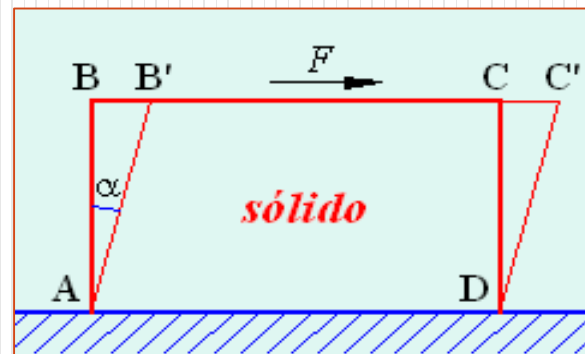
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- Mass conservation principle and Bernoulli
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What is a fluid?

Liquids	Gases
Molecules are near, cohesive forces are strong.	Molecules are separated, cohesive forces are weak.
Uncompressible, conserve their volume.	Can be compressed, no defined volume.
	



Properties of fluids

Viscosity

Quantitative measurement of the **resistance of a fluid to movement.**



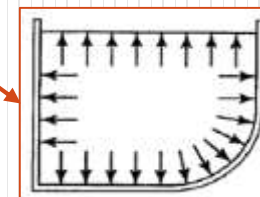
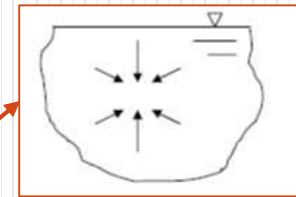
Compresibility

Volume change that occurs in a fluid when a pressure change is applied.

$$\beta = -\frac{1}{V} \frac{\partial V}{\partial p}$$

Pressure

The **amount of force** applied on a unit area (Surface).



Volumetric and mass flow

Mass flow

Amount of fluid in mass that crosses a surface per unit of time.

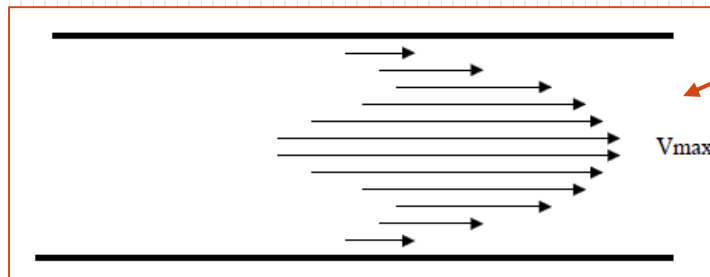
$$Q = \frac{\dot{m}}{\rho}$$

Volumetric flow

Amount of fluid in volume that crosses a surface per unit of time.

$$Q = \frac{dV}{dt}$$

$$Q = v_m A$$

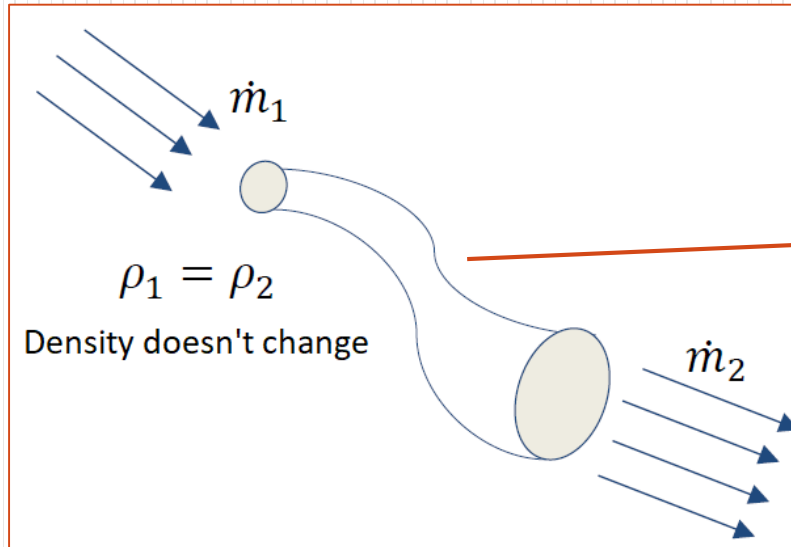


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Mass conservation principle

The mass that enters the system is equal to the mass that exits the system.



System with no loss

$$\dot{m}_1 = \dot{m}_2 \quad \dot{m} = \rho v_m A$$

$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2$$

$$v_1 A_1 = v_2 A_2$$

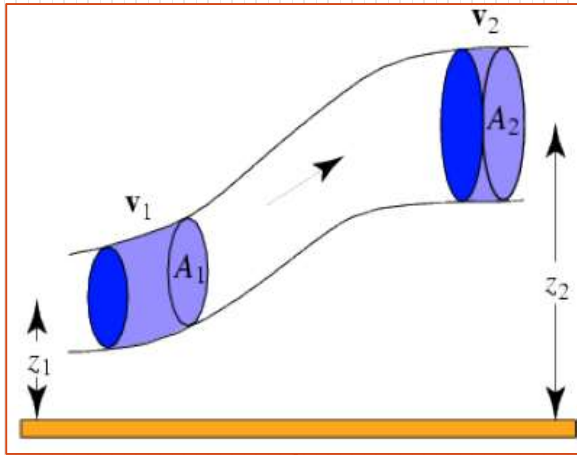
$$Q_1 = Q_2$$

Uncompressible
fluid (density
doesn't change)

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Bernoulli equation



Three types of energies considered

$$E_c = \frac{1}{2}mv^2$$

$$E_p = mgh$$

$$F = pA$$

$$W = Fd \quad V = Ad \quad E_f = pV$$

$$E_1 = E_2 \rightarrow \text{Energy conservation}$$

$$E_{c1} + E_{p1} + E_{f1} = E_{c2} + E_{p2} + E_{f2}$$

$$\frac{1}{2}mv_1^2 + mgz_1 + \frac{mp_1}{\rho} = \frac{1}{2}mv_2^2 + mgz_2 + \frac{mp_2}{\rho}$$

Bernoulli equation

$$\frac{v_1^2}{2} + gz_1 + \frac{p_1}{\rho} = \frac{v_2^2}{2} + gz_2 + \frac{p_2}{\rho}$$

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Bernoulli equation example

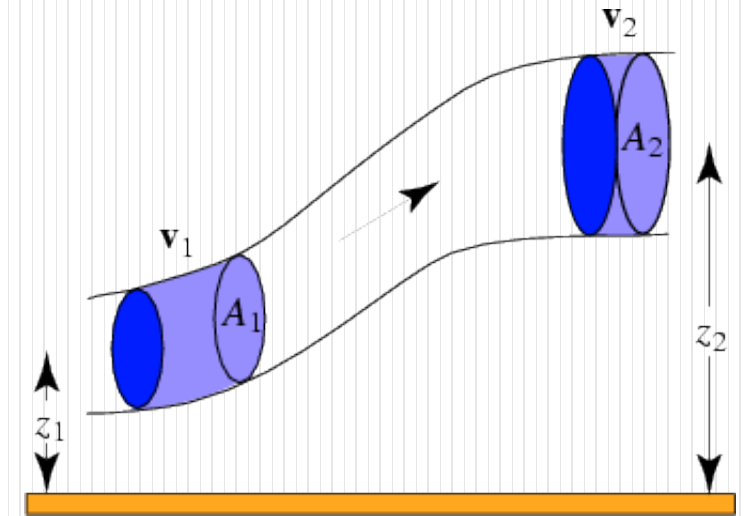
$$\frac{v_1^2}{2} + gz_1 + \frac{p_1}{\rho} = \frac{v_2^2}{2} + gz_2 + \frac{p_2}{\rho}$$

$$\frac{10^{-6^2}}{2} + 9.81 \cdot 0 + \frac{p_1}{1000} = \frac{5^{-6^2}}{2} + 9.81 \cdot 1 + \frac{5}{1000}$$

$$\frac{p_1}{1000} = \frac{5^{-6^2}}{2} + 9.81 \cdot 1 + \frac{5}{1000} - \frac{10^{-6^2}}{2}$$

$$p_1 = 1000 \cdot \left(\frac{5^{-6^2}}{2} + 9.81 \cdot 1 + \frac{5}{1000} - \frac{10^{-6^2}}{2} \right)$$

$$p_1 = 9815 \text{ Pa}$$



$$v_1 = 10 \frac{\mu\text{m}}{\text{s}} \quad v_2 = 5 \frac{\mu\text{m}}{\text{s}}$$

$$z_1 = 0\text{mm} \quad z_2 = 1\text{mm}$$

$$p_1 = ??? \quad p_2 = 5\text{Pa}$$



Bernoulli equation and its limitations

- Flux that is in a stable equilibrium point
- Constant density fluids (must be incompressible, so only liquids)
- There is no loss of energy by friction

We need an equation that can be used in the nano and micro scales

Viscosity must be taken into account



Real fluids and viscosity

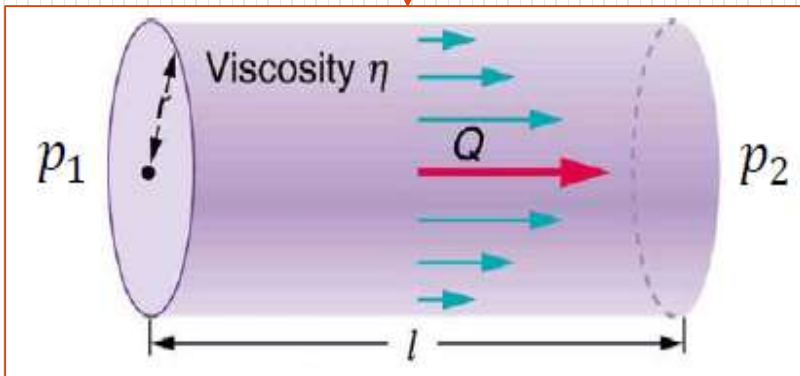
Viscosity

Responsible for the friction forces acting between the fluid layers.

Depends on temperature, not pressure

$$\frac{v_1^2}{2} + gz_1 + \frac{p_1}{\rho} - \Delta h_{12} = \frac{v_2^2}{2} + gz_2 + \frac{p_2}{\rho}$$

No-slip condition: the fluid in the border with the wall has 0 velocity



Forces in a fluid

Laminar flow	Turbulent flow
Viscous forces dominate, flux has an ordered movement.	Inertia forces dominate, flux has a disordered movement.
Particles move in parallel lines and there is no intermixing.	Mixing between layers
	

In nano and micro scale the viscous forces dominate so the flow will be mainly **laminar**.



Reynolds number

$$Re = \frac{\textit{Inertia forces}}{\textit{Viscous forces}}$$

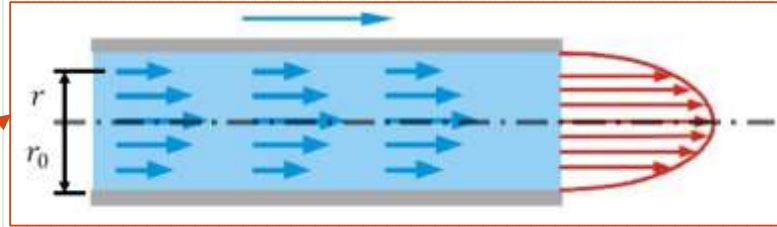
- If $Re < 2000$ laminar flow (high viscosity and low speed)
- If $2000 < Re < 4000$ transient flow (can't be determined if it's laminar or turbulent)
- If $4000 < Re$ turbulent flow (low viscosity and high speed)



Velocity profiles

Laminar flow

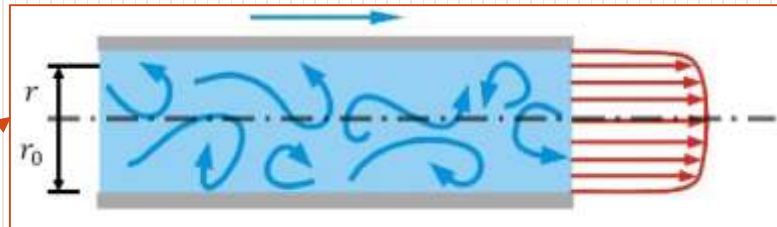
Parabolic shape



$$U = 2v \left[1 - \left(\frac{r}{r_0} \right)^2 \right]$$

Turbulent flow

Linear shape



$$U = v \left[1 + 1.43\sqrt{f} + 2.15\sqrt{f} \log_{10} \left(1 - \frac{r}{r_0} \right) \right]$$



Drag force

Force that acts at the particles in suspension, important to model sedimentation.

$$F_a = C_a \left(\frac{\rho v^2}{2} \right) A$$

Pressure component: Due to particles disturbing fluid flow.

Friction component: Due to shear forces in the surface of the particles.

Friction coefficient

Dynamic pressure

Depends on the body shape, RE , surface size and the influence of other bodies or nearby surfaces.



Stokes' law

Nano and
micro world

Laminar flow and slow
velocities, drag depends
only on friction.

Ca is simplifi



2

$$F_a = 12\mu v \pi D$$

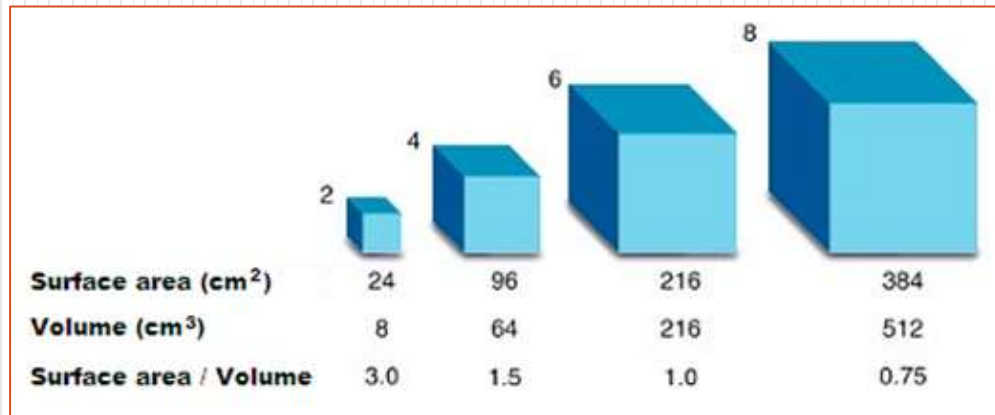
tion velocity can be calculated by
the forces that act on the particle
ion and the weight of the particle.

$$v_s = \frac{(\rho_c - \rho)gD^2}{18\mu}$$

ρ_c = particle density
 ρ =fluid density

D =diameter
 μ =viscosity coefficient

Capillary effects



Dominance of capillary effects due to high surface to volume ratio

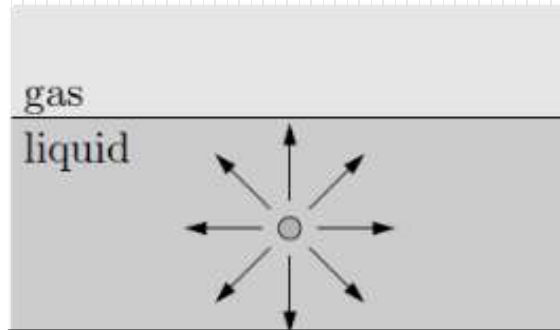
$$\Delta G = \Delta H - T\Delta S$$

Minimize the Gibbs free energy to reach an equilibrium

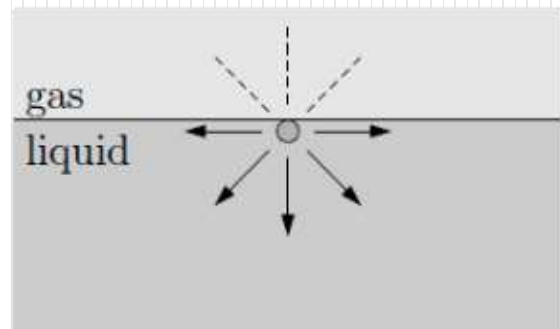
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Surface tension



Bonds in all directions,
resultant force is 0.



Some chemical bonds are
missing in the surface.

The energy of the surface
molecules is higher than
the ones inside the liquid.

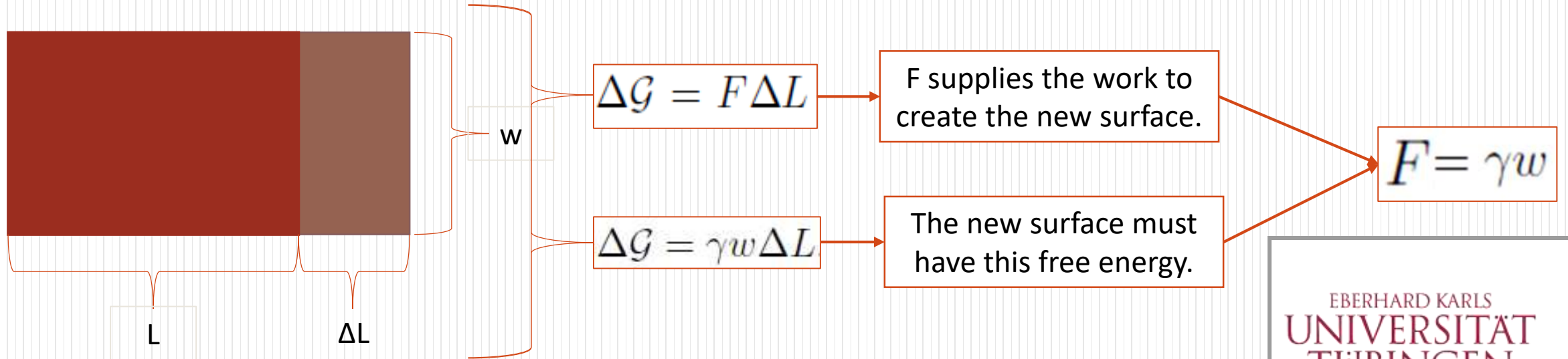
Energy and surface must be
minimized to reach stability.



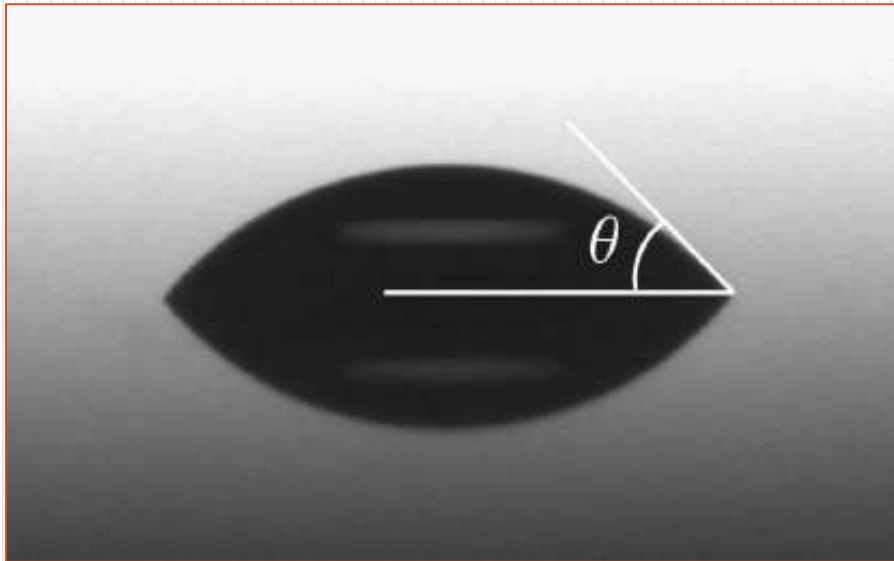
Surface tension

$$\gamma \equiv \left(\frac{\partial \mathcal{G}}{\partial \mathcal{A}} \right)_{p,T}$$

Surface tension is defined as Gibbs free energy divided by area.



Contact angle



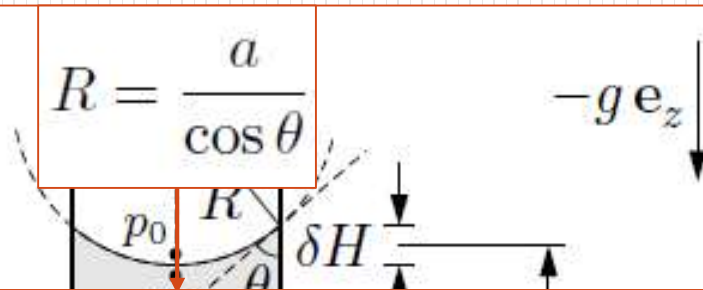
- Contact angle θ is the angle between the solid-liquid surface and the gas-liquid surface.
- Appears in the contact line between three phases.
- Contact angle depends on three surface tensions.

$$\cos \theta = \frac{\gamma_{sg} - \gamma_{sl}}{\gamma_{lg}}$$

Angle in equilibrium
(minimize energy)

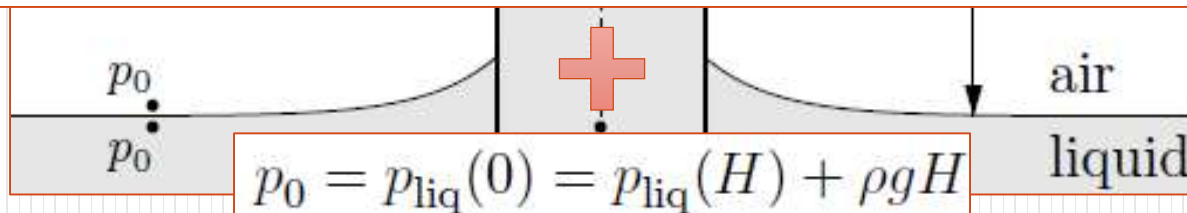


Capillary rise



$$p_{\text{liq}}(H) = p_0 - \Delta p_{\text{surf}} = p_0 - \frac{2\gamma}{R} = p_0 - \frac{2\gamma}{a} \cos \theta.$$

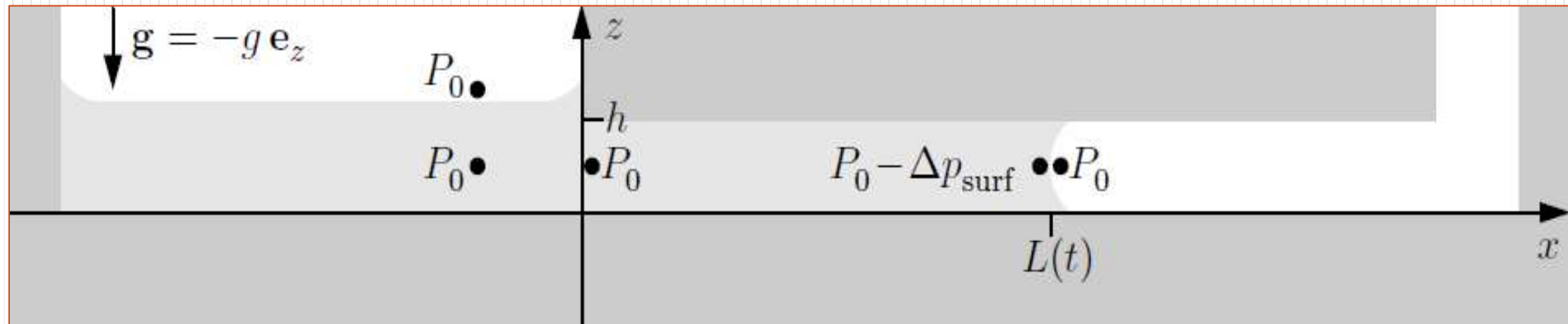
Can we determine the capillary rise height H ?



$$H = \frac{2}{\rho g a} (\gamma_{\text{sg}} - \gamma_{\text{sl}})$$



Capillary pumps



- Is a passive mechanism, it does not rely on an external energy source.
- There is no gravity to stop the “rise”, so it continues until the tube ends.
- Capillary flow rate generally depends on the liquid sample viscosity and surface energy.



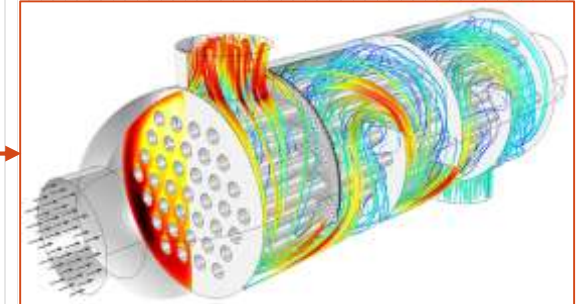
FEM and numerical methods

Finite elements
method (FEM)

Most physical systems can be described by **partial derivative equations (PDE)**.

$$\frac{\partial u}{\partial x}(x, y) = 0$$

Solving PDEs with analytical methods is impossible for arbitrary geometries.



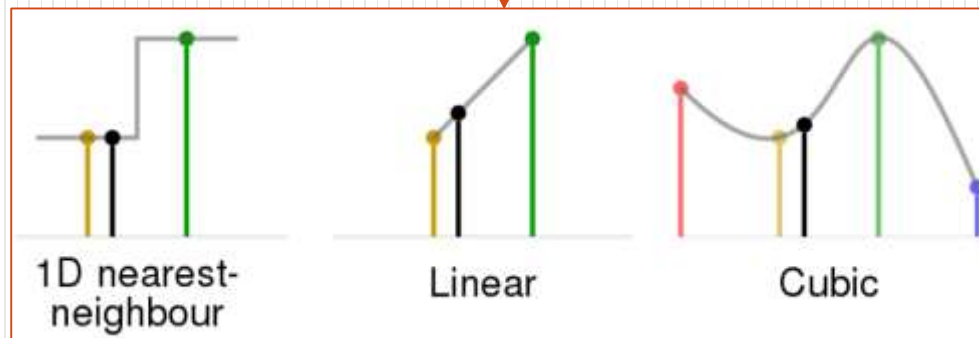
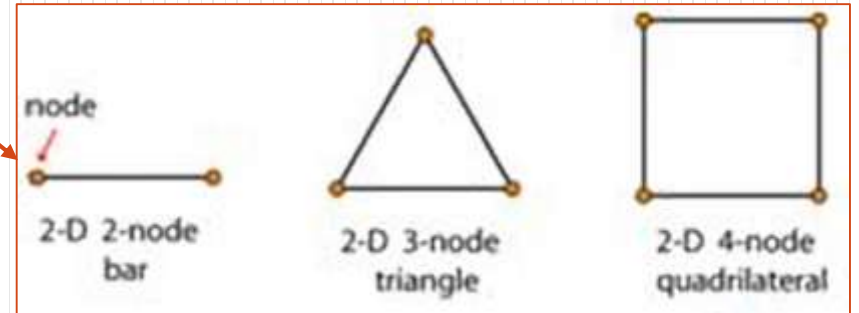
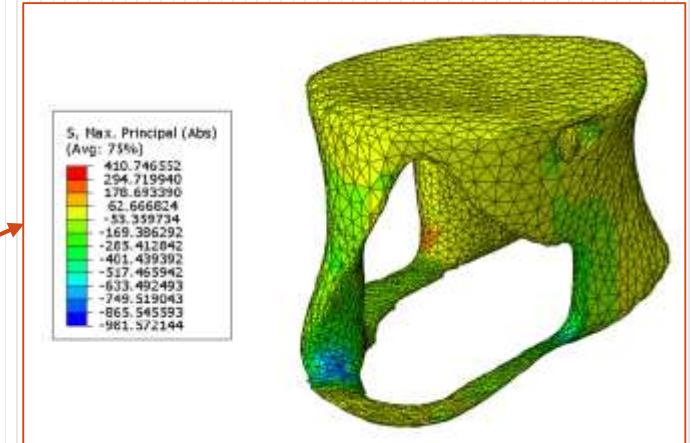
FEM is a **numerical technique** for solving PDEs in an approximate way using simulations.



FEM basic idea

FEM divides the object to analyze in a **set of finite elements**, with a certain number of nodes

FEM provides a methodology for solving PDEs at the nodes. To get information outside the nodes -> **Interpolation**



FEM basic idea

The idea of FEM is **solving a linear system of equations** with a number of unknowns equal to the number of nodes in the system.

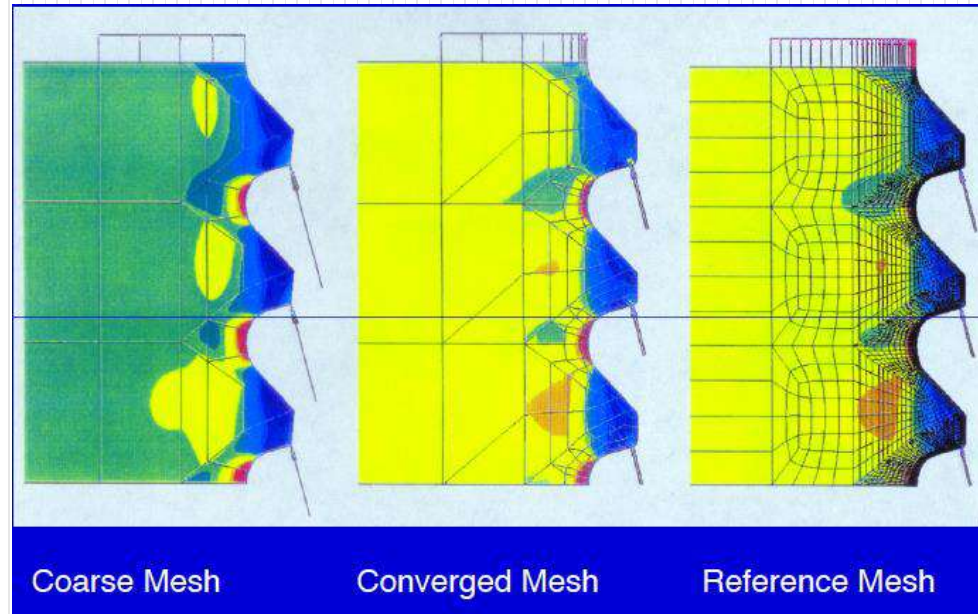
$$\begin{aligned} \bullet 8a - b &= 9 \\ 4a + 9b &= 7 \end{aligned}$$

$$\begin{aligned} \bullet 3a - b + 14c &= 7 \\ 2a + 2b + 3c &= 0 \\ a - 12b - 18c &= 33 \end{aligned}$$

$$\begin{aligned} \bullet 3r + s - 7t &= 15 \\ r - 12s + t &= 0 \\ 5s - 4t &= 8 \end{aligned}$$

$$\begin{aligned} \bullet x + 7y &= 9 \\ 2x - y &= 18 \end{aligned}$$

To obtain a more accurate solution more elements and nodes are needed, the mesh need to be fine enough.



A FEM application example

Implementation of an innovative prosthesis for osteoporosis

Pau Pomes, Quim Marti, Judith Torras, Mitar Nikolic

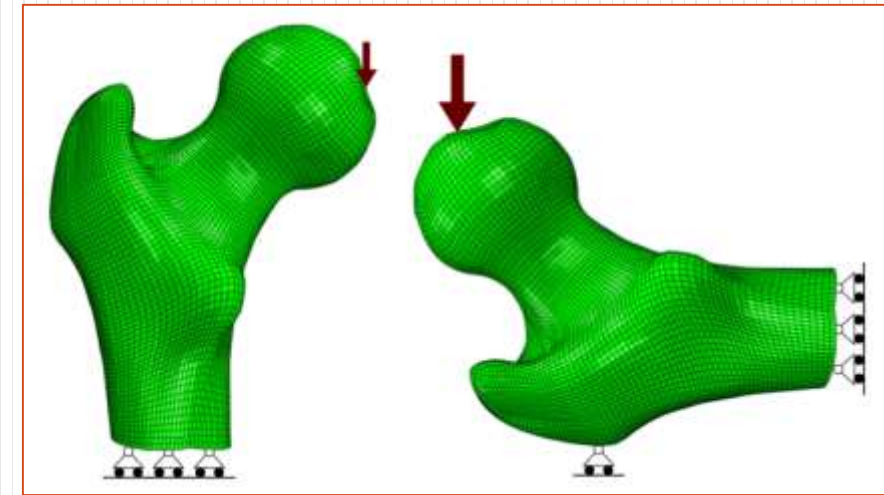
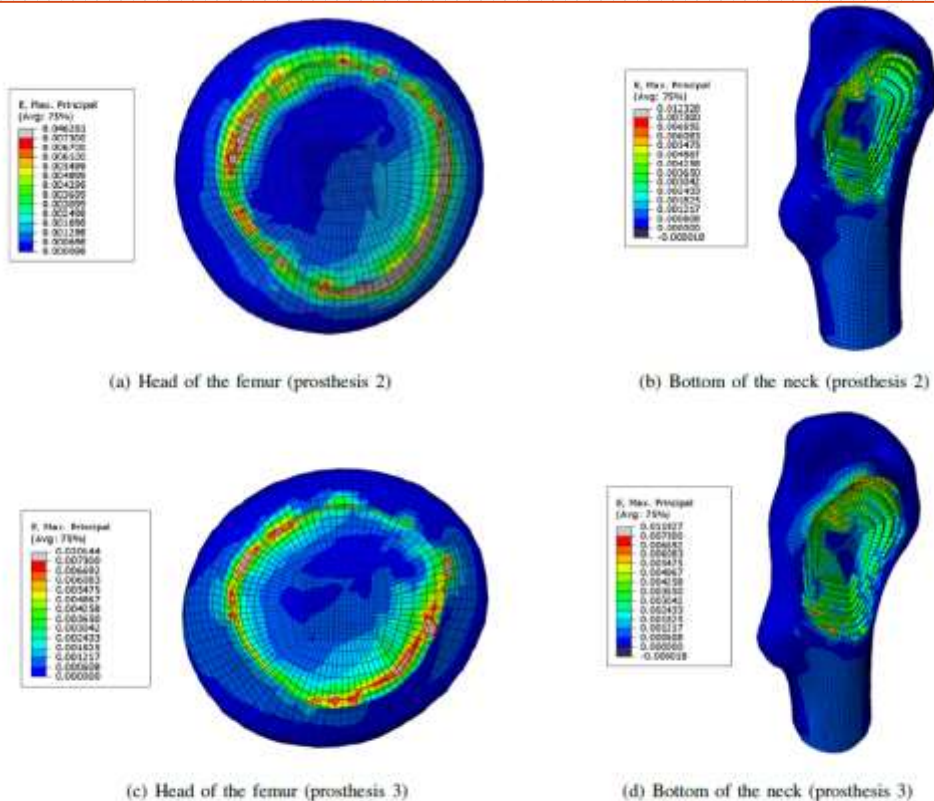
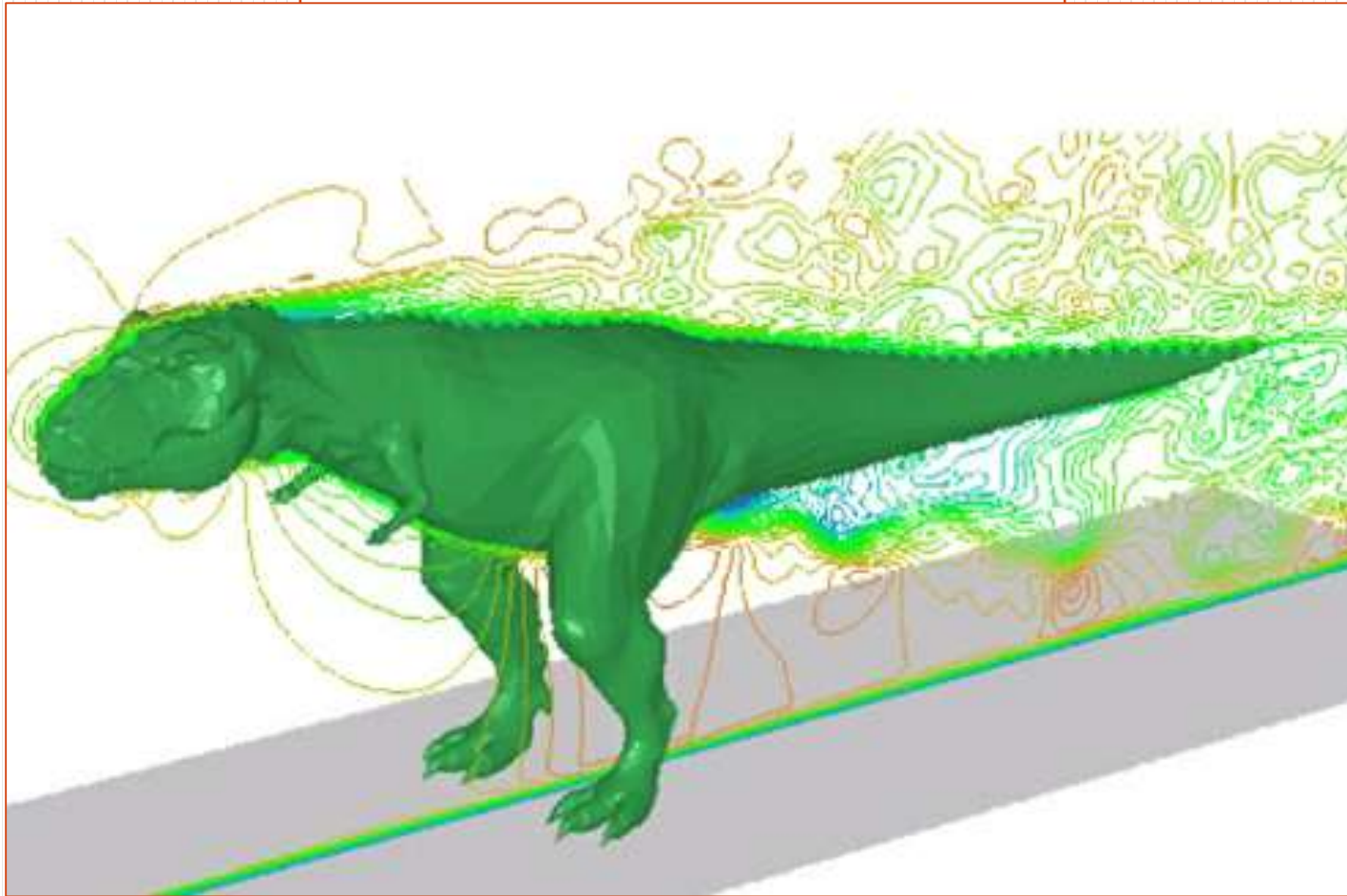


Fig. 7: Maximum Principal strains in the upper and lower regions of the bone in contact with the prosthesis. Regions that surpassed the limit tensile strains are marked in gray.

FEM question

What's wrong with this simulation?



Thanks for your attention

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Bibliography

- *Henrik Bruus*, Theoretical microfluidics (third edition) 2006
- *Carlos Ruiz Wills*, Mecánica de fluidos y aplicación de dinámica de fluidos en el cuerpo humano (lecture at UPF)
- *Pau Pomes, Quim Marti, Judith Torras, Mitar Nikolic*, Implementation of an innovative prosthesis for osteoporosis 2019

