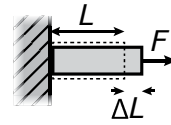


The Elastic Moduli

- Young's modulus:

$$E := \frac{\overset{\text{tensile stress}}{\sigma}}{\underset{\text{extensional strain}}{\varepsilon}} = \frac{F/A}{\Delta L/L} \quad \text{or} \quad \frac{F}{A} = E \cdot \frac{\Delta L}{L}$$



Typical $E \approx 100 \text{ GPa}$

10 GPa

1 GPa

1 – 100 MPa

1 – 1000 kPa

0.1 – 10 kPa

for

metals

bone, woods

proteins, plastics

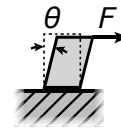
rubber, soft polymers

tissue

single cells

- Shear modulus:

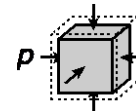
$$G := \frac{\overset{\text{shear stress}}{\tau}}{\underset{\text{shear strain}}{\gamma}} = \frac{F/A}{\frac{\Delta x/L}{\tan \theta \approx \theta}}$$



Typically $E \approx 2 \dots 3 \cdot G$

- Bulk modulus:

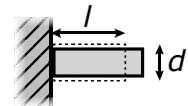
$$K := - \frac{\overset{\text{pressure}}{P}}{\underset{\text{rel. volume change}}{\Delta V/V}}$$



- Three moduli are related by $E = 2(1 + \nu) \cdot G = 3(1 - 2\nu) \cdot K$

with Poisson ratio ("Querkontraktionszahl") $\nu = - \frac{\Delta d/d}{\Delta l/l}$

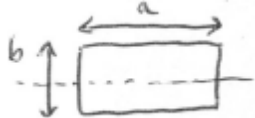
typically $0.2 < \nu < 0.5$ (= incompressible)



Second Moment of Area

("Flächenträgheitsmoment") for selected cross sections:

rectangle: $I = \frac{1}{12} ab^3$



ellipse: $I = \frac{\pi}{4} ab^3$



circle: $I = \frac{\pi}{4} r^4$

