

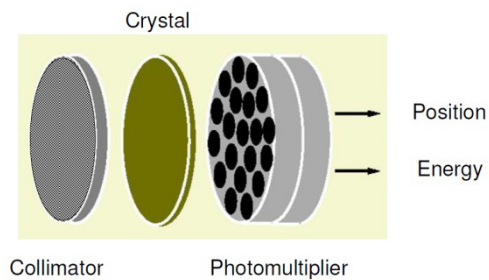


Image reconstruction and correction methods in nuclear medicine

J. Kupferschläger, Dipl.-Phys.

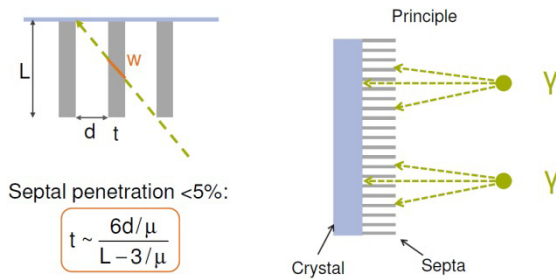
Before we start

Gamma Camera – H. Anger 1964



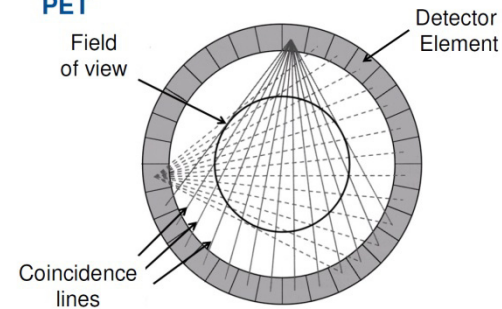
Collimator – Parallel Hole

Septum thickness dependent of energy and attenuation coefficient μ

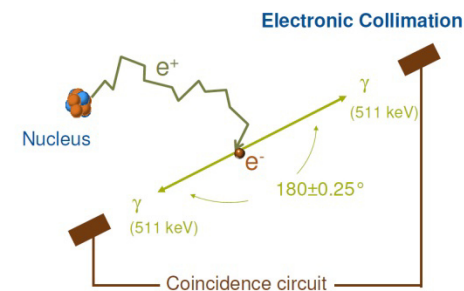


single photon

PET



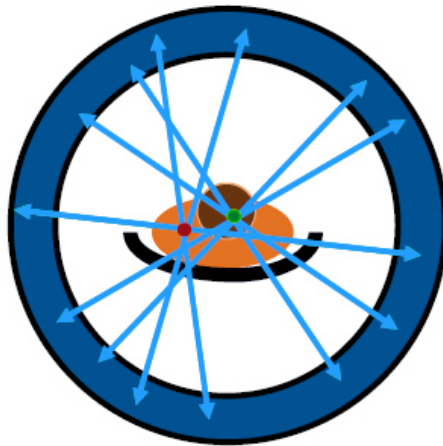
PET Measuring Principle



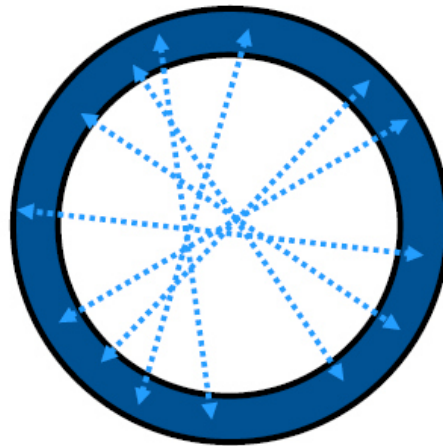
pair of annihilation photons

The Reconstruction Problem

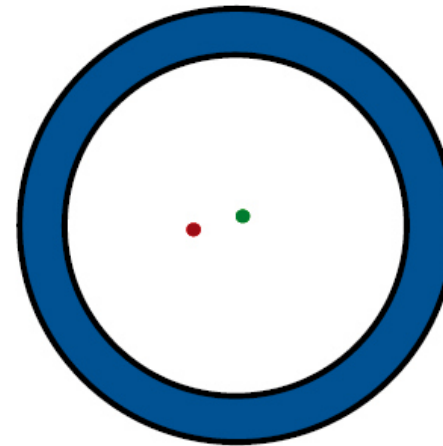
This is...



what
happened.

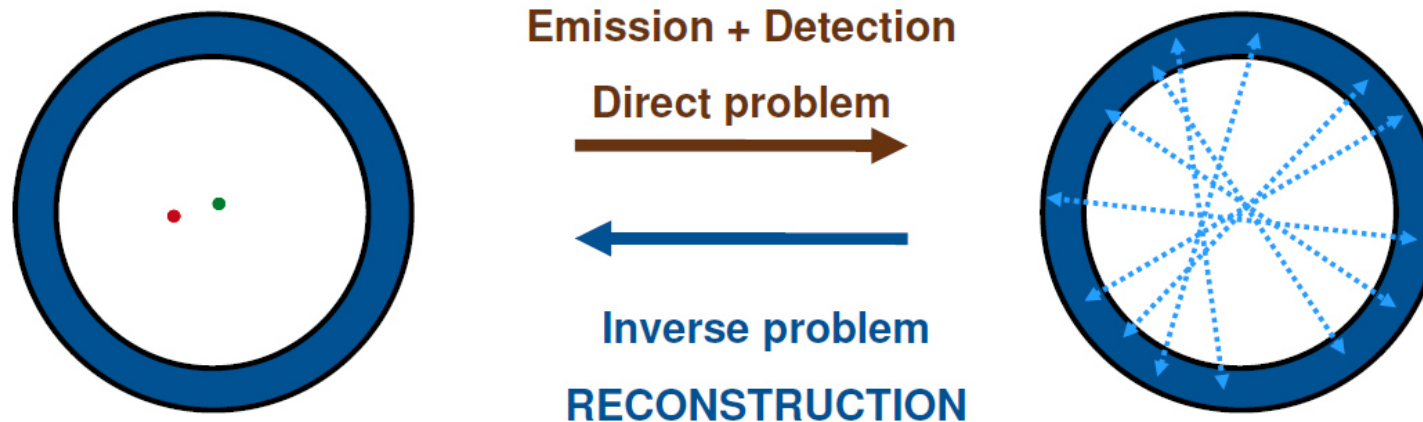


what we
measured.



what we are
looking for!

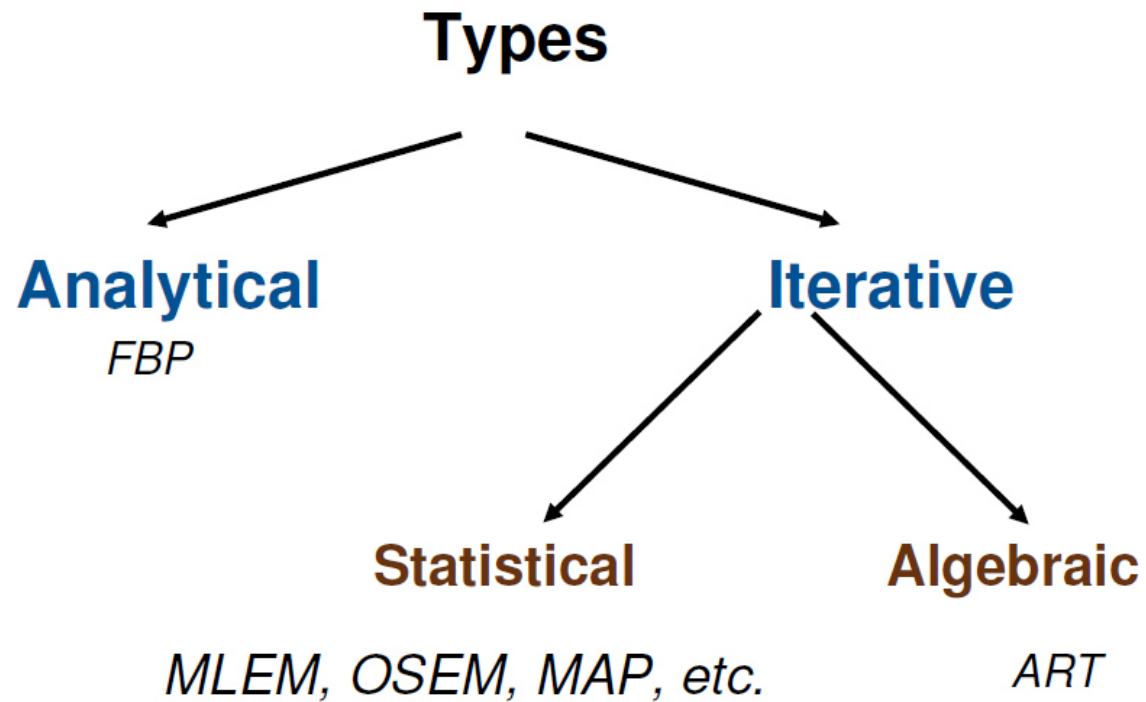
The Reconstruction Problem



ill-posed problem:

Small variation of experimental data cause
large variations of possible solutions

Reconstruction methods



Problem formulation

Continuous-discrete problem

- Unknown, $\mathbf{a}(x,y,z)$:

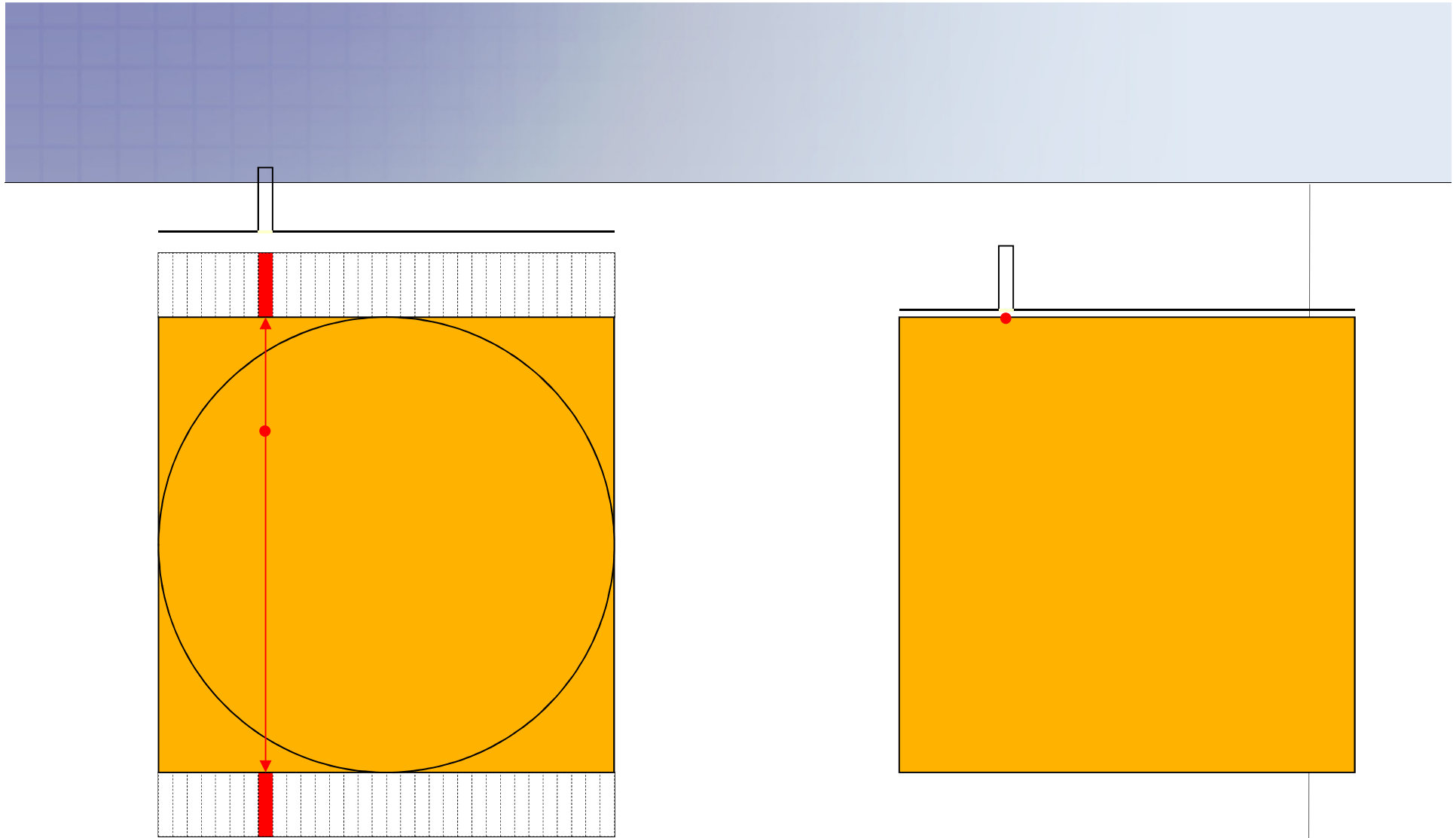
Spatial distribution of radioisotope (continuous function)

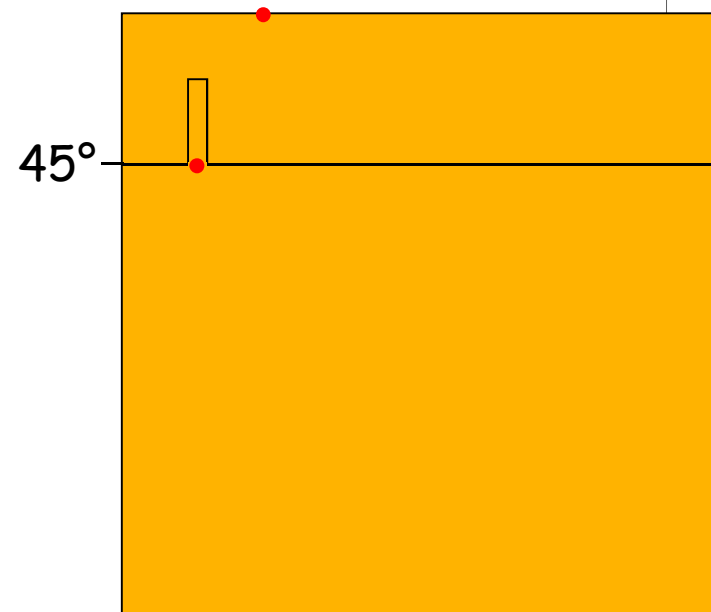
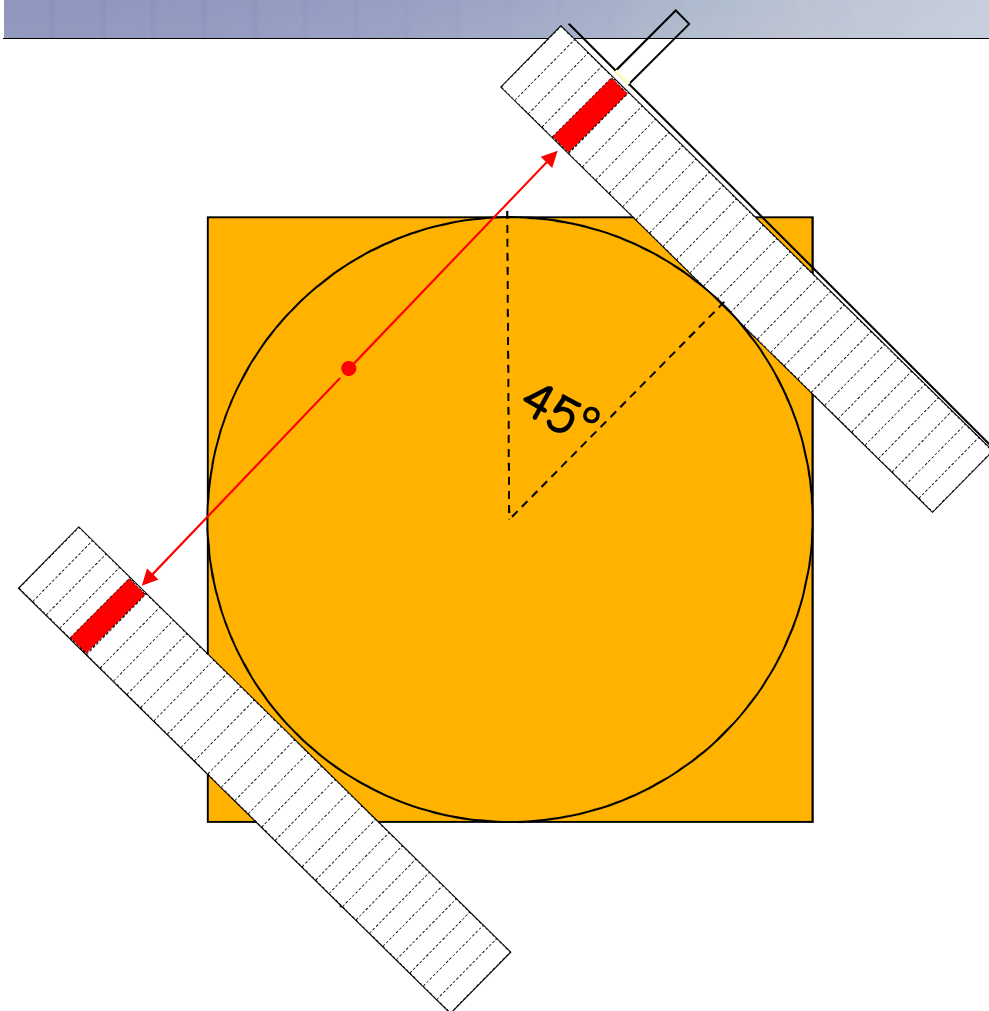
- Measured data, $\mathbf{M}$:

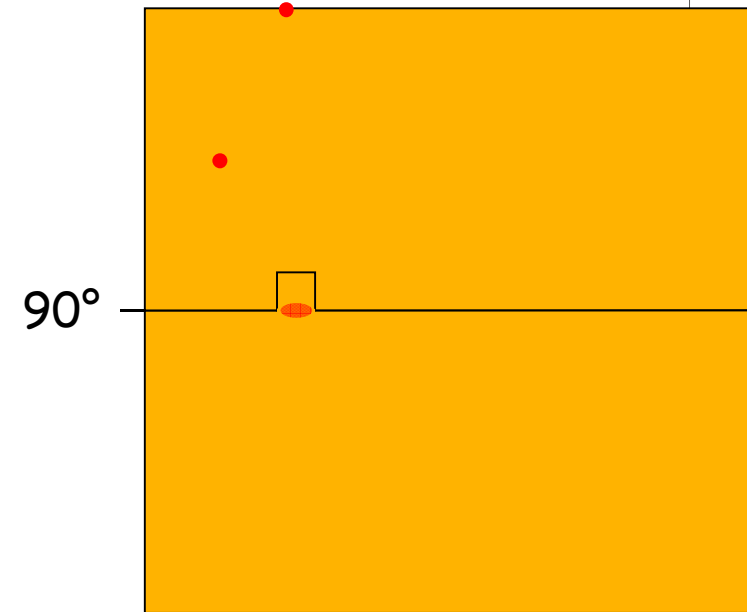
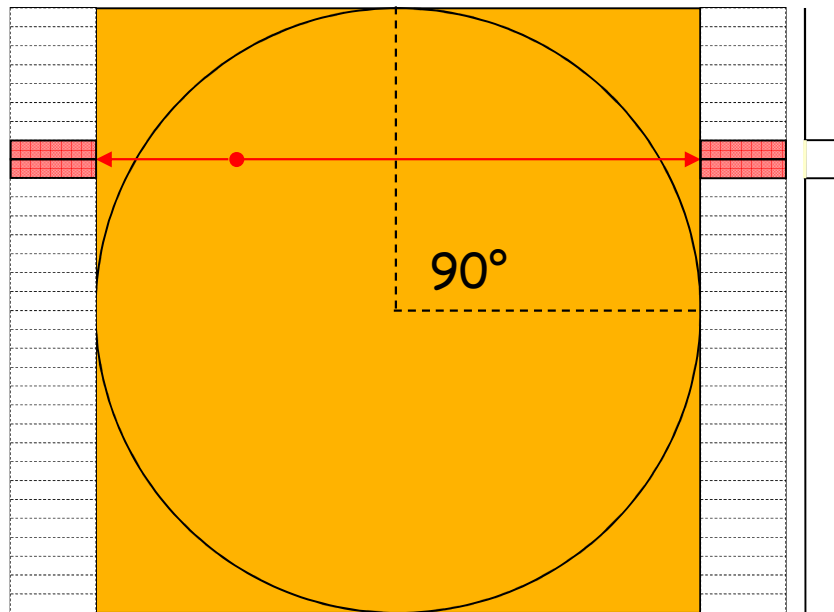
M_i : Number of events registered by detector i (discrete)

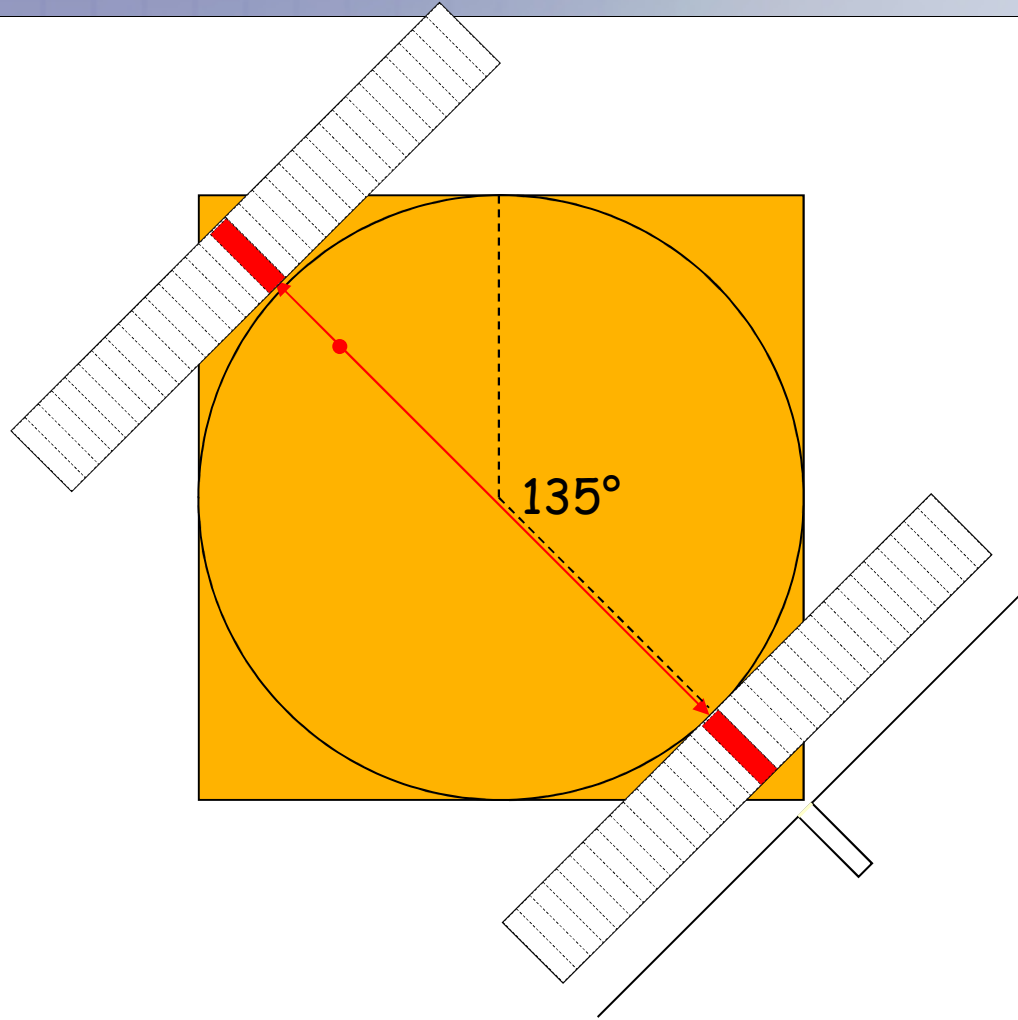
- System response, $h_i(x,y,z)$:

Contribution of radioactivity located at (x,y,z) to measured counts by detector i

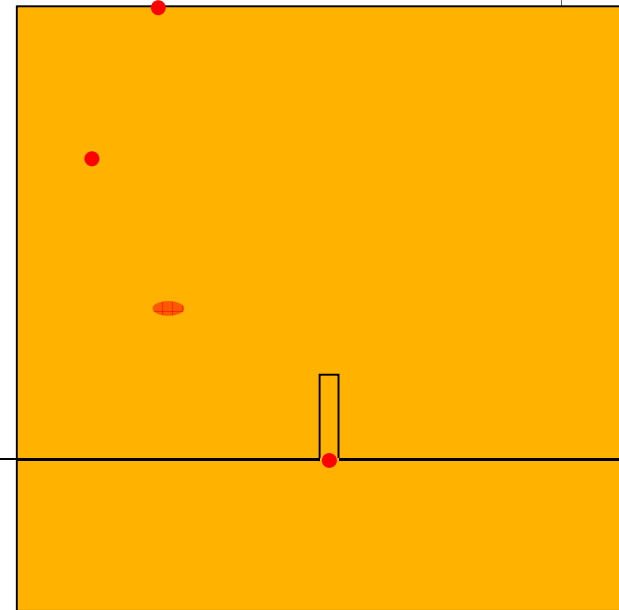


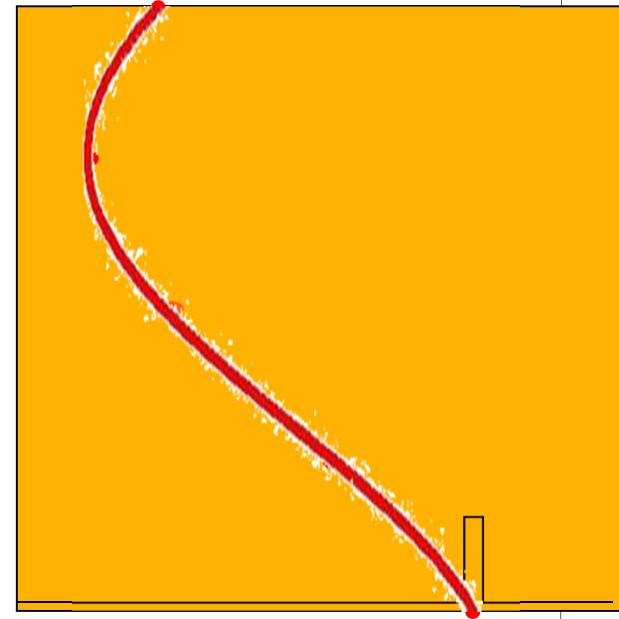
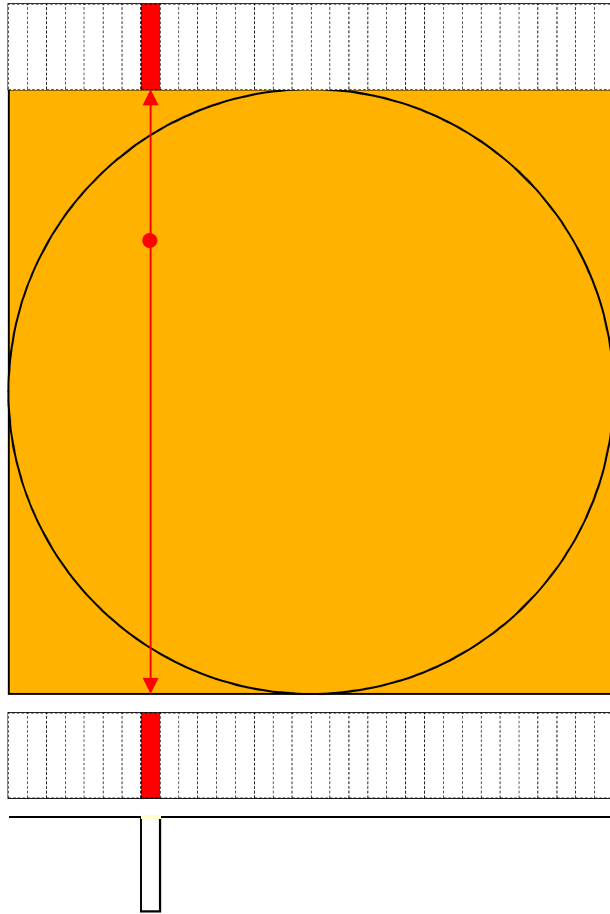


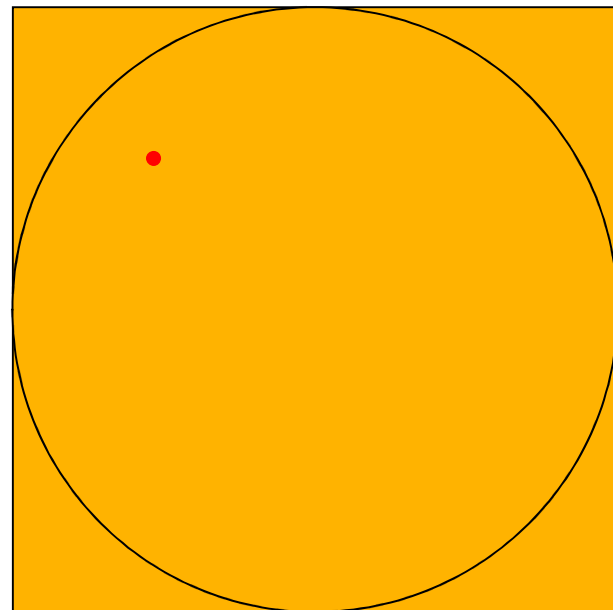




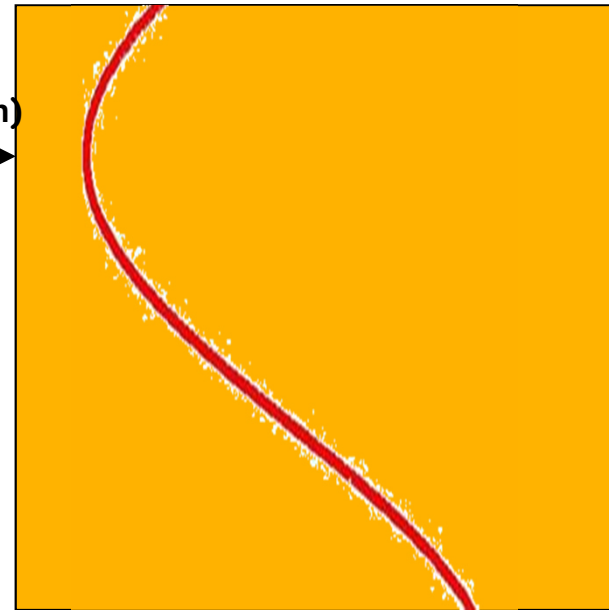
135°







forward projection
(Radon Transformation)

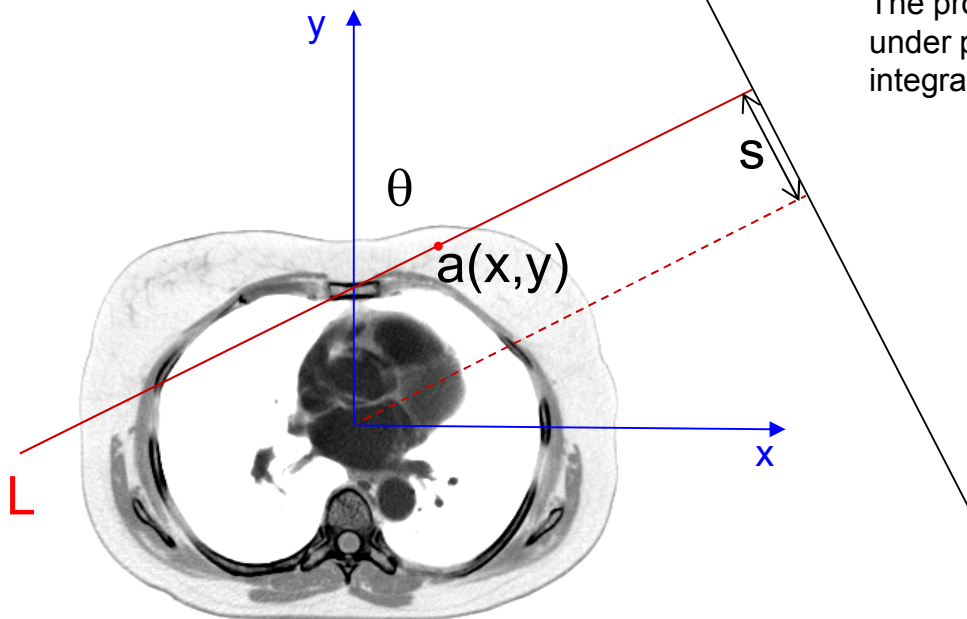


The Radon transformation

Transformation of an image to an angular projection

activity distribution $a(x,y)$

projection line L



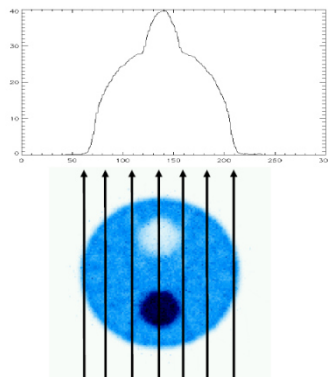
$$s = x \cos \theta - y \cdot \sin \theta$$

$$p(s, \theta) = \int_L a(x, y) dL = \int a(x, y) \cdot \delta(s + y \sin \theta - x \cos \theta) dx dy$$

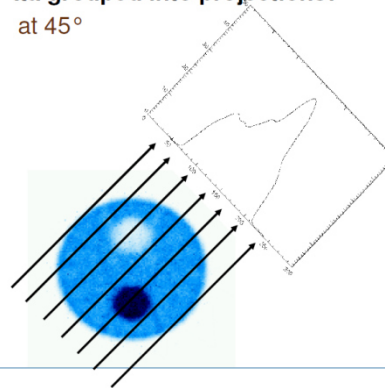
The projection $p(s, \theta)$ of an activity distribution a under projection angle θ is given by the line integral along the projection line L .

Measured data grouped into projections:

Projection
at 0°



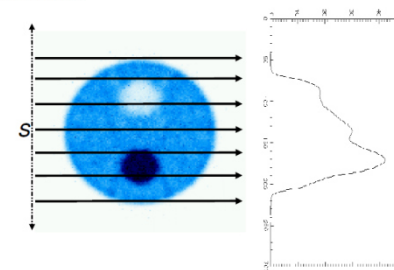
Measured data grouped into projections:
at 45°

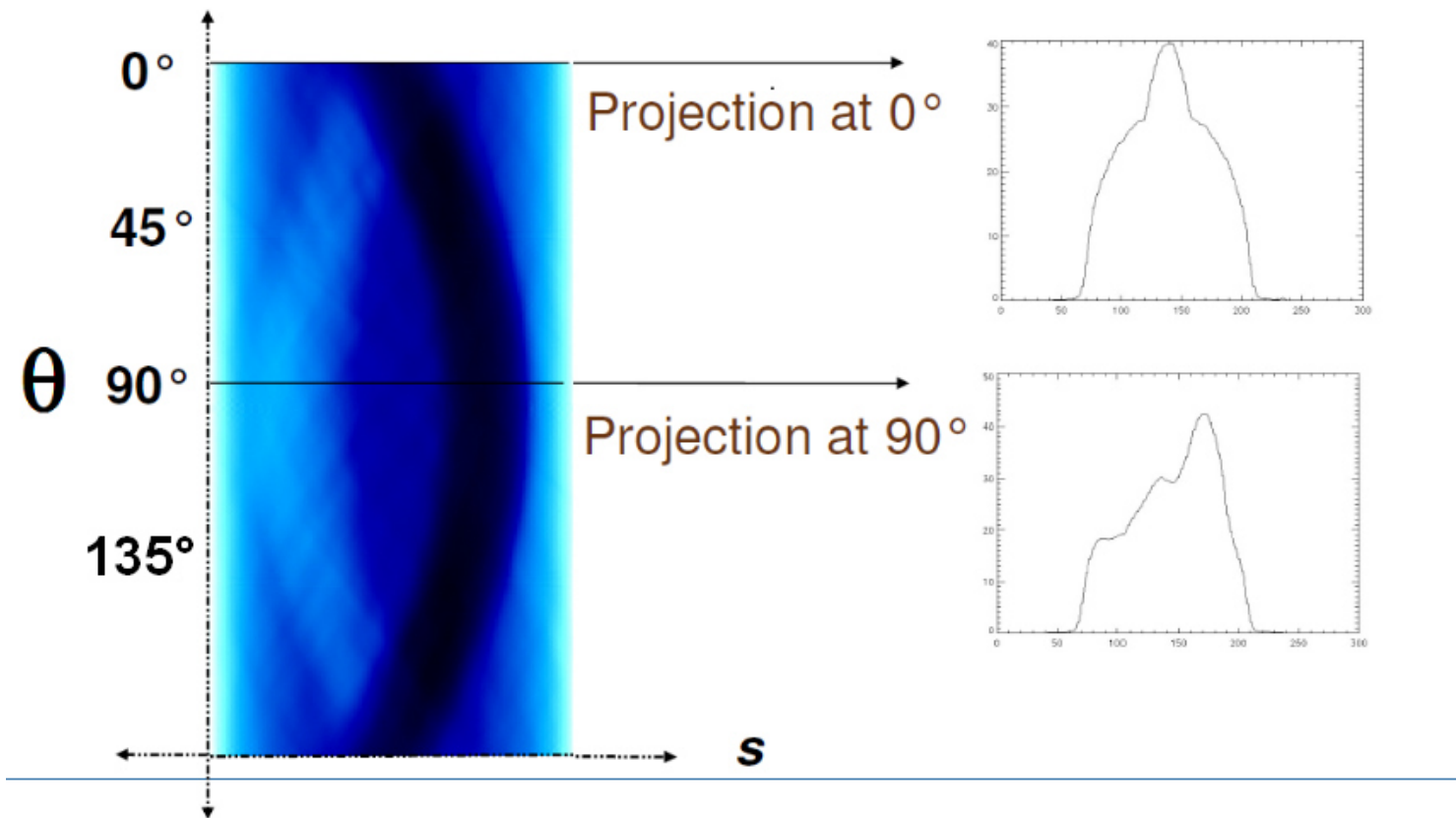


25.01.2011

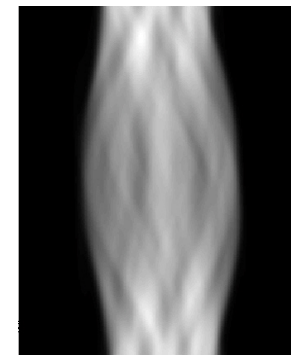
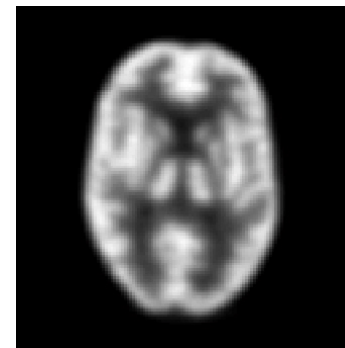
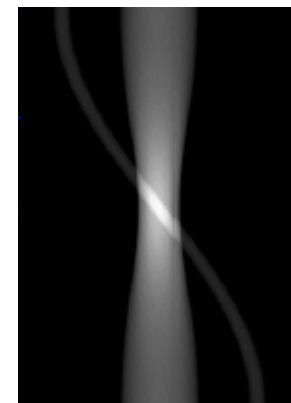
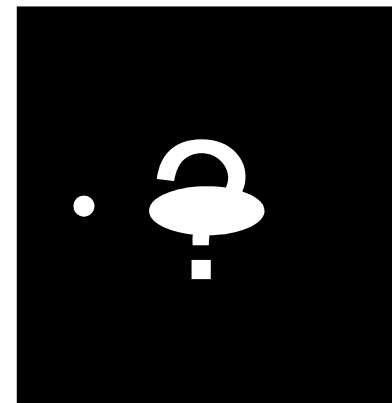
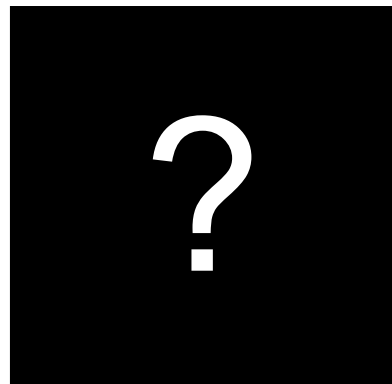
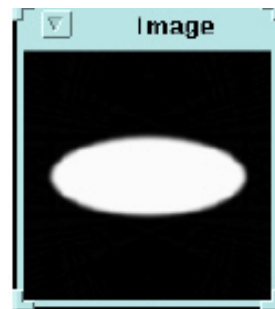
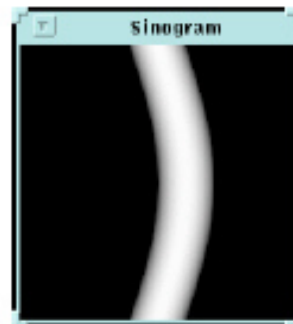
Measured data grouped into projections:

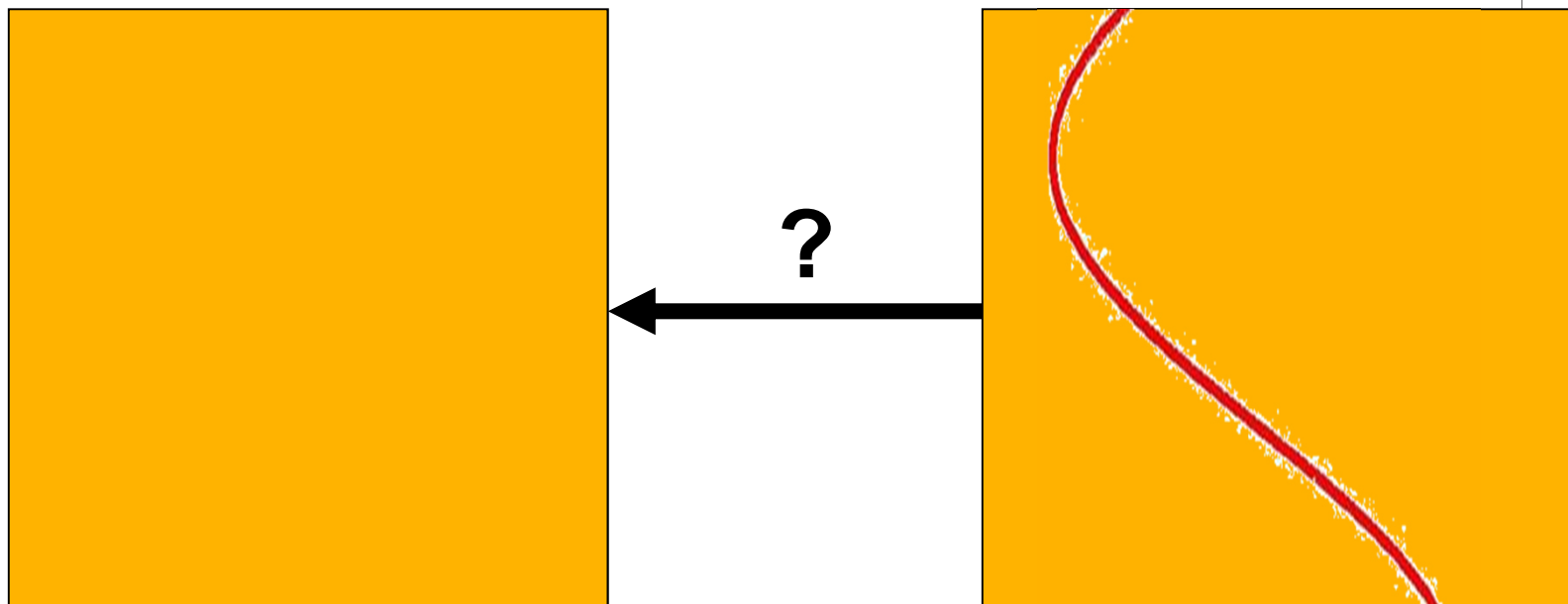
Projection at 90°



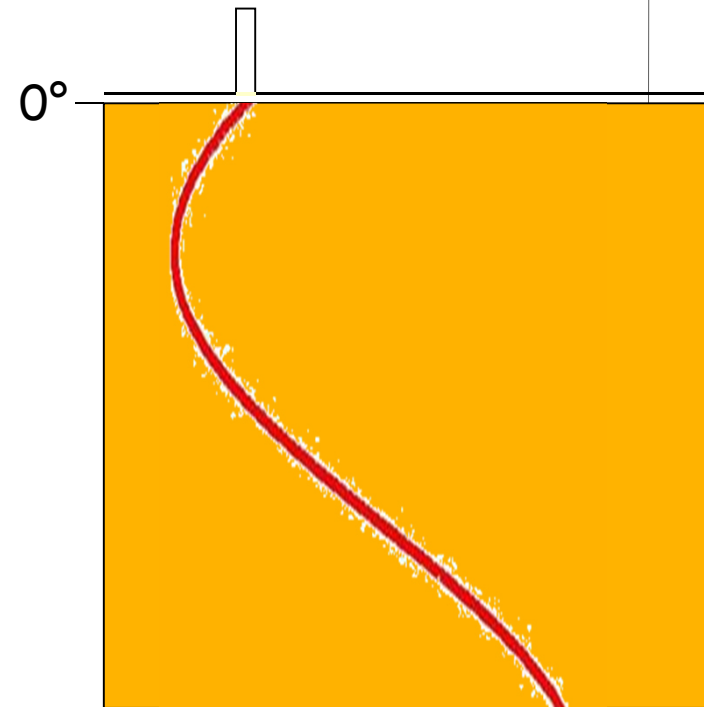
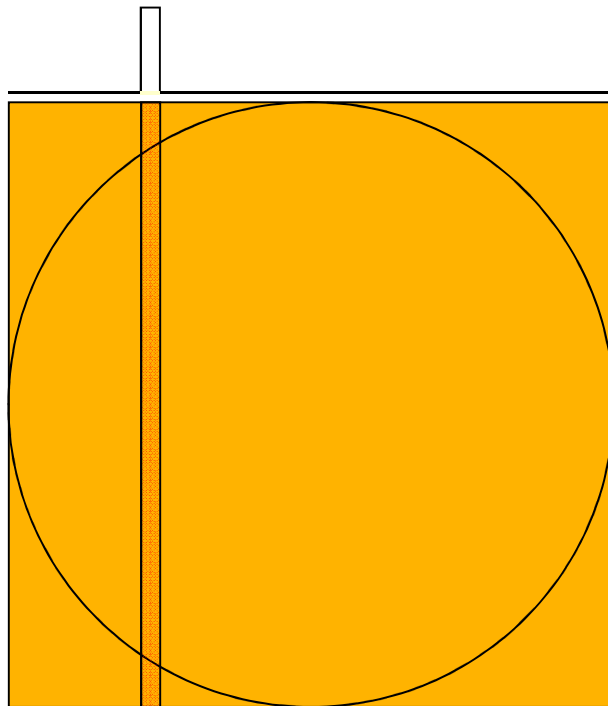


sino puzzles

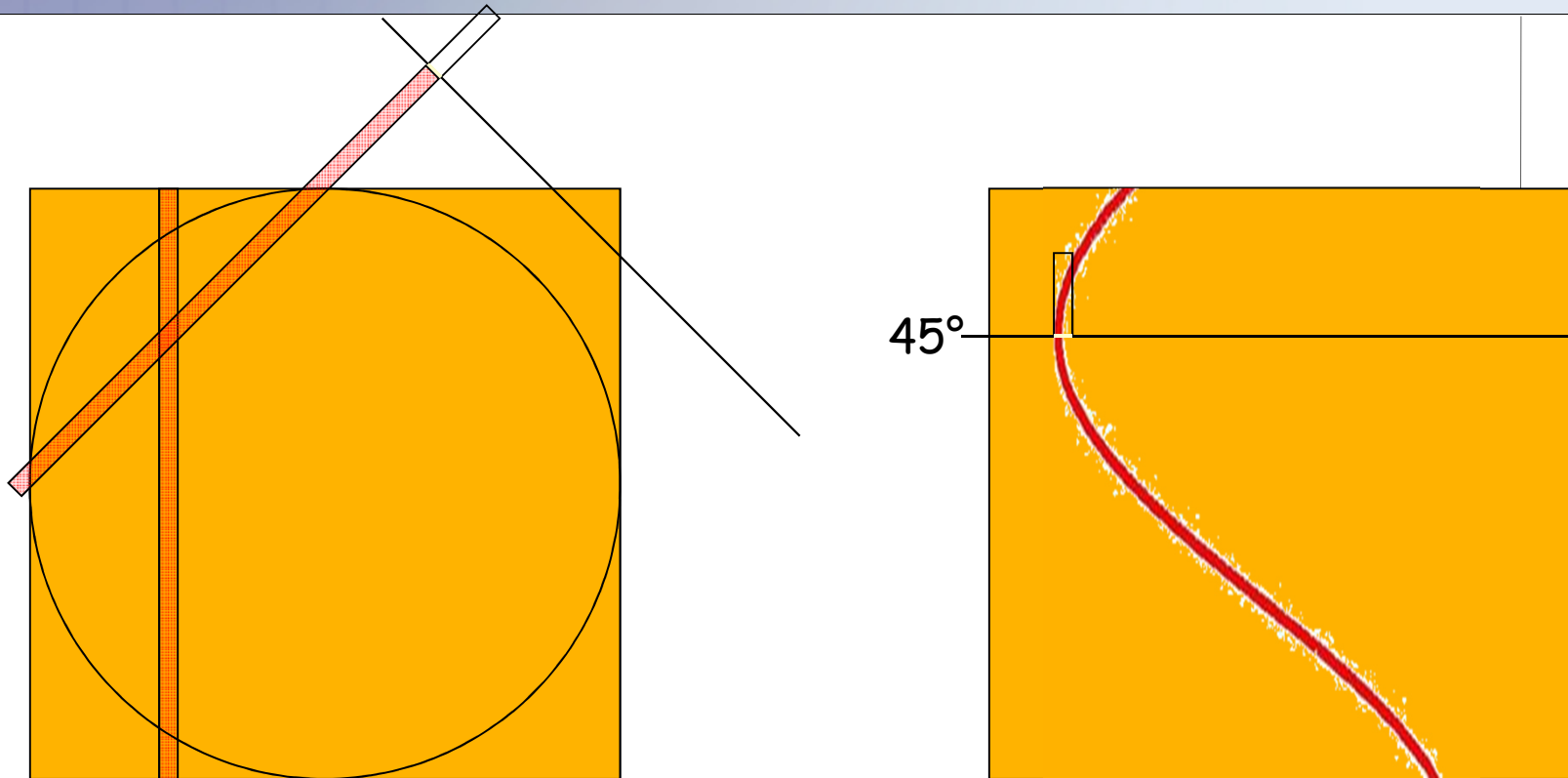




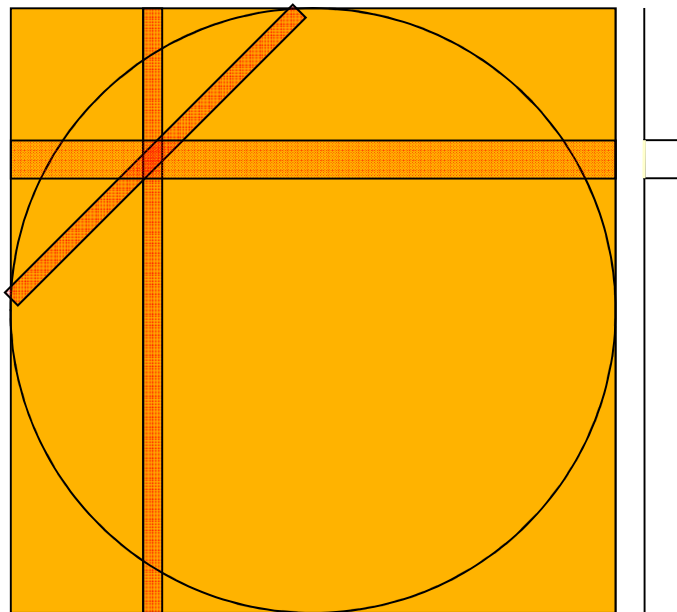
back projection



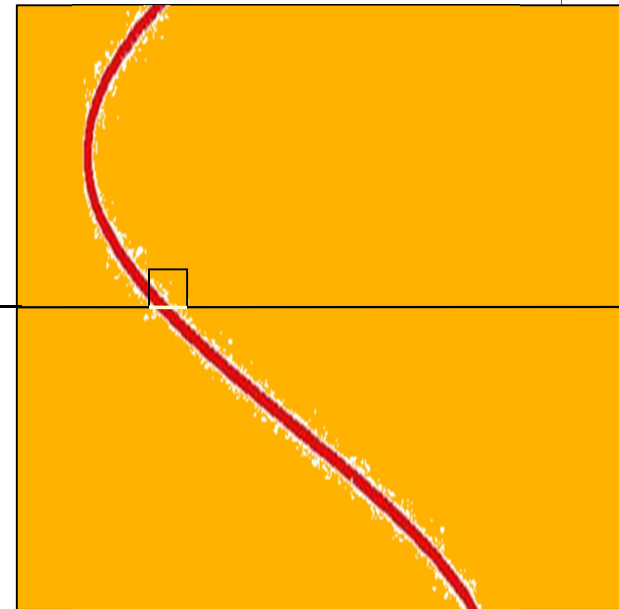
back projection



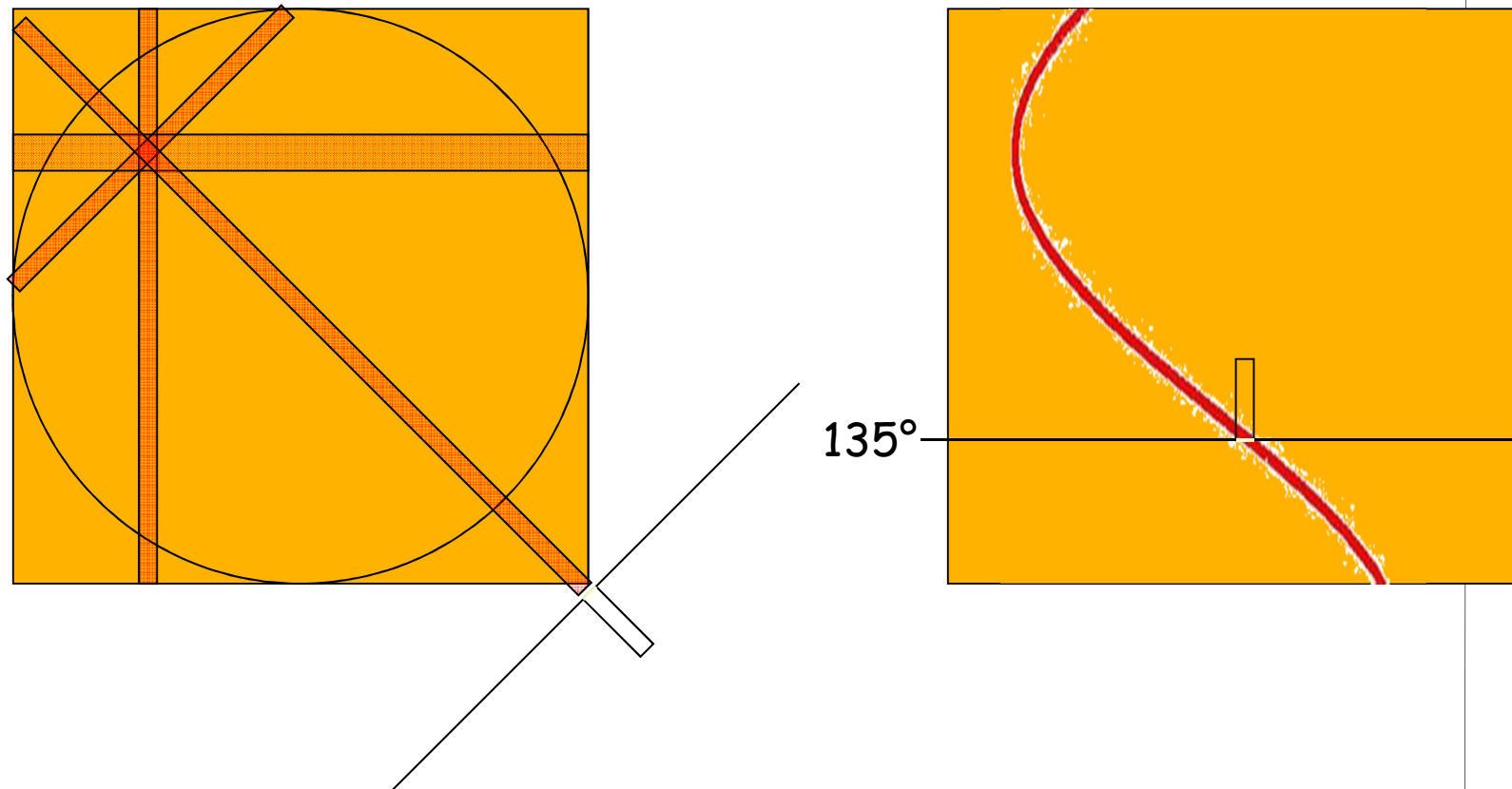
back projection



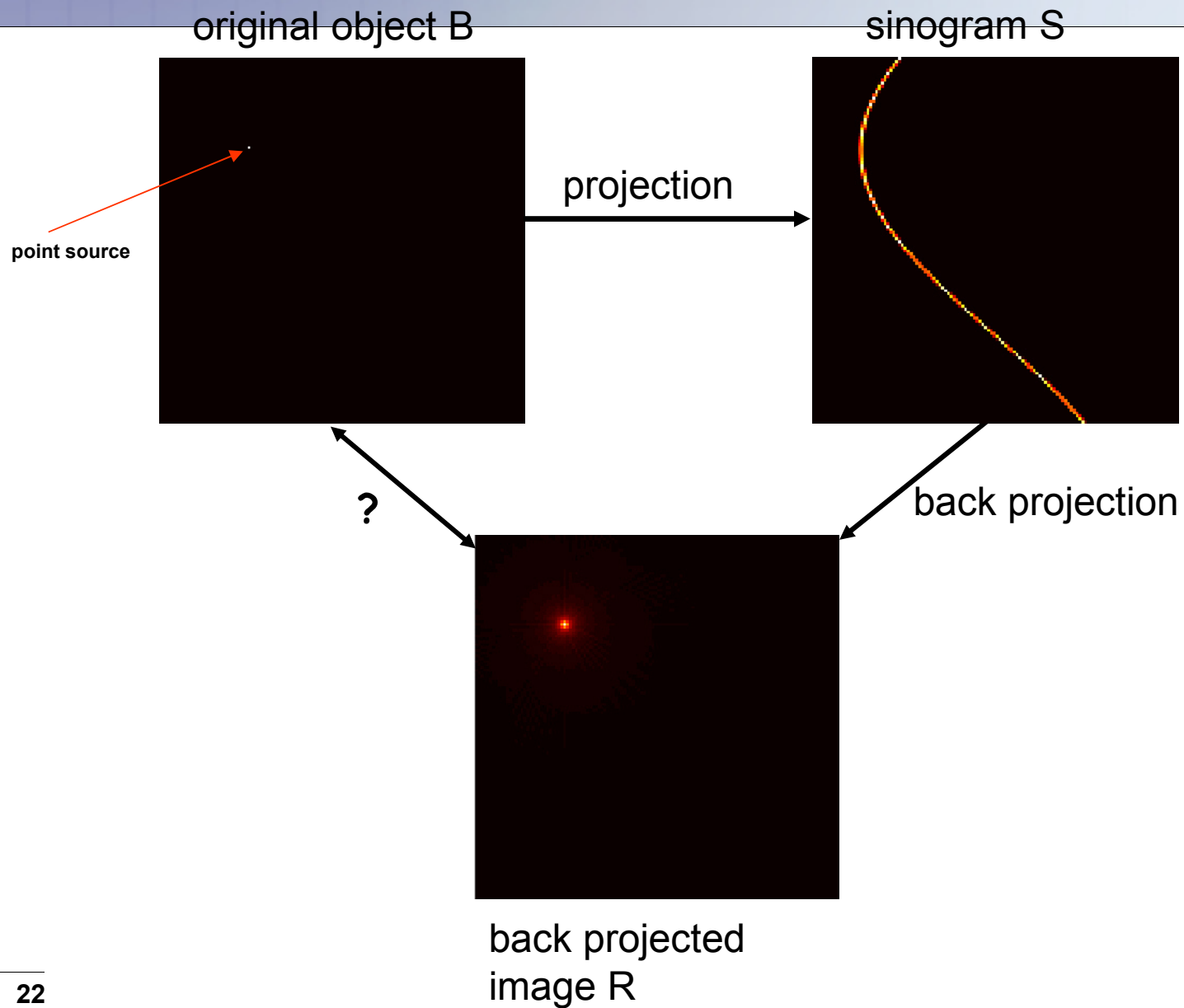
90°



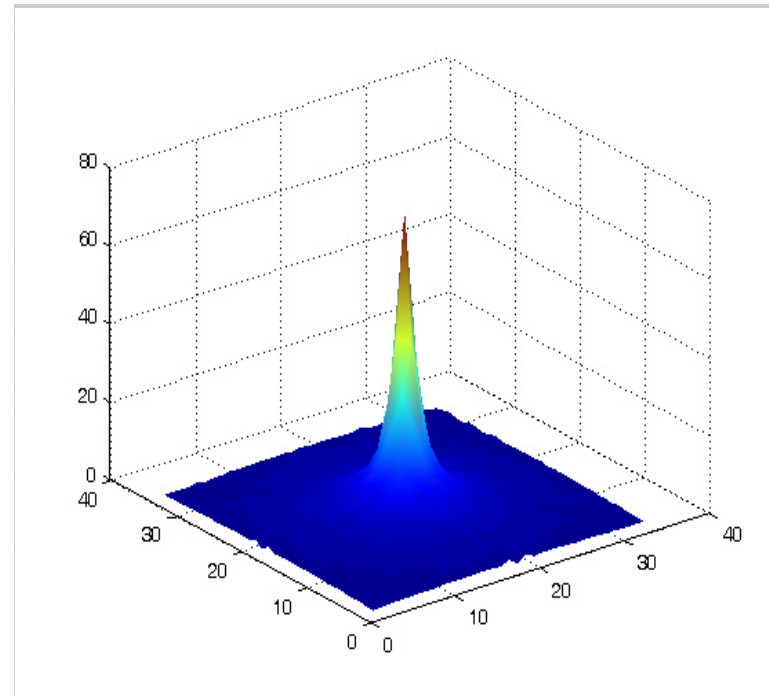
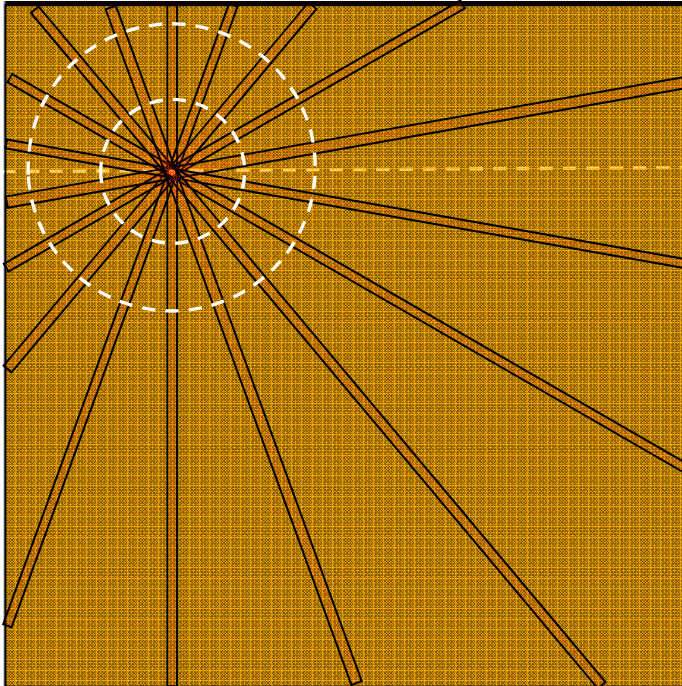
back projection



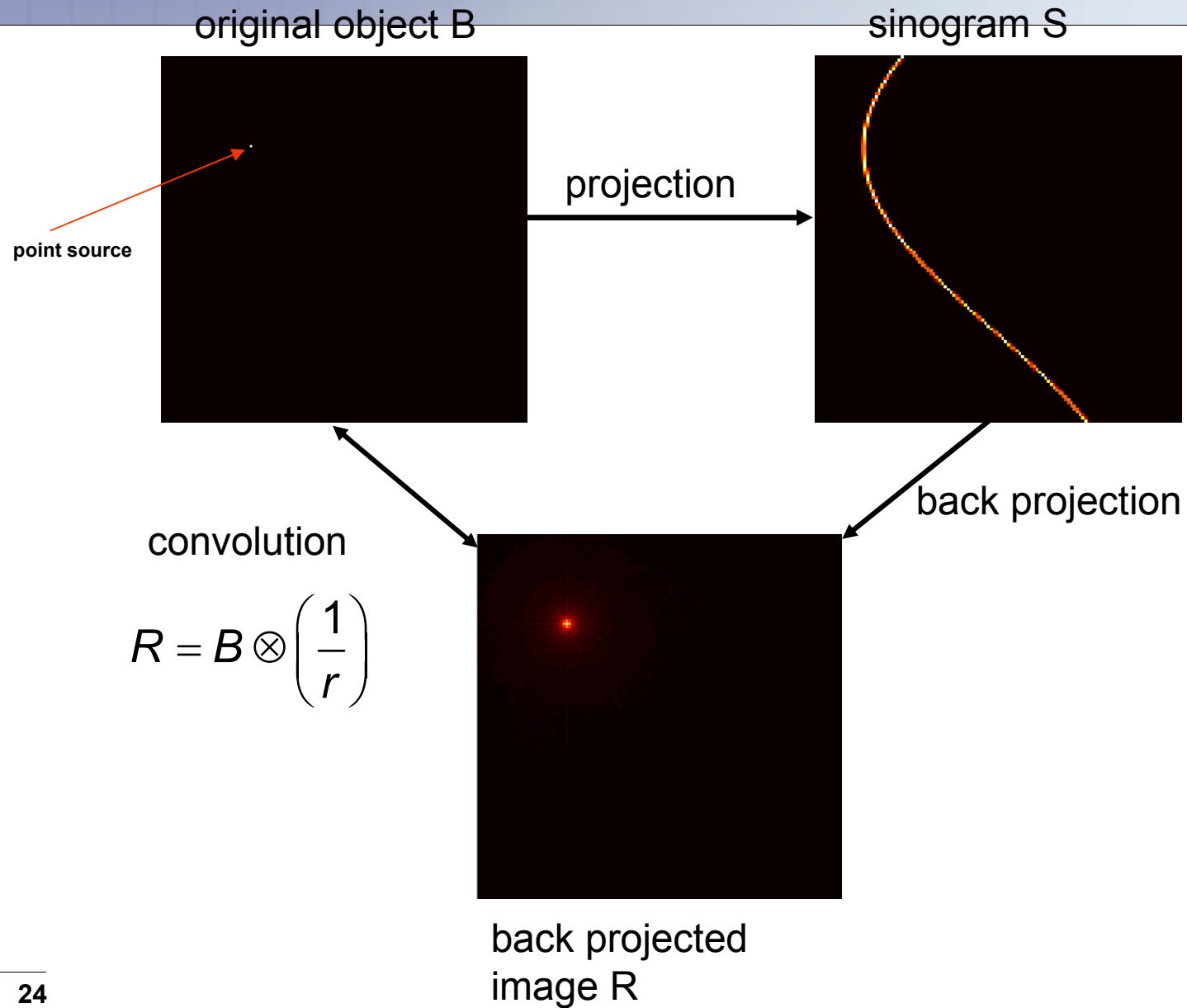
The truth



intensity distribution of the backprojected image R



intensity profile



The back projected image is identical to the convolution of the original object with the $(1/r)$ -function

$$R = B \otimes \left(\frac{1}{r} \right)$$

Inverse operation ?

Yes: deconvolution

$$\Rightarrow B = R \otimes \left(\frac{1}{r} \right)^{-1} = R \otimes F \quad F : \text{Filterfunction}$$

The original object can be restored by filtering the back projected image: filtered backprojection

filtered back projection FBP

We already know, that

$$R = B \otimes \left(\frac{1}{r} \right) \text{ or, using Fourier transformation}$$

$$F(R) = F(B) \cdot F\left(\frac{1}{r}\right) = F(B) \cdot \frac{1}{|k|}$$

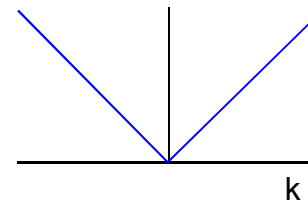
$$\Rightarrow F(B) = |k| \cdot F(R) \text{ (ramp filter)}$$

$$\Rightarrow B = R \otimes F^{-1}(|k|)$$

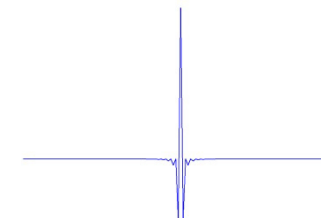
Strategy:

- backprojection R
- 2D Fourier transformation (FFT) of R
- Multiplication with frequency
- Inverse FFT (that's it!)

frequency domain



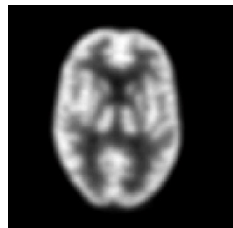
spatial domain



Problem: The measure is a random process

$\Rightarrow$ NOISE!

$\Rightarrow$ increasing of the noise caused by multiplication with the frequency



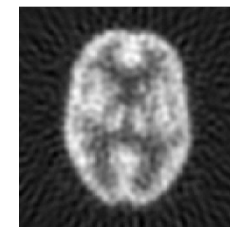
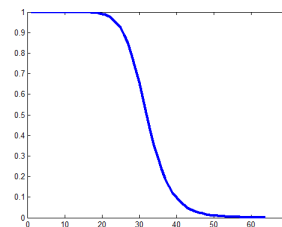
original



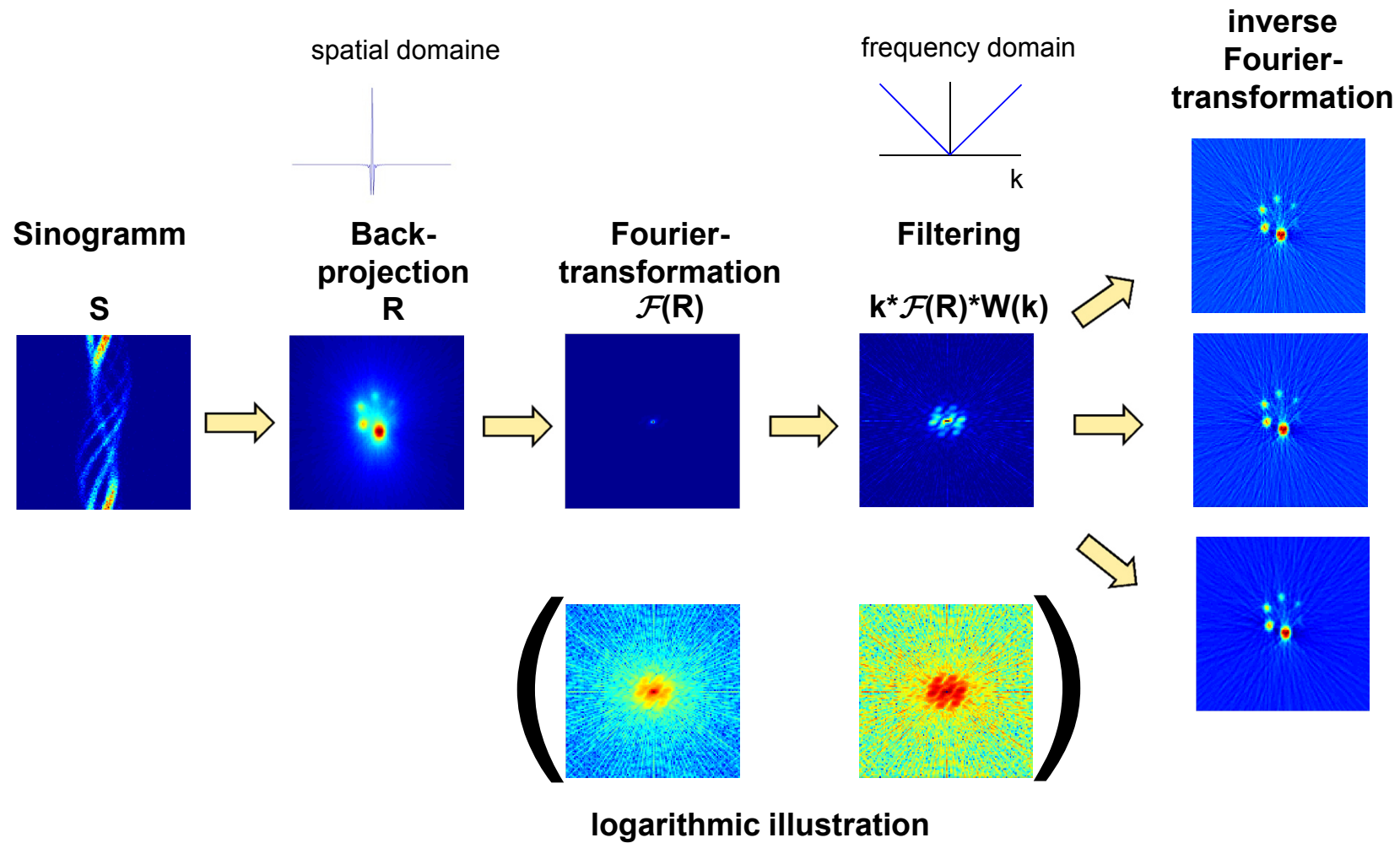
FBP (ramp)

Solution: Defining of Window filter functions $W(k)$ to suppress the behaviour at high frequencies

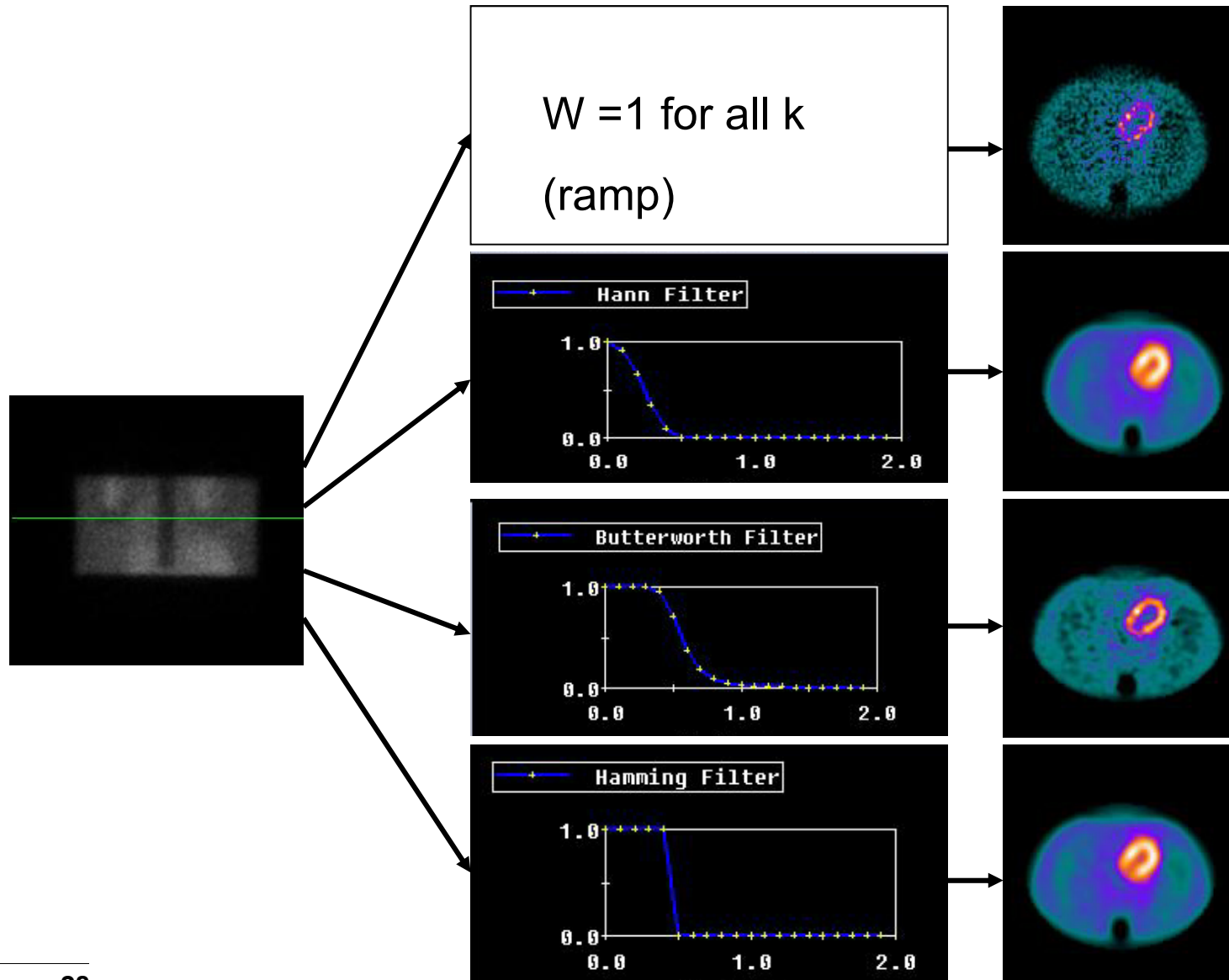
$$\Rightarrow \text{Filterfunction } F(k) = W(k) \cdot k \Rightarrow F(\tilde{B}) = F(k) \cdot F(R)$$



FBP step by step

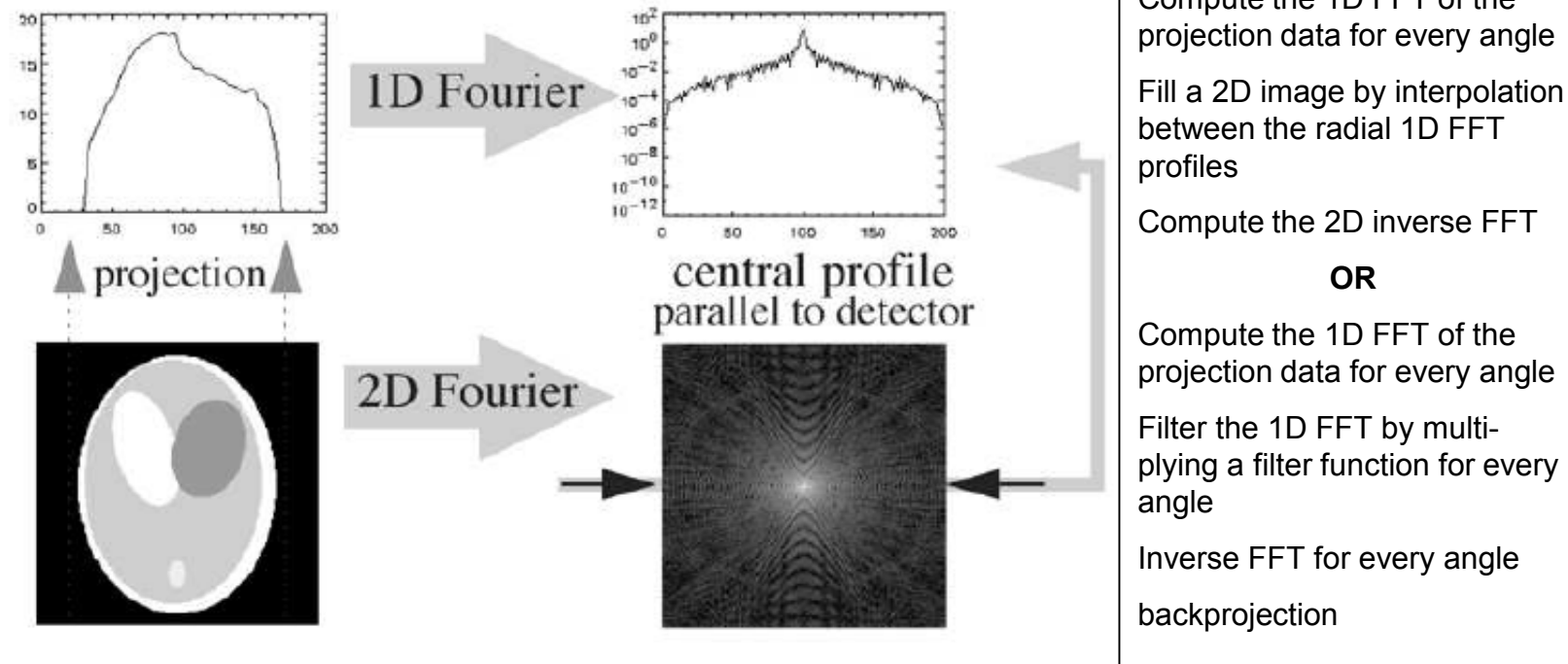


FBP: an example



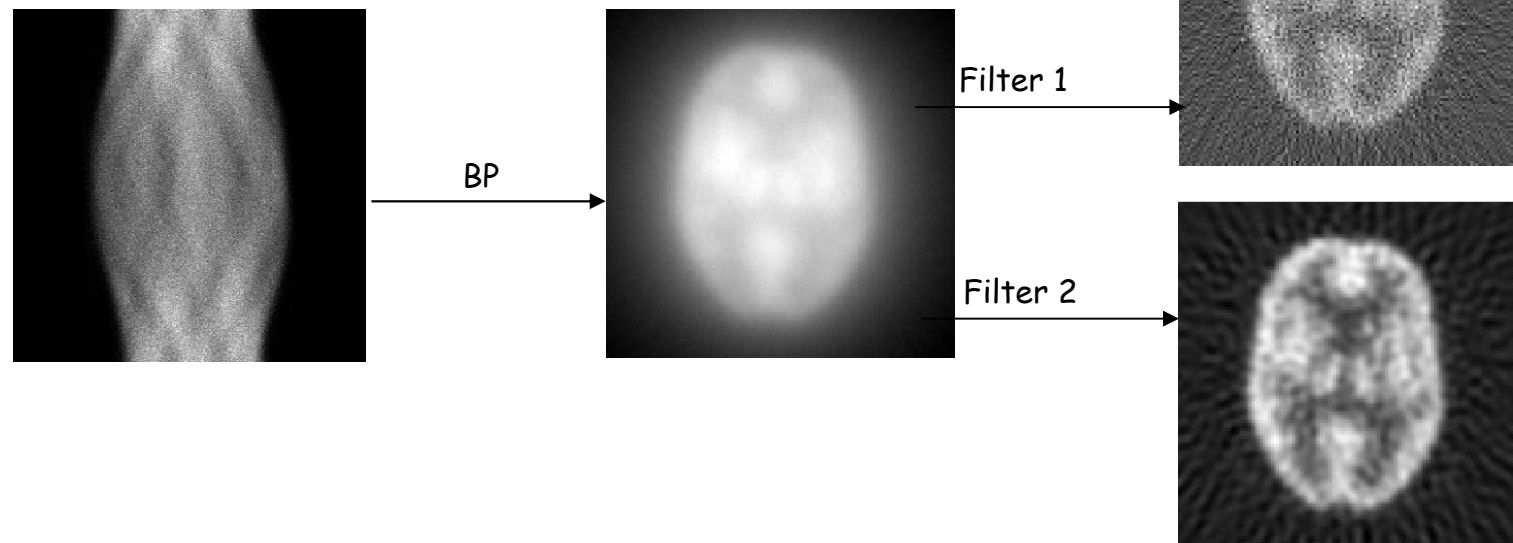
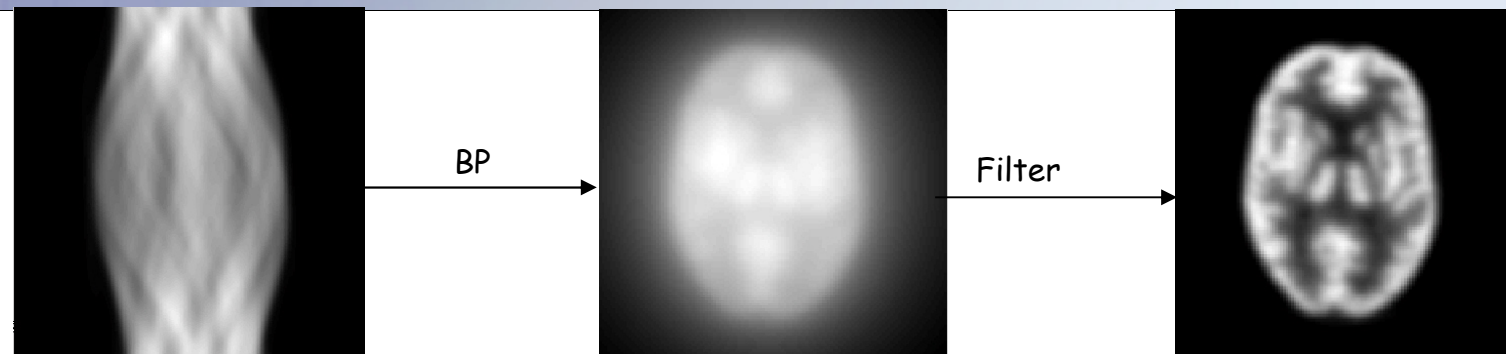
The second & third way of FBP

The central slice theorem



The 1-D Fouriertransform of the projection acquired for angle θ is identical to a central profile along the same angle through the 2-D Fourier transform of the original distribution.

Is there an optimal filter?



filtered backprojection

easy to implement

fast

quantitative

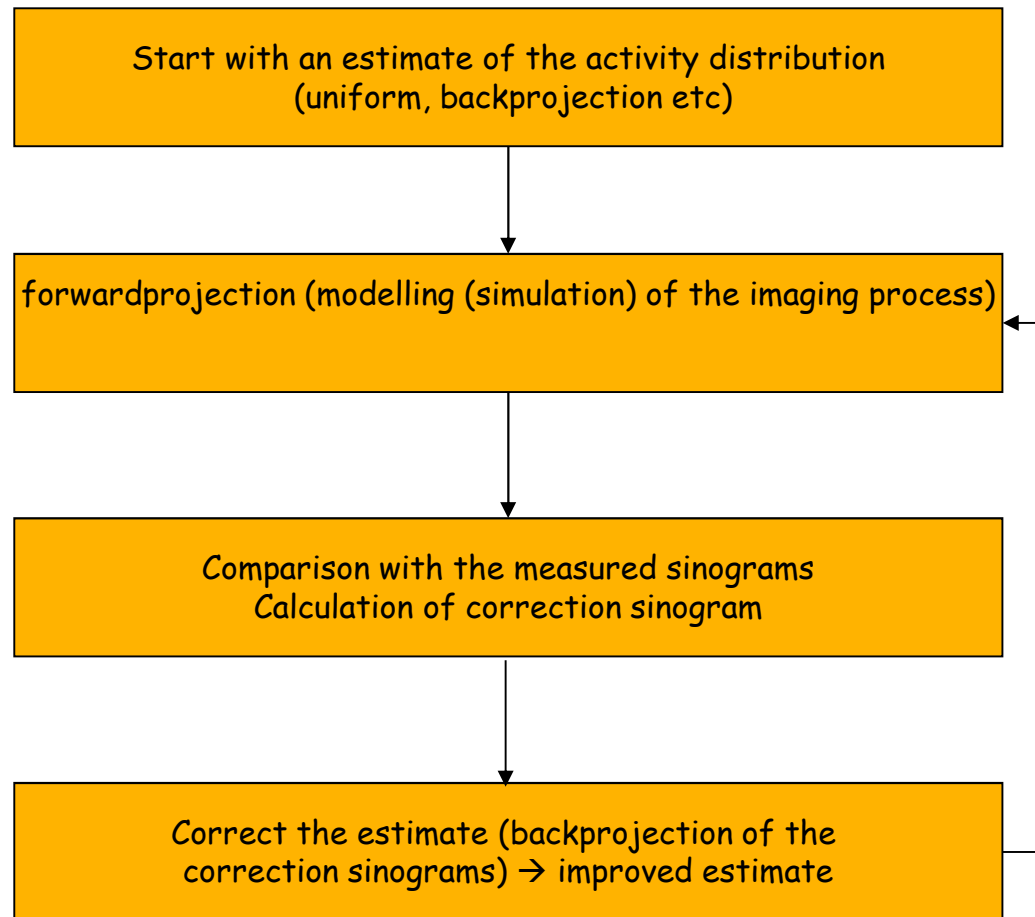
But:

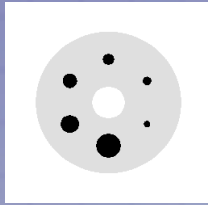
activity outside the object

increasing noise

The choice of the filter and it's parameters depends on the user: There is no optimal filter in general!

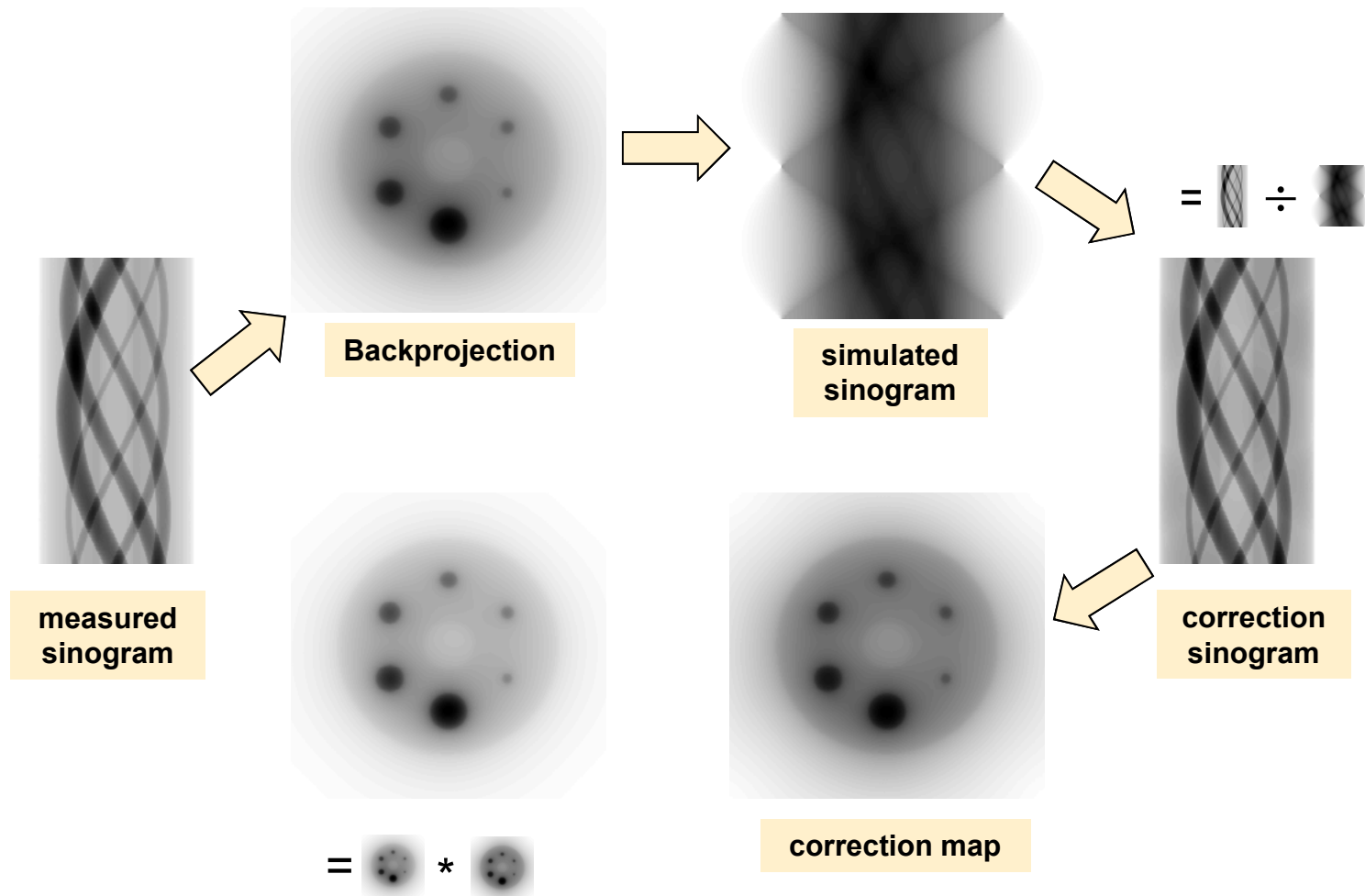
An alternative: Iterative reconstruction

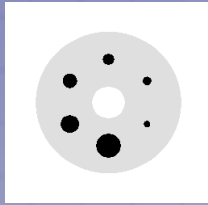




original
(unknown)

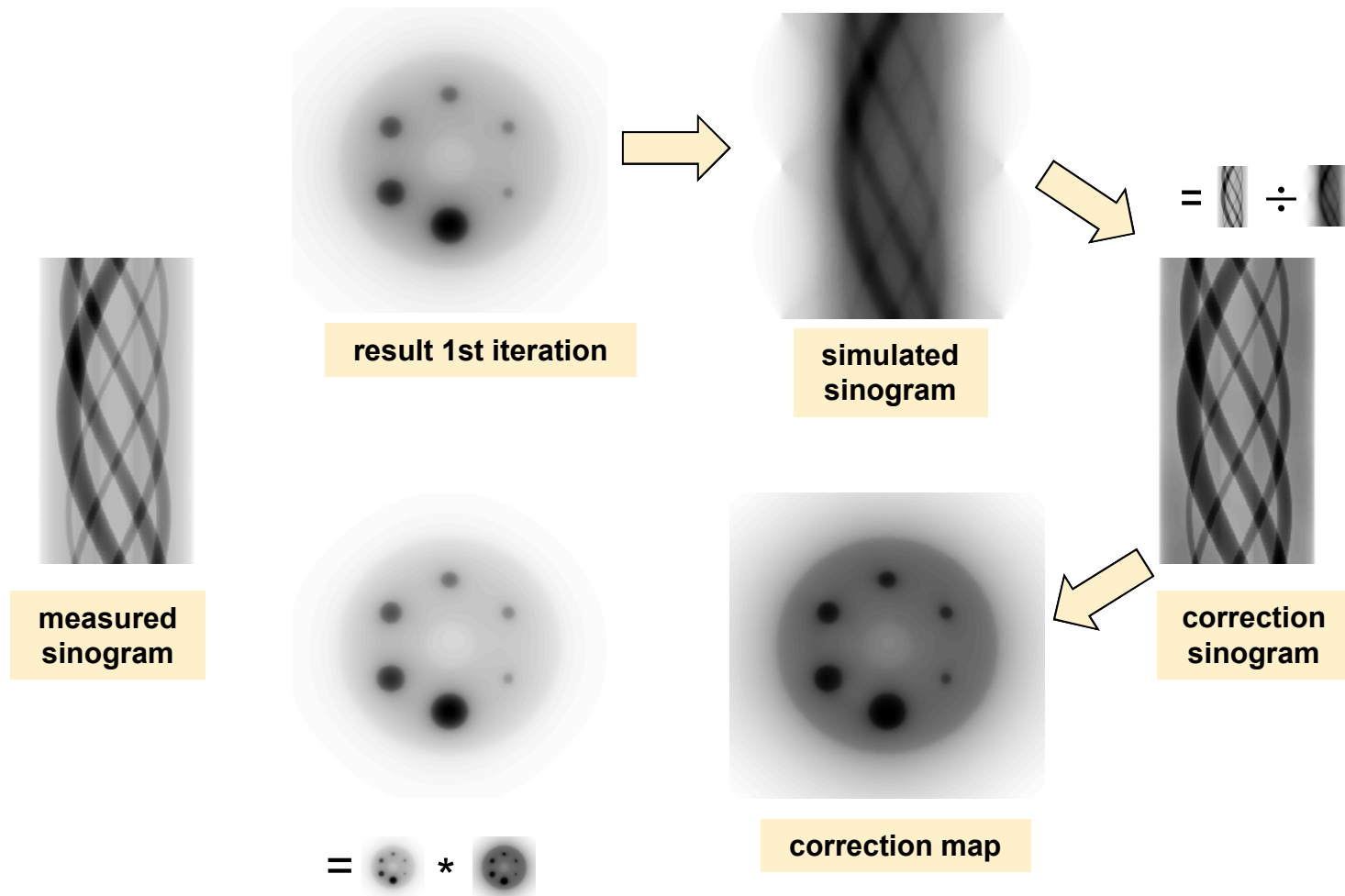
1st Iteration

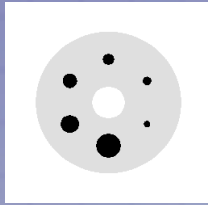




original
(unknown)

2nd iteration



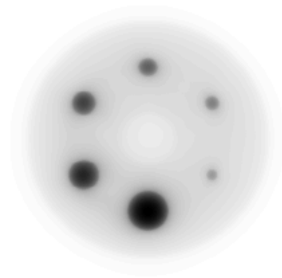


original
(unknown)

5th iteration



measured
sinogram

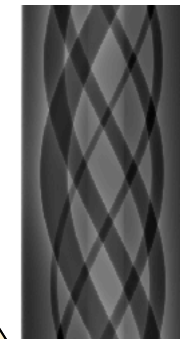


result 4th iteration

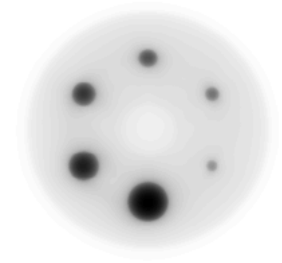


simulated
sinogram

$$= \text{measured sinogram} \div \text{simulated sinogram}$$

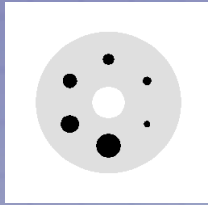


correction
sinogram



correction map

$$= \text{result 4th iteration} * \text{correction map}$$

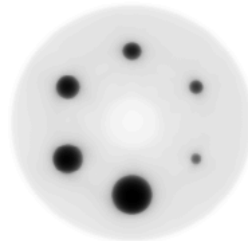


original
(unknown)

10th iteration



measured
sinogram



result 9th iteration

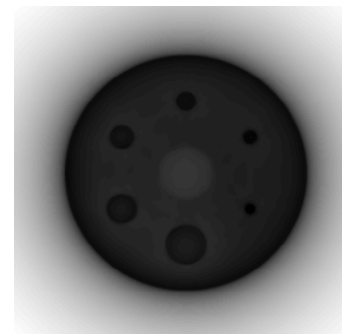


simulated
sinogram

$$= \text{measured sinogram} \div \text{simulated sinogram}$$

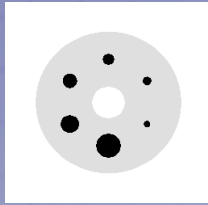


correction
sinogram



correction map

$$= \text{result 9th iteration} * \text{correction map}$$

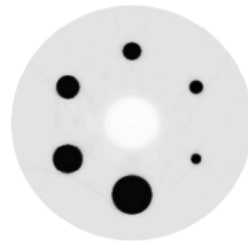


original
(unknown)

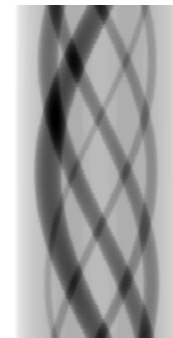
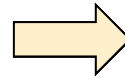
40th iteration



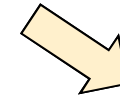
measured
sinogram



result 39th iteration



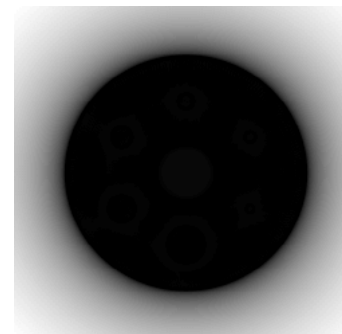
simulated
sinogram



$$= \text{measured sinogram} \div \text{simulated sinogram}$$



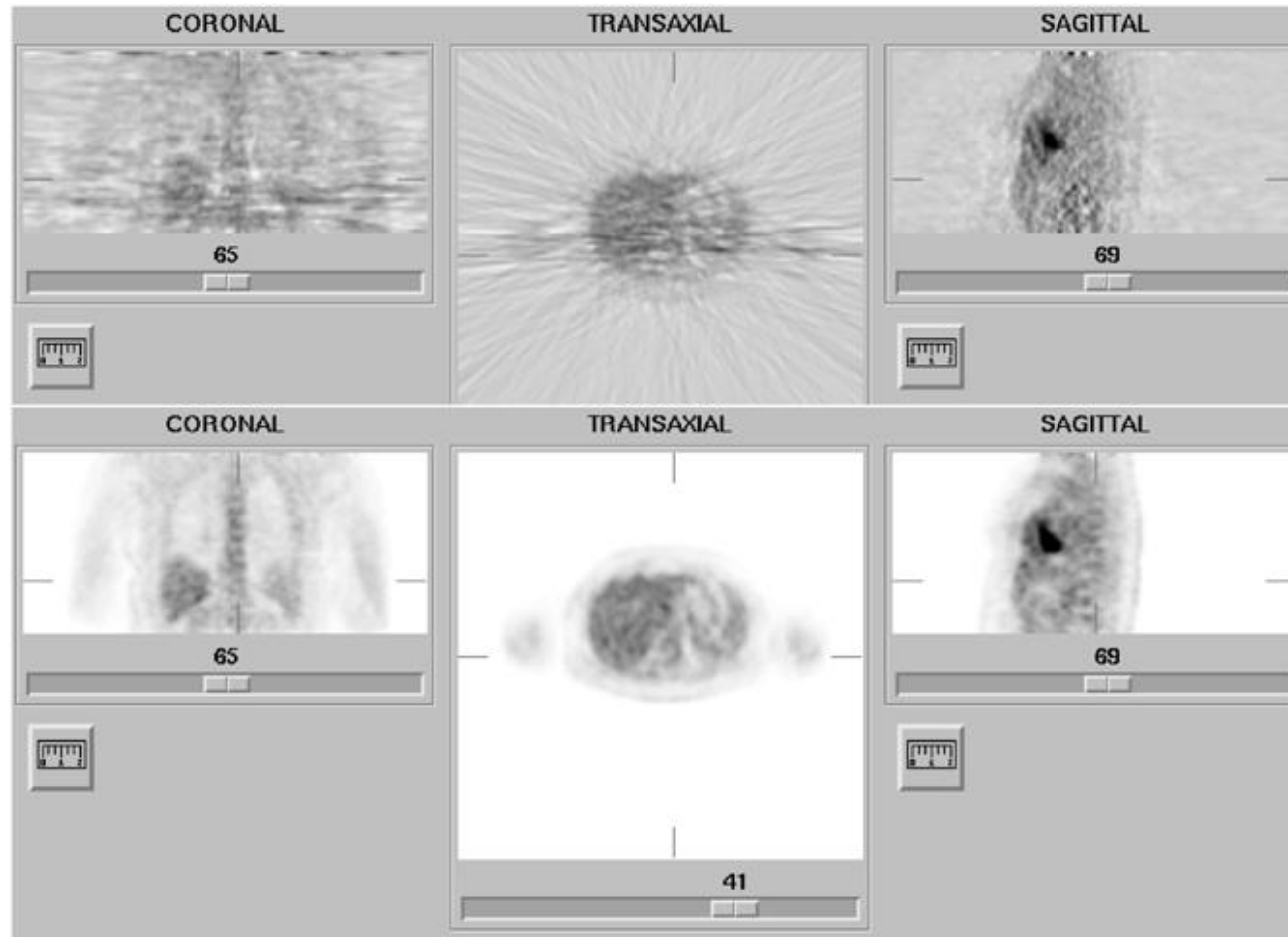
correction
sinogram



correction map

$$= \text{result 39th iteration} * \text{correction map}$$

Iterative reconstruction vs. FBP



Iterative reconstruction

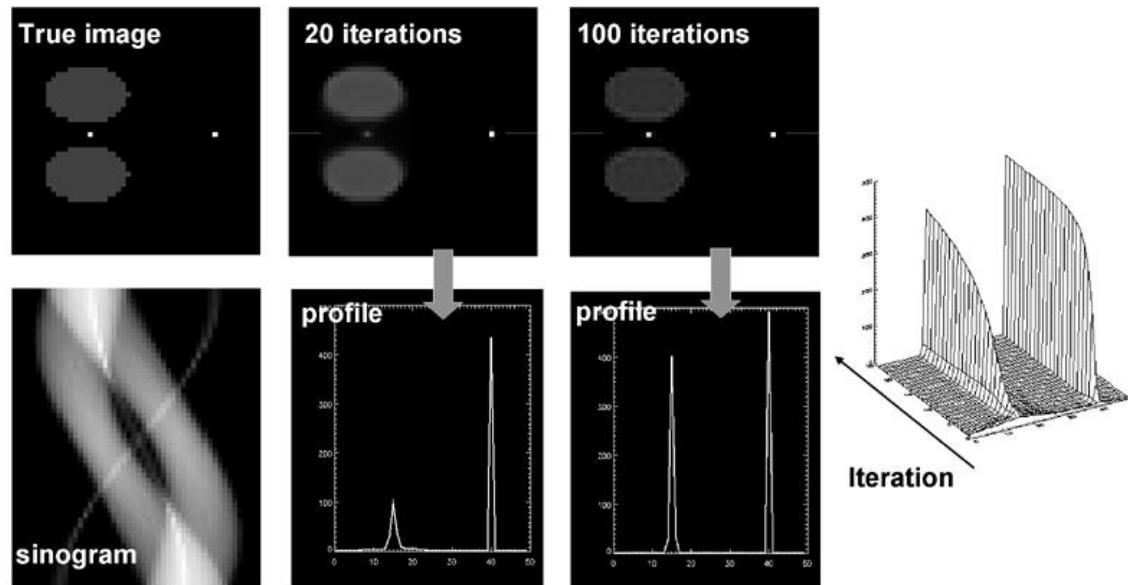
slow

high computation effort

convergence ?

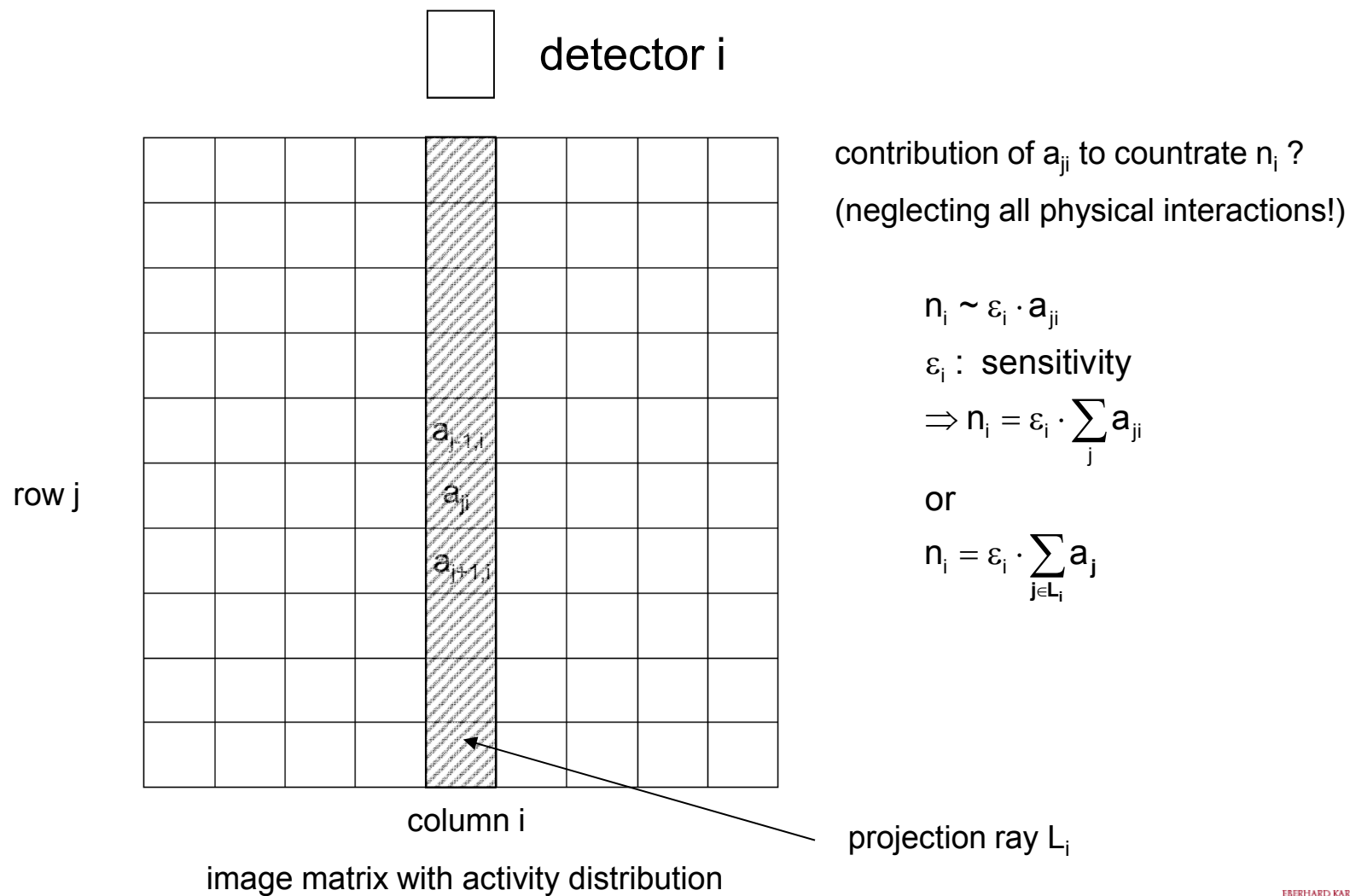
result depends on the number of iterations

→ limited quantitation

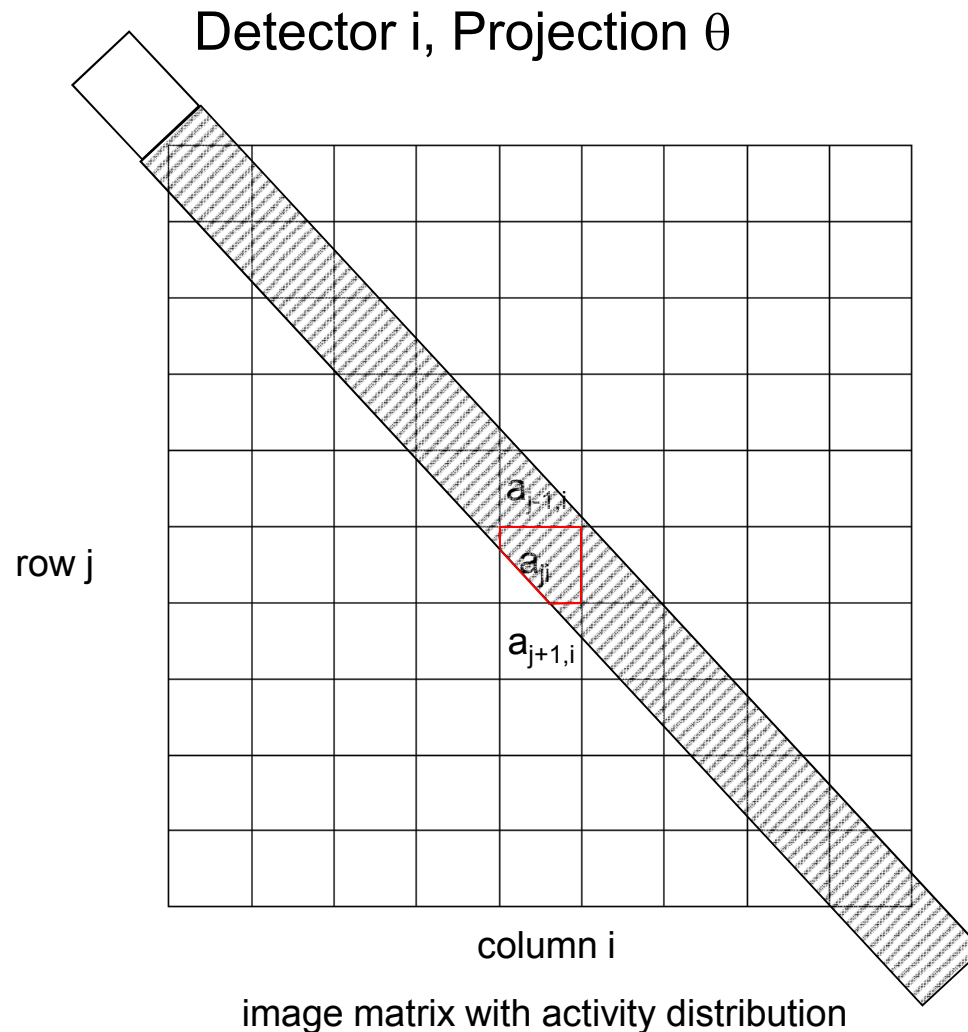


Bildnachweis: J. Nuyts, K.U. Leuven

Inside iterative reconstruction: forward projection



Inside iterative reconstruction: forward projection



contribution of a_{ji} to countrate n_i ?
(neglecting all physical interactions!)

$$n_i(\theta) \sim \varepsilon_i \cdot a_j \quad (j \in L_i)$$

introduction of a weight w_{ij} ,

$$n_i(\theta) \sim \varepsilon_i \cdot w_{ij} \cdot a_j$$

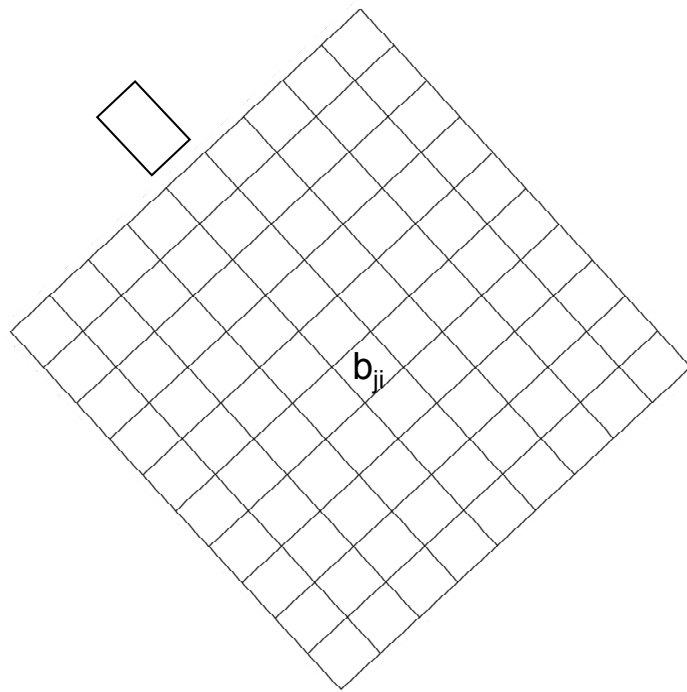
$$\Rightarrow n_i(\theta) = \varepsilon_i \cdot \sum_{j \in L_i} w_{ij} \cdot a_j$$

Now we are able to integrate the
all physical interactions in the
forward projection:

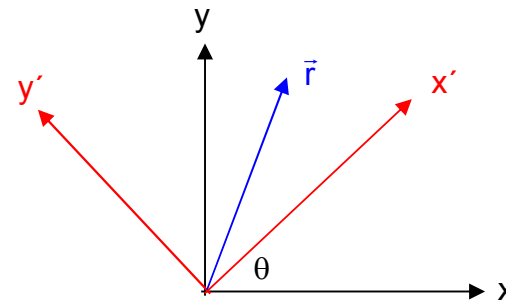
$$w_{ij} = \text{geom}_{ij} \cdot \text{atten}_{ij} \cdot \text{PSF}_{ij} \cdot \text{scatter}_{ij}$$

construction of the forward projector: Rotation of the image matrix in the projection direction

Detector i, Projection θ

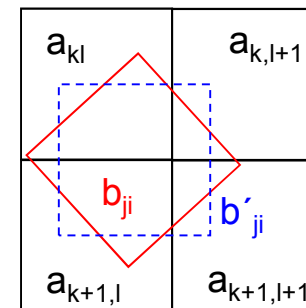


rotated Matrix M' with activity distribution



$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\vec{r}' = D \cdot \vec{r} \Rightarrow D^{-1} \cdot \vec{r}' = \vec{r}$$



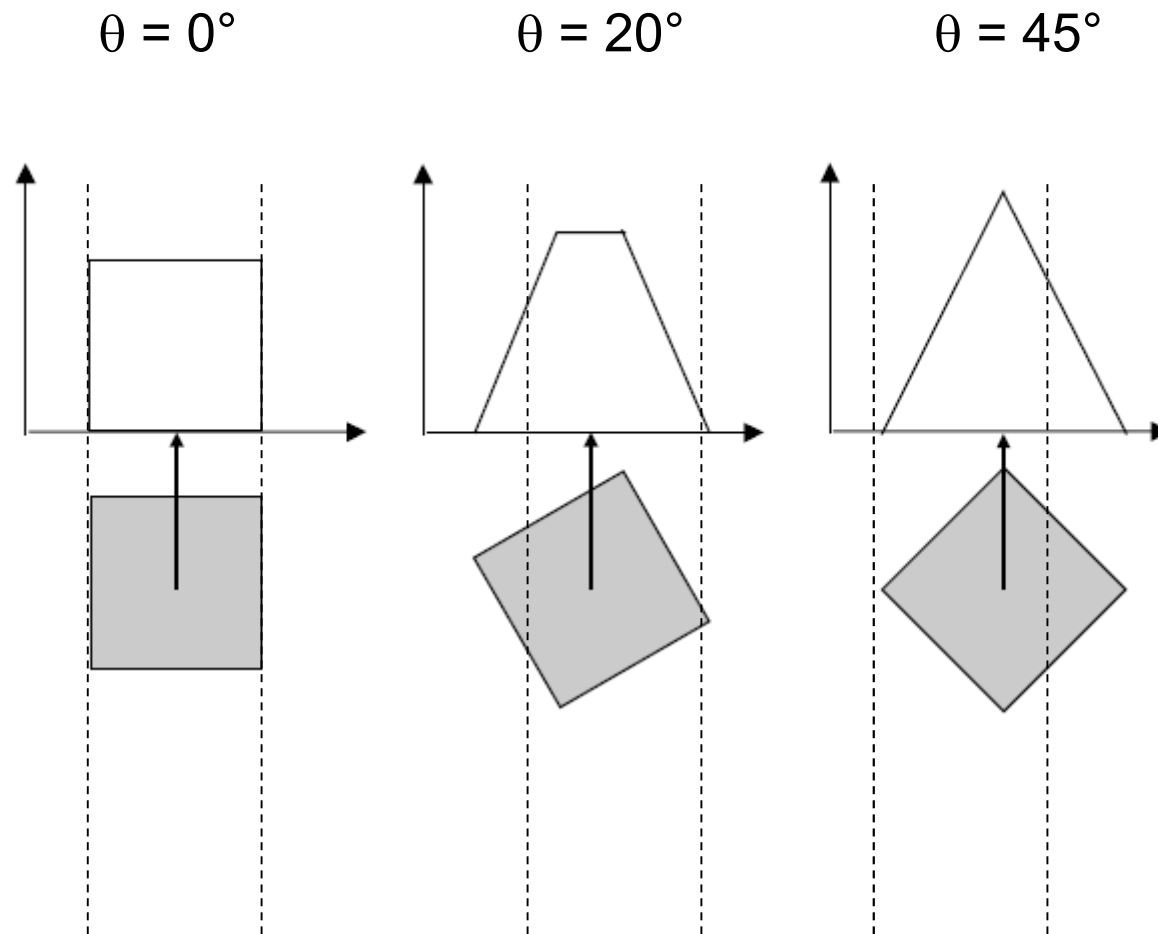
$$b'_{ji} = \sum_{k,l} w_{kl} \cdot a_{kl}$$

$$b_{ji} \approx b'_{ji} \quad (\text{really ?})$$

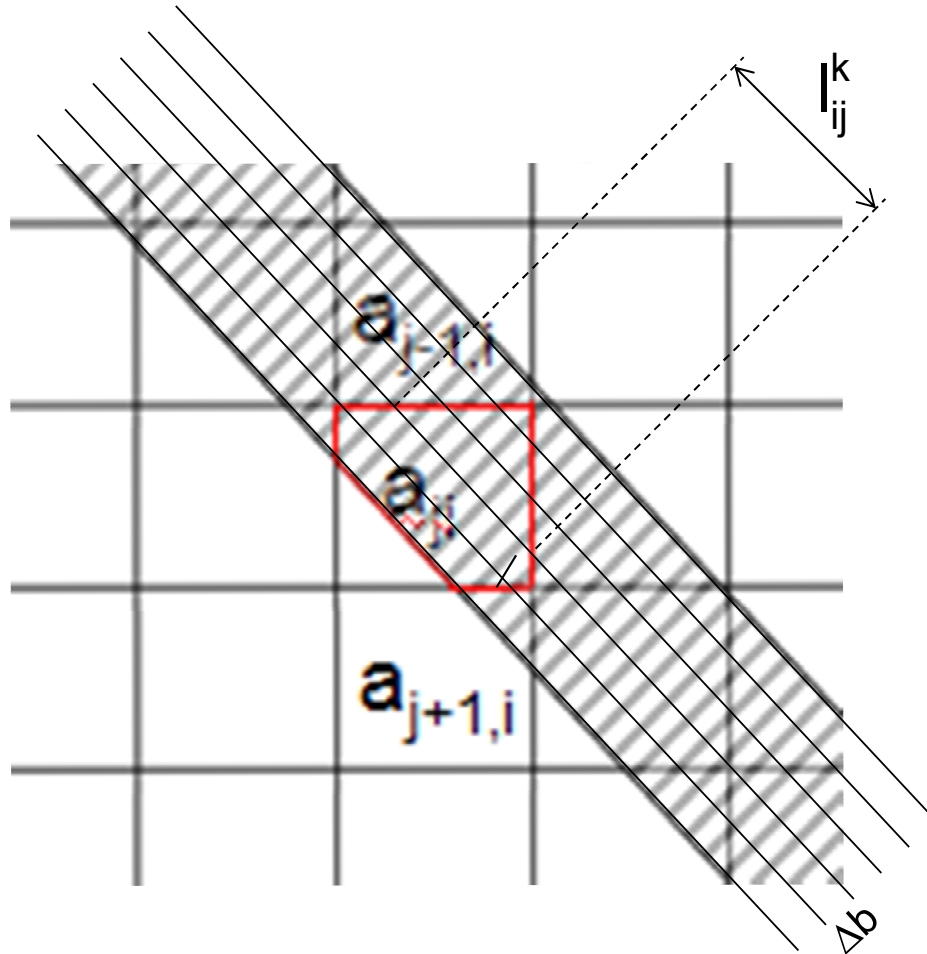
bilinear Interpolation

→ blurring !!!

construction of the forward projector: projection of a pixel to a pixel profile



construction of the forward projector: „fine-binning“

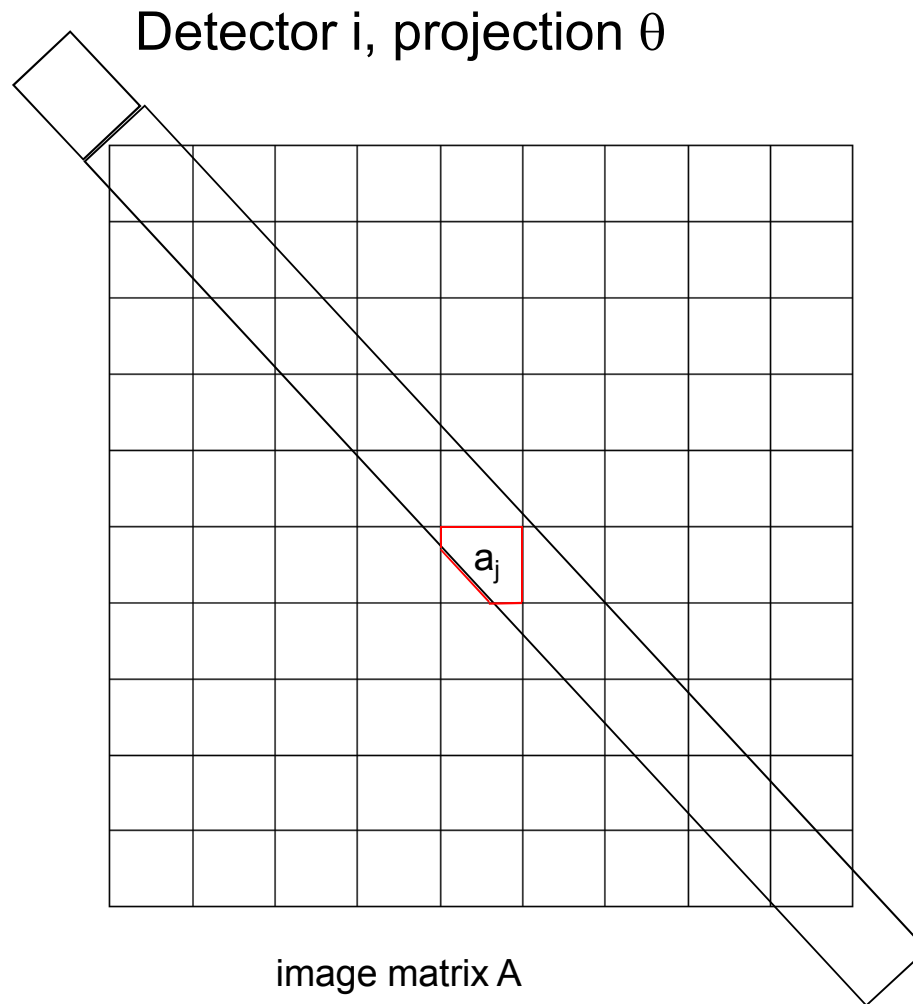


$$w_{ij} \approx \Delta b \cdot \sum_{k=1}^N \frac{1}{2} (l_{ij}^k + l_{ij}^{k+1})$$

$$w_{ij} \approx \Delta b \cdot \sum_{k=1}^{N+1} l_{ij}^k - \frac{1}{2} (l_{ij}^1 + l_{ij}^{N+1})$$

$$\Delta b = \frac{B}{N}$$

Image representation: pixel-(voxel) based



$$a(x,y,z) = \sum_j a_j \cdot H(x - x_j, y - y_j, z - z_j)$$

$$\text{where } H(x,y,z) = \begin{cases} 1 & \text{for } |x|, |y|, |z| \leq \frac{\Delta}{2} \\ 0 & \text{otherwise} \end{cases}$$

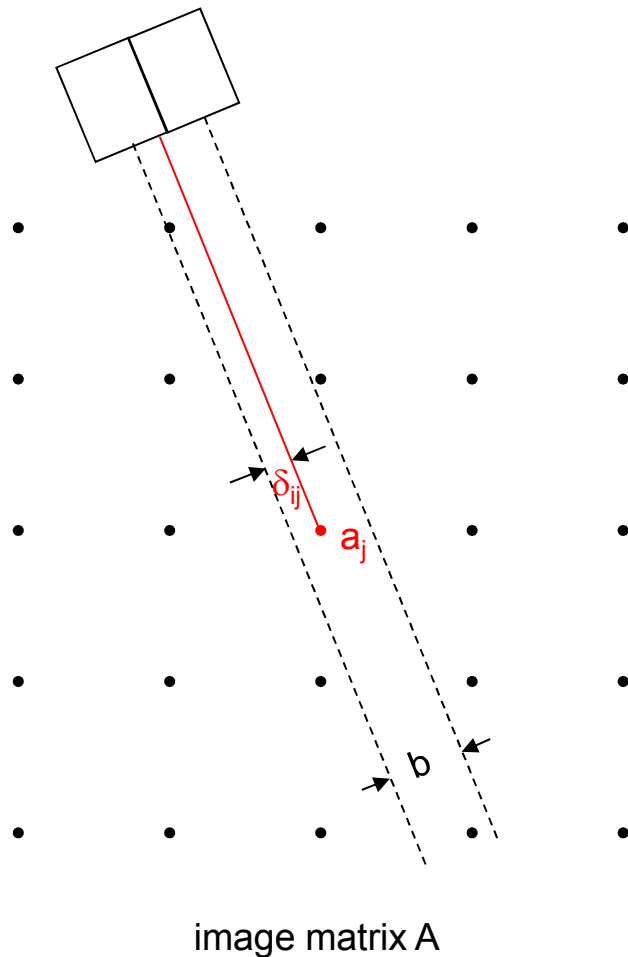
(voxel basis function)

$$n_i(\theta) = \varepsilon_i \cdot \sum_{j \in L_i} w_{ij} \cdot a_j$$

needs to calculate the weights w_{ij} to compute the countrate $n_i(\theta)$.

Image representation: grid point-based

Detector i, i+1, projection θ



$$a(x, y, z) = \sum_j a_j \cdot H(x - x_j, y - y_j, z - z_j)$$

$$\text{where } H(x, y, z) = \begin{cases} 1 & \text{for } |x|, |y|, |z| = 0 \\ 0 & \text{otherwise} \end{cases}$$

(grid basis function)

$$n_i(\theta) = \varepsilon_i \cdot \sum_j \left(1 - \frac{\delta_{ij}}{b}\right) \cdot a_j$$

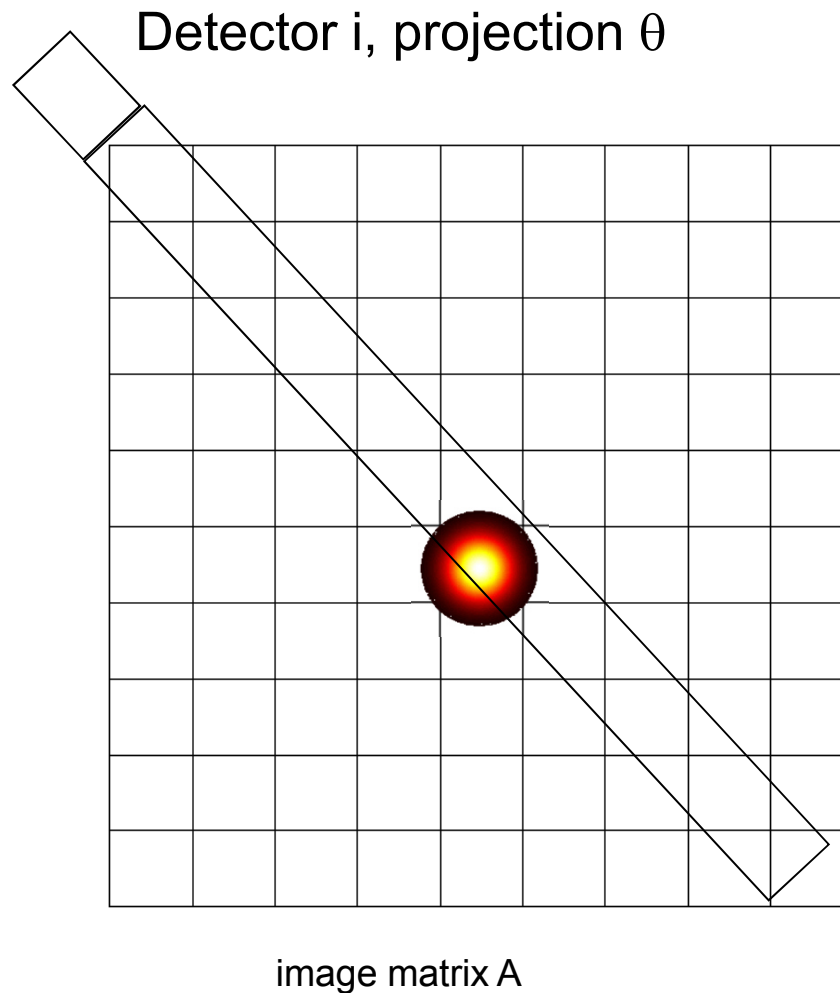
$$n_{i+1}(\theta) = \varepsilon_i \cdot \sum_j \frac{\delta_{ij}}{b} \cdot a_j$$

Easy calculation of δ_{ij} to

compute the countrate $n_i(\theta)$

but simplified model

Image representation: blob-based (spherically symmetric basis functions)



$$a(\vec{r}) = \sum_j a_j \cdot H(|\vec{r} - \vec{r}_j|)$$

$$\text{where } H(|\vec{r}|) = \begin{cases} \text{spherically symmetric for } |\vec{r}| \leq R \\ 0 \text{ otherwise} \end{cases}$$

(blob basis function)

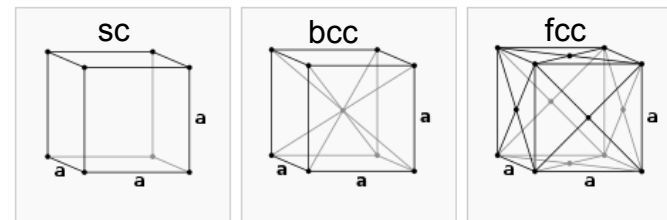
for example

$$H(|\vec{r} - \vec{r}_j|) = \begin{cases} 1 - \left(\frac{|\vec{r} - \vec{r}_j|}{R} \right)^2 & \text{for } |\vec{r}| \leq R \\ 0 & \text{otherwise} \end{cases}$$

$$n_i(\theta) = \varepsilon_i \cdot \sum_j w_{ij} \cdot a_j$$

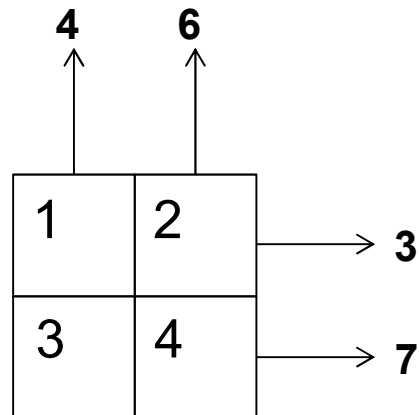
Easy and accurate calculation of the weights w_{ij} to compute the countrate $n_i(\theta)$.

Changing to other basis systems:

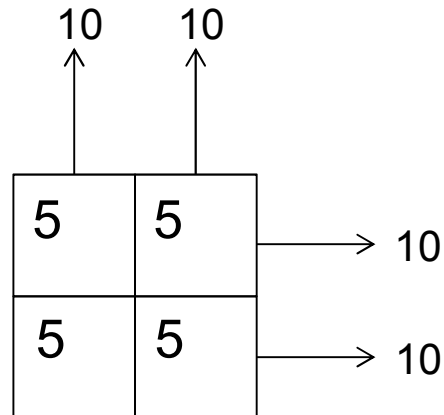


A numeric example

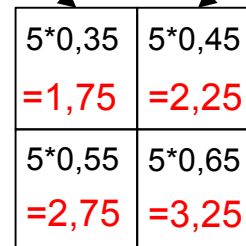
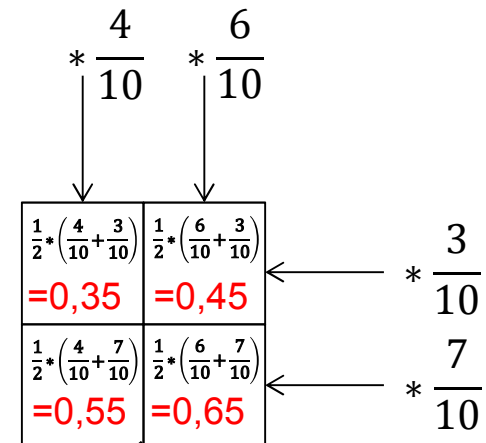
Initial
(measurement)



Current estimate &
forward projection



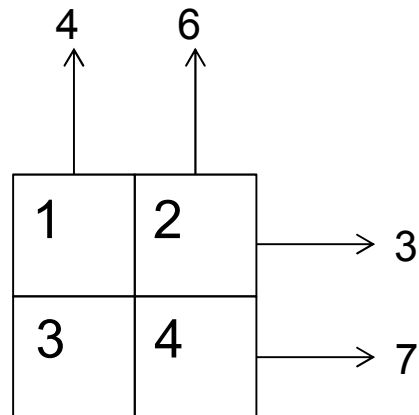
Correction



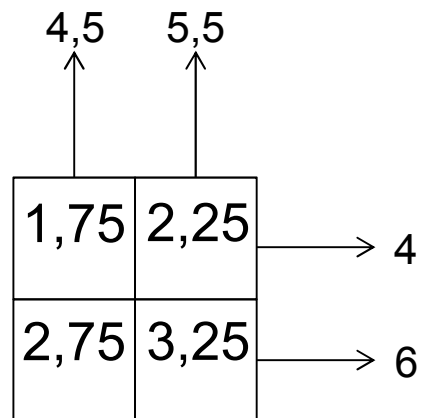
next estimate

A numeric example

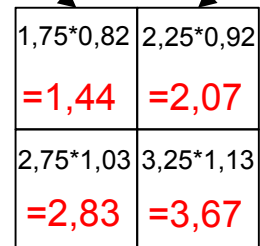
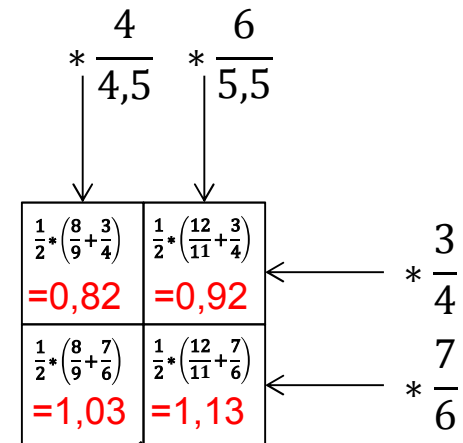
Initial
(measurement)



Current estimate &
forward projection



Correction



next estimate

iterative reconstruction: mathematical description

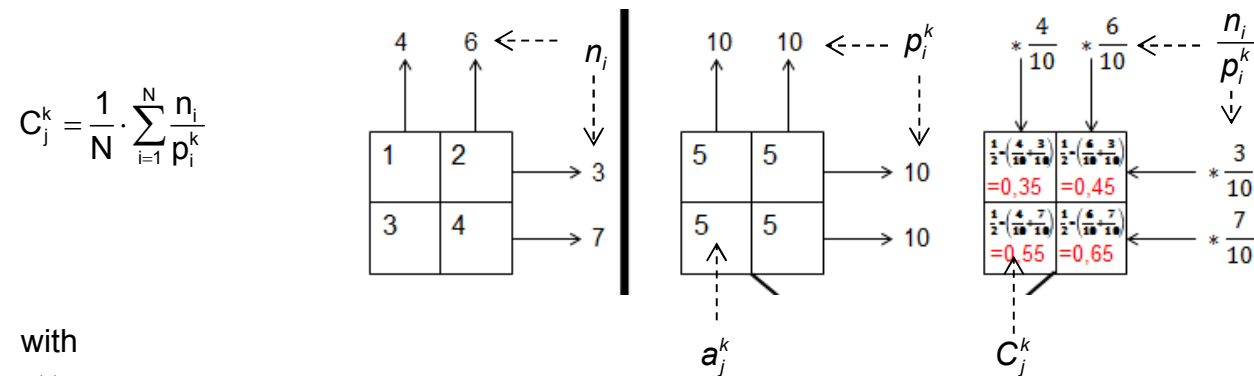
iterative reconstruction:

stepwise improvement of the image by comparing measured with simulated sinograms

correction in the (k+1) iterationstep for pixel j

$$a_j^{(k+1)} = a_j^{(k)} \cdot C_j^{(k)}$$

How looks the $C_j^{(k)}$ like? For Example, average values of the ratios measured vs. simulated



with

$$n_i^{(k)} = \sum_j w_{ij} \cdot a_j^{(k)} \text{ (forwardprojection)}$$

N_j : Number of projection rays, which contain pixel j

iterative reconstruction:

Different reconstruction algorithms

- ART: Algebraic reconstruction technique
 - technique for solving a set of linear equations of high order ($\sim 10^6$)
 - use average values (arithmetic, geometric)
- MLEM: Maximum Likelihood Expectation Maximization
 - statistical procedure regarding the statistical nature of nuclear disintegrations
 - maximize the likelihood for the measurement handling the forward projections as expectation values for the Poisson statistic
 - use weighted means
- MAP: Maximum a posteriori
 - similar to MLEM but control noise and resolution
 - maximize the likelihood for the validity of the reconstructed image regarding the validity of the measurement (posterior)
 - need to define the likelihood for the image using a prior term
 - Examples
 - anatomic based prior: images with 2 kidneys are more likely than images with 10 kidneys
 - szintigraphic based prior: large fluctuations in the neighborhood of a pixel are more unlikely than moderate fluctuations

iterative reconstruction: Different reconstruction algorithms

normal

ART:
$$C_j^k = \frac{1}{N} \cdot \sum_{i=1}^N \frac{n_i}{p_i^k}$$

MLEM:
$$C_j^k = \frac{1}{\sum_i w_{ij}} \cdot \sum_{i=1}^N w_{ij} \cdot \frac{n_i}{p_i^k}$$

MAP:
$$C_j^k = \frac{1}{\sum_i w_{ij} + \beta \cdot \frac{\partial}{\partial a_j} \underbrace{U(a_j, a_k)}_{\text{a priori term}}} \cdot \sum_{i=1}^N w_{ij} \cdot \frac{n_i}{p_i^k}$$

accelerated

ART:
$$C_j^k = \left(\frac{1}{N} \cdot \sum_{i=1}^N \frac{n_i}{p_i^k} \right)^\alpha$$

OSEM:
$$C_{js}^k = \frac{1}{\sum_i w_{ijs}} \cdot \sum_{i=1}^N w_{ijs} \cdot \frac{n_{is}}{p_{is}^k}$$

MAP-OS:
$$C_{js}^k = \frac{1}{\sum_i w_{ijs} + \beta \cdot \frac{\partial}{\partial a_j} \underbrace{U(a_j, a_k)}_{\text{a priori term}}} \cdot \sum_{i=1}^N w_{ijs} \cdot \frac{n_{is}}{p_{is}^k}$$

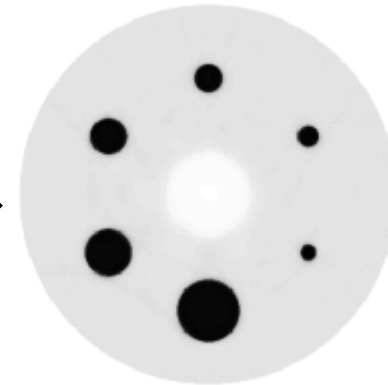
Inside OSEM I

180 projections
1° steps
0°, 1°, 2° ... 179°



40 iterations

87 sec

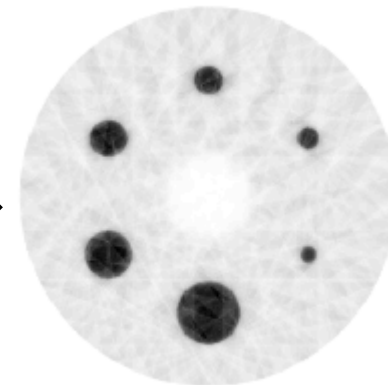


18 projections
10° steps
0°, 10°, 20° ... 170°



40 iterations

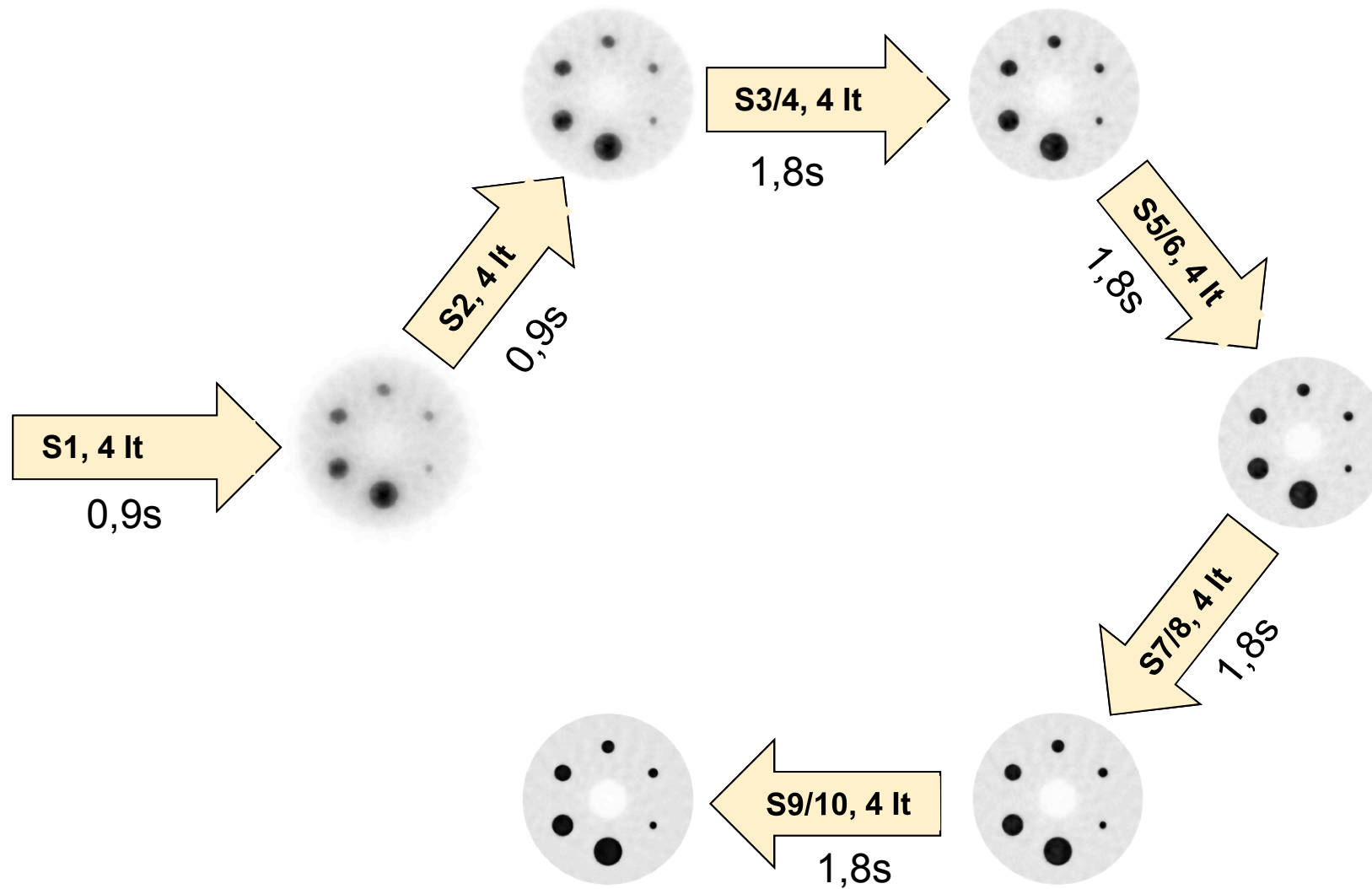
9,1 sec



= Subset

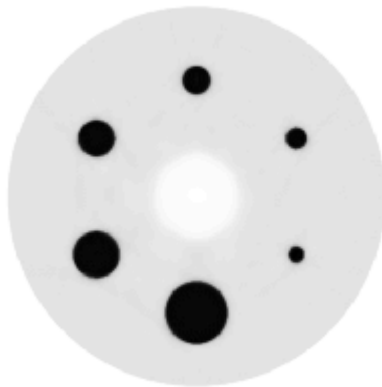
Not so bad !!!

Inside OSEM II



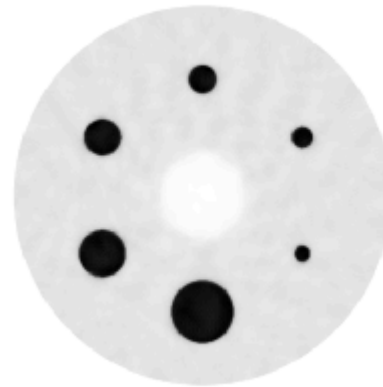
comparison MLEM/OSEM

MLEM
40 iterations



87 sec

OSEM
4 iterations, 10 subsets

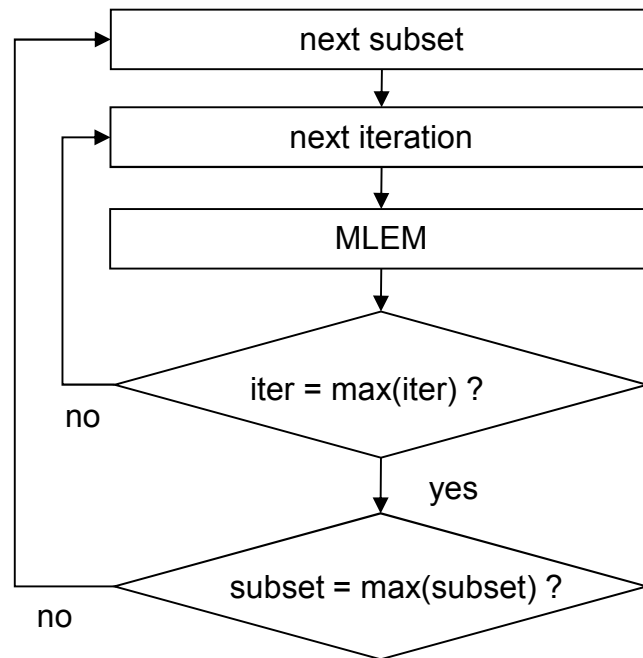


9 sec

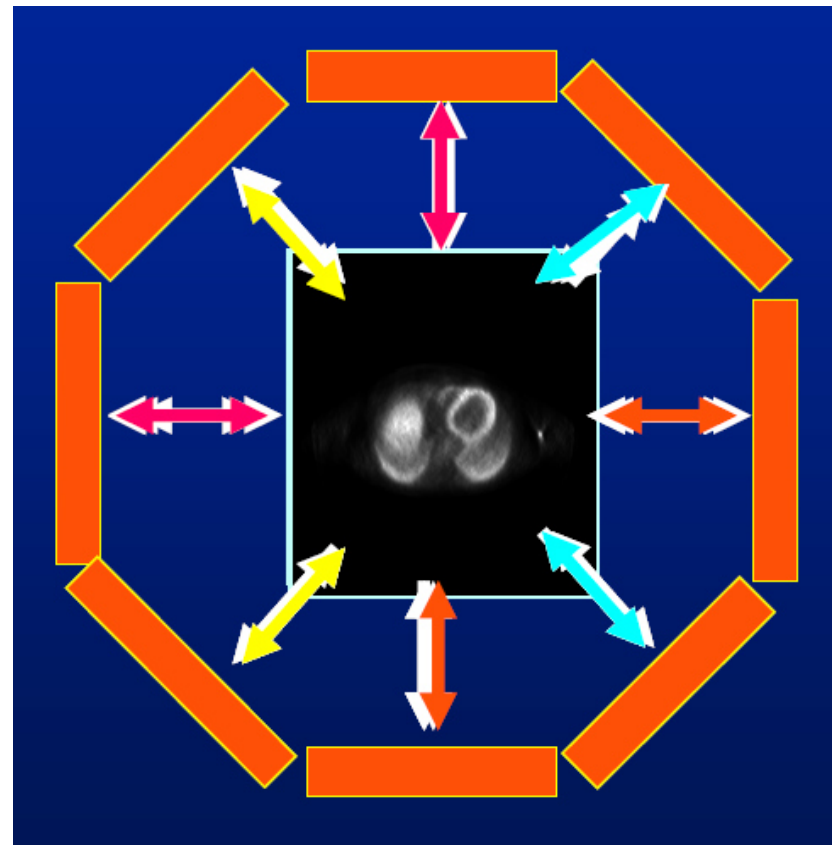
Iterative reconstruction OSEM

OSEM (Ordered Subset Expectation Maximisation)

Subdivide the projectionset in ordered subsets



→ acceleration



inside MAP

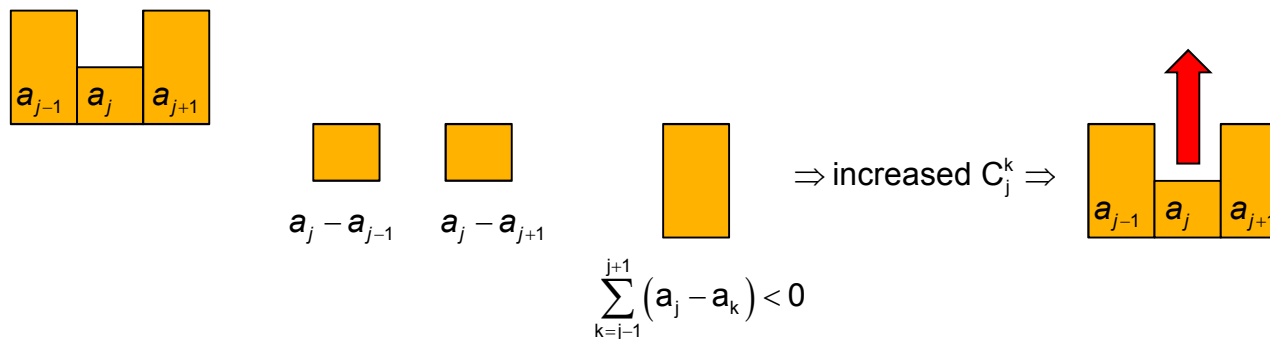
$$a_j^{(k+1)} = a_j^{(k)} \cdot C_j^k$$

$$C_j^k = \frac{1}{\sum_i w_{ij} + \underbrace{\beta \cdot \frac{\partial}{\partial a_j} U(a_i, a_k)}_{\text{noise \& resolution control}}} \cdot \sum_{i=1}^N w_{ij} \cdot \frac{n_i}{p_i^k}$$

Example: quadratic prior

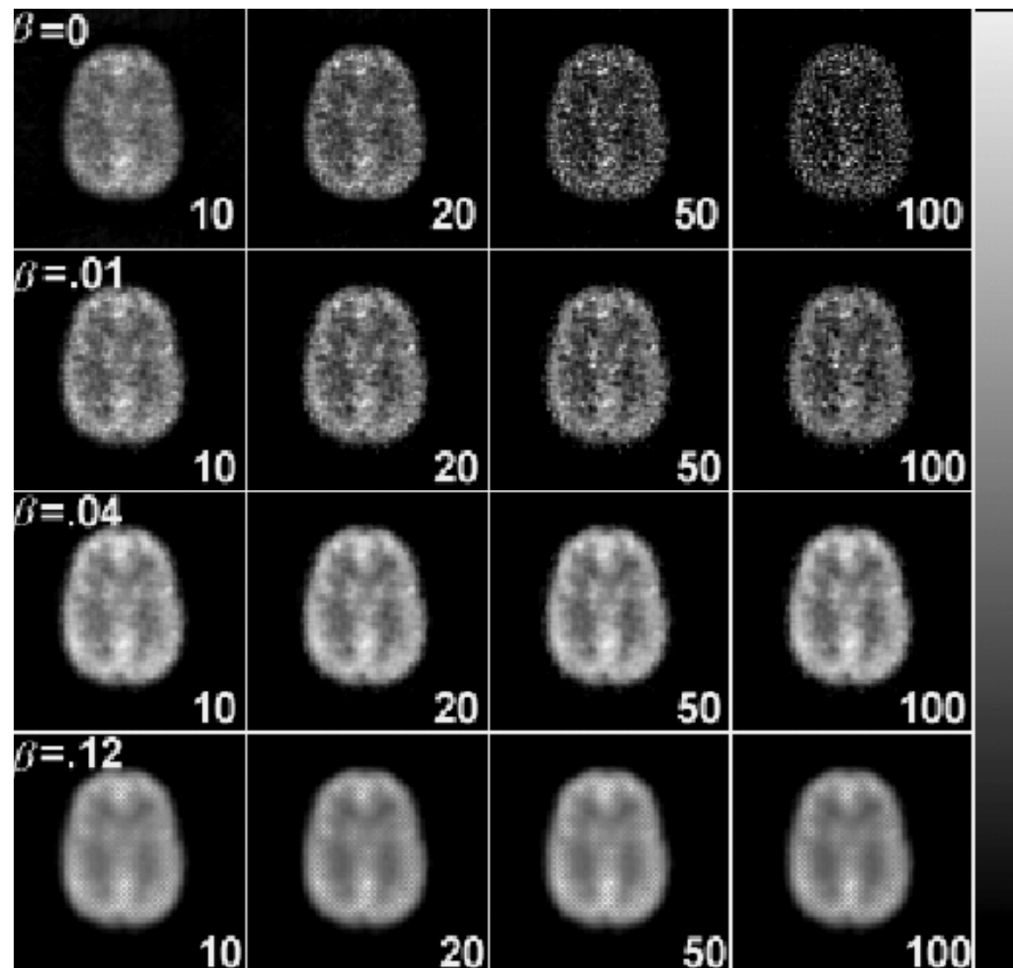
$$U(a_i, a_k) = \sum_k \sum_{i \in \text{Neighborhood of } k} (a_i - a_k)^2$$

$$\Rightarrow \frac{\partial}{\partial a_j} U(a_i, a_k) \sim \sum_{k \in \text{Neighborhood of } j} (a_j - a_k)$$

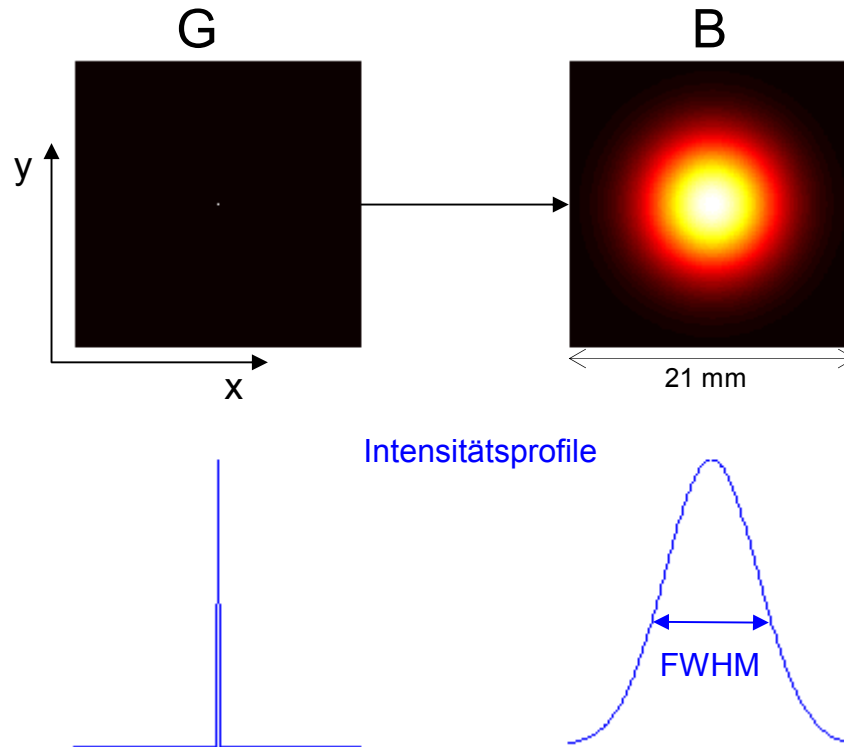


Iterative reconstruction MAP

- ‘strength’ of prior defined by β (hyperparameter)
- degree of smoothing versus edge preserving defined by type of prior



point spread function (PSF) and spatial resolution



An image in nuclear medicine can be described as a convolution of the original image with the PSF.

$$B(x, y) = G(x, y) \otimes PSF(x, y)$$

$$B(x, y) = \iint_{\square^2} G(x', y') \cdot PSF(x - x', y - y') dx' dy'$$

The FWHM depends on the distance d source-detector

$$FWHM(d) = a + m \cdot d$$

empirical value $m \approx 0,5 \text{ mm/cm distance}$

direct influence on image quality and quantity

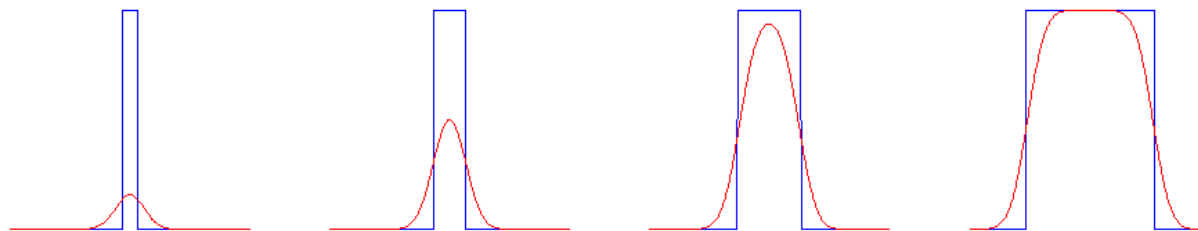
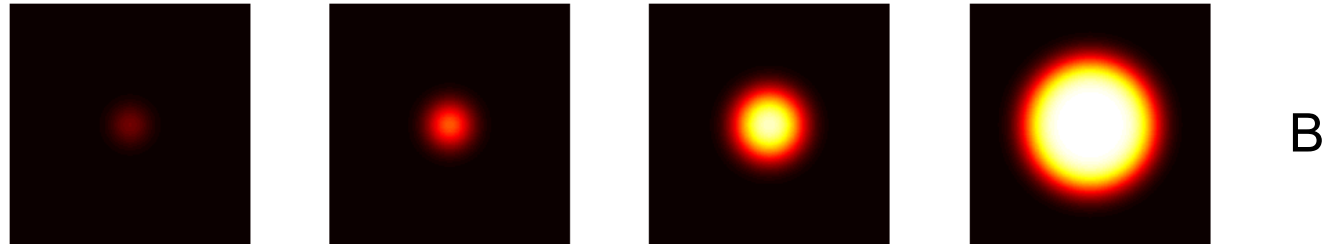
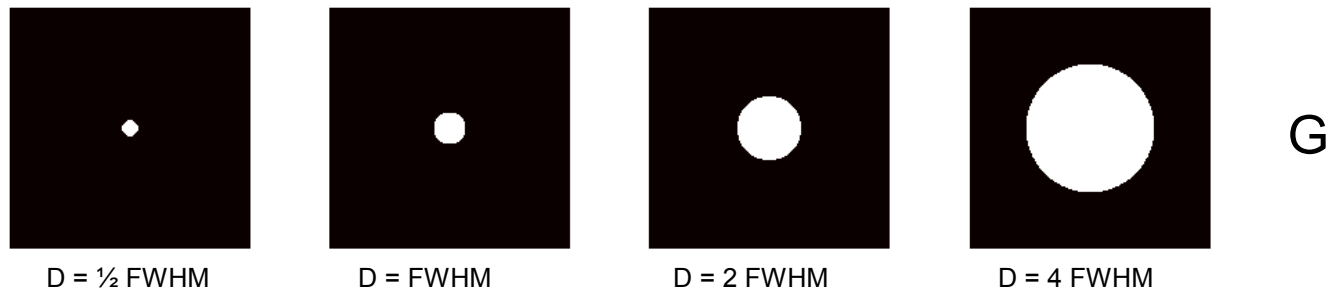
spatial resolution = FWHM of the PSF

$$PSF(x, y) = \frac{1}{2\pi\sigma_x\sigma_y} e^{-\left(\frac{x^2}{2\sigma_x^2} + \frac{y^2}{2\sigma_y^2}\right)} \quad (2 \text{ dimensional})$$

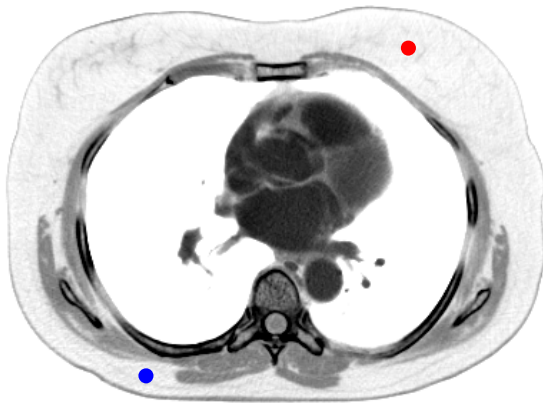
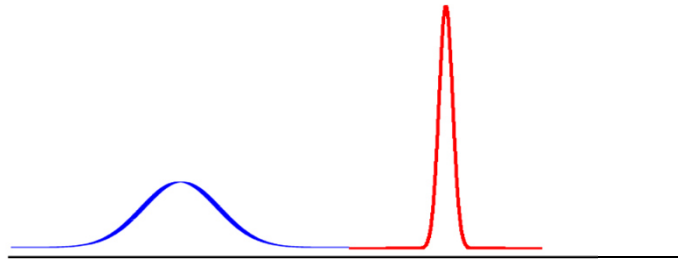
$$\text{mit } \iint_{\square^2} PSF(x, y) dx dy = 1$$

partial volume effekt (PVE)

original distribution: homogenous activity distribution in lesions G with different diameters



Spatial resolution (PSF)



Correction:

PSF computation of all image points to all possible projections (Calculation of contribution probabilities p_{ij})

Storage in a lookup-table (LUT)

Calculation of the total contribution weights

$$w_{ij} = \begin{cases} w_{ij}^{\text{geom}} \cdot p_{ij} & \text{if } w_{ij}^{\text{geom}} \neq 0 \\ p_{ij} & \text{otherwise} \end{cases}$$

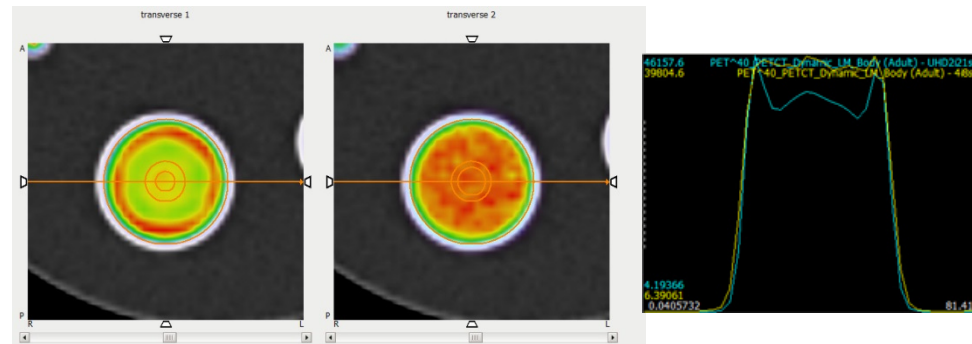
and apply iterative reconstruction

$$a_j^{(k+1)} = a_j^{(k)} \cdot \frac{1}{\sum_i w_{ij}} \cdot \sum_i w_{ij} \cdot \frac{m_i}{n_i^{(k)}}$$

or: iterative deconvolution in a postprocessing step

→ increased resolution, increased contrasts

→ but: artefacts (Gibbs-oscillations, ringing)



PVE Correction

Definition:

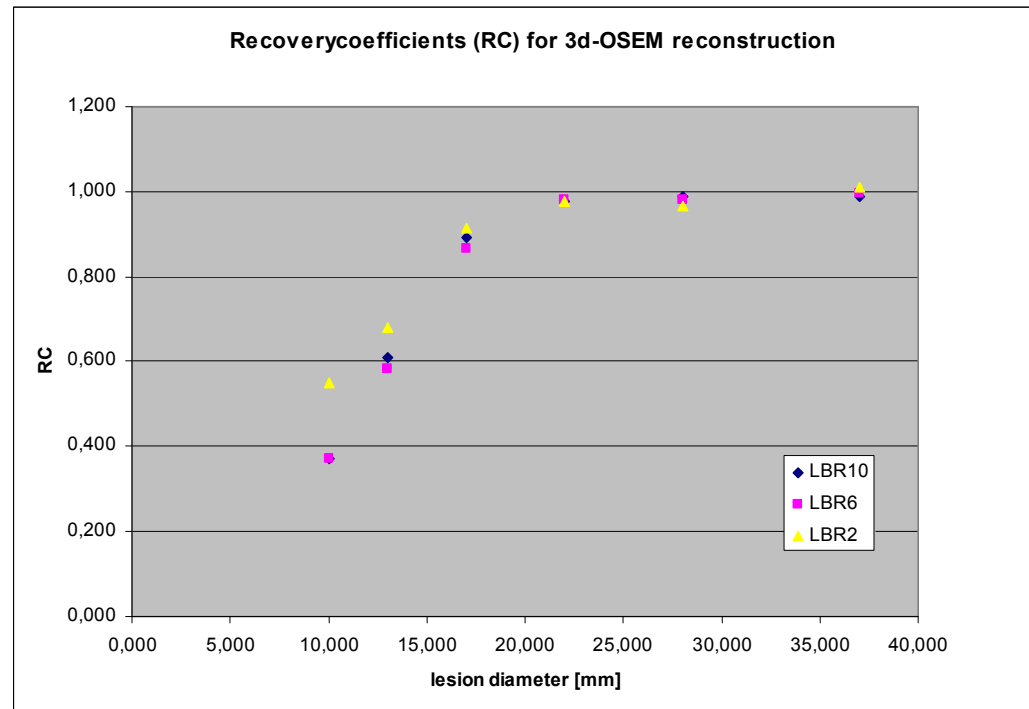
Recovery coefficient RC

$$RC = \frac{\text{measured activity concentration}}{\text{real activity concentration}} \leq 1$$

for a defined volume of interest (VOI)

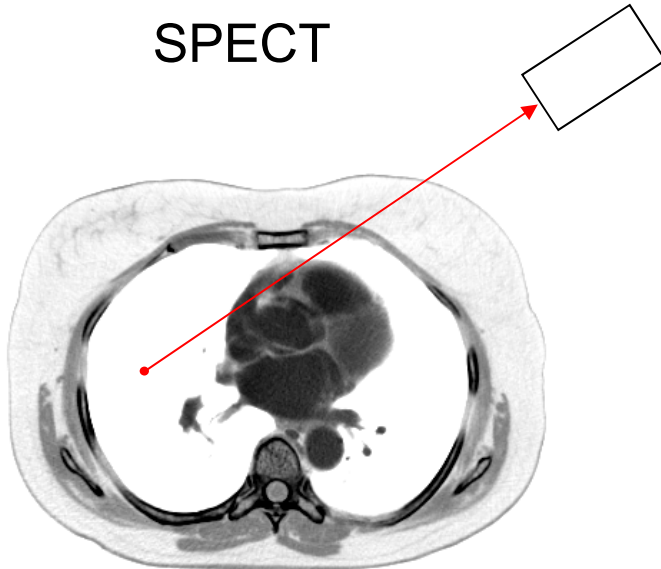
Creation of lookup tables for defined lesions in dependance of

- lesion size
- lesion geometry
- contrast (LBR)
- reconstruction method
- VOI size
- maximum and avg concentration



attenuation

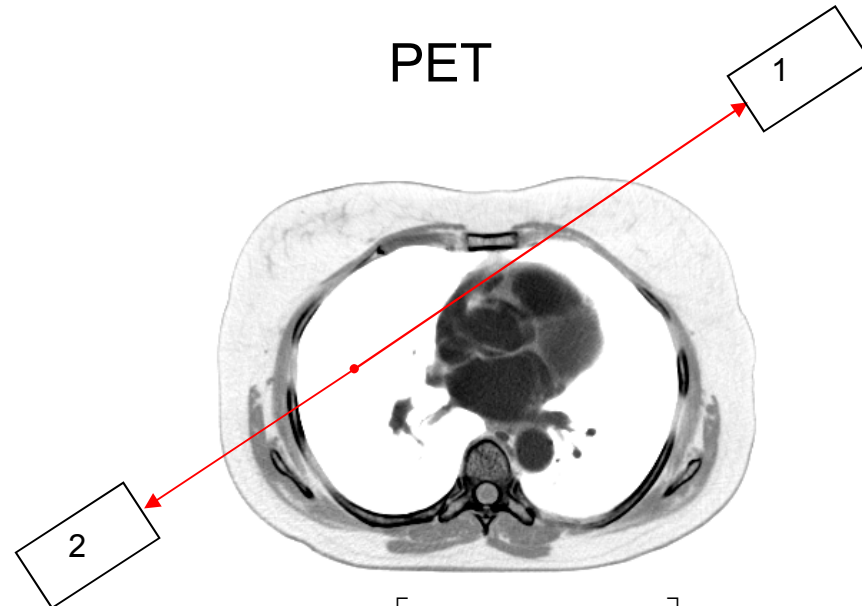
SPECT



$$f(x, y) = \exp \left[- \int_{(x,y)}^{\text{Detektor}} \mu(x', y') dL \right]$$

depends on the location of the source

PET



$$f_1(x, y) = \exp \left[- \int_{(x,y)}^{\text{Detektor1}} \mu(x', y') dL \right]$$

$$f_2(x, y) = \exp \left[- \int_{(x,y)}^{\text{Detektor2}} \mu(x', y') dL \right]$$

$$f = f_1 \cdot f_2 = \exp \left[- \int_{\text{Detektor1}}^{\text{Detektor2}} \mu(x', y') dL \right]$$

independent of the source location

Generation of the map of linear attenuation coefficients (μ -map)

CT

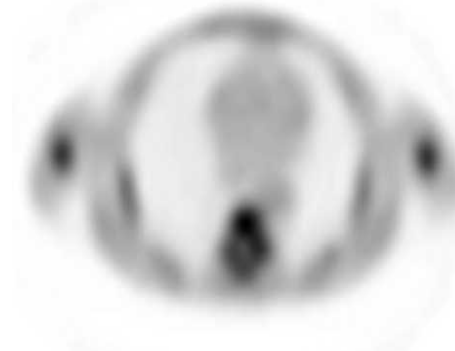


- HU
 - calculate μ -map from the HU values
 - apply an energy correction
- Warning by applying contrast agents

Transmission measurement with radioactive sources

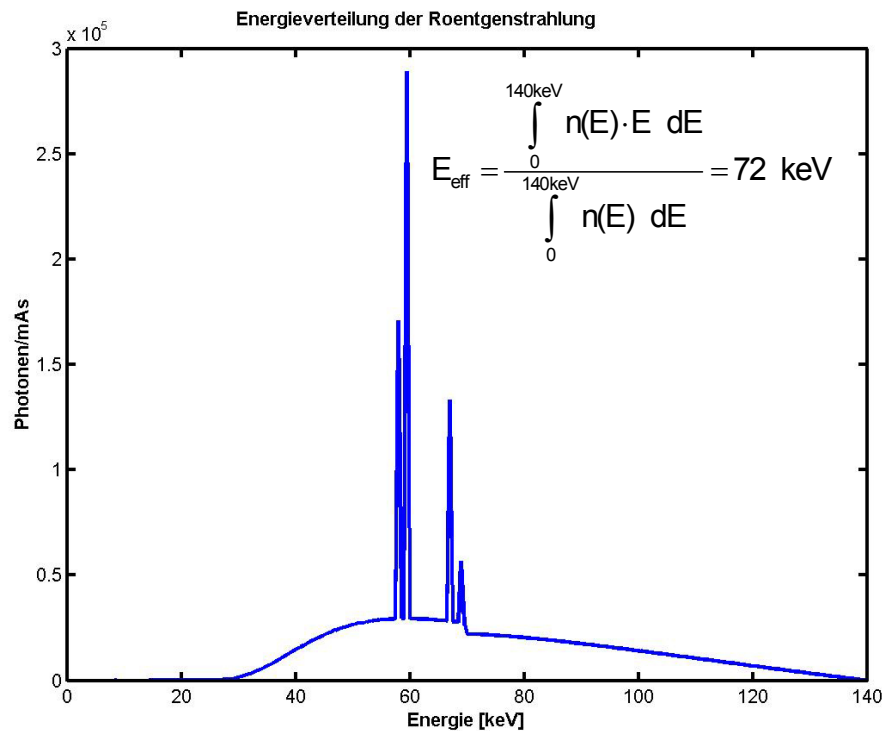
PET : ^{68}Ge

SPECT: ^{153}Gd



- reconstruction results directly in the μ -map (for transmission energy)
 - needs energy correction
- expensive, noisy

Energydistribution of X-ray radiation and calculation of the μ -map from the HU-values



$$HU = \frac{\mu_{\text{tissue}} - \mu_{\text{H}_2\text{O}}}{\mu_{\text{H}_2\text{O}}} \cdot 1000$$

energycorrection :

$HU < 0$ (mixture of air and water)

$$\mu(E) = \mu_{\text{H}_2\text{O}}(E) + \frac{(\mu_{\text{H}_2\text{O}}(E) - \mu_{\text{air}}(E)) \cdot HU}{1000}$$

$HU \geq 0$ (mixture of bone and water)

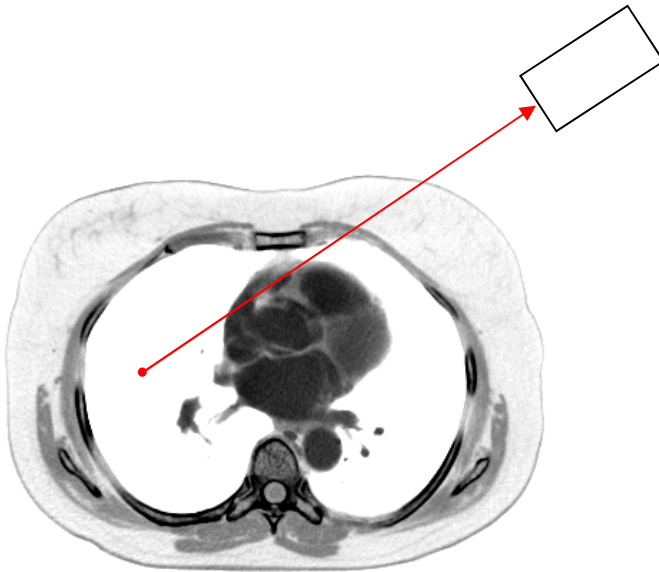
$$\mu(E) = \mu_{\text{H}_2\text{O}}(E) + \frac{(\mu_{\text{bone}}(E) - \mu_{\text{H}_2\text{O}}(E)) \cdot HU}{1000} \cdot Q$$

$$Q = \frac{\mu_{\text{H}_2\text{O}}(E_{\text{eff}})}{\mu_{\text{bone}}(E_{\text{eff}}) - \mu_{\text{H}_2\text{O}}(E_{\text{eff}})}$$

attenuation correction for SPECT

Needs a μ -map

- Transmission measurement
- CT



$$f(x, y) = \exp \left[- \int_{(x,y)}^{\text{Detektor}} \mu(x', y') dL \right]$$

correction (not analytical):

Calculation of the attenuation factors for every point in the image and every projection angle

Calculation of the contribution weights

$$w_{ij} = \text{geom}_{ij} \cdot \text{atten}_{ij}$$

and apply an iterative reconstruction

$$a_j^{(k+1)} = a_j^{(k)} \cdot \frac{1}{\sum_i w_{ij}} \cdot \sum_i w_{ij} \cdot \frac{m_i}{n_i^{(k)}}$$

Alternative (und not so accurate): Chang's Method

Calculation of the attenuation factors for every point in the image and every projection angle

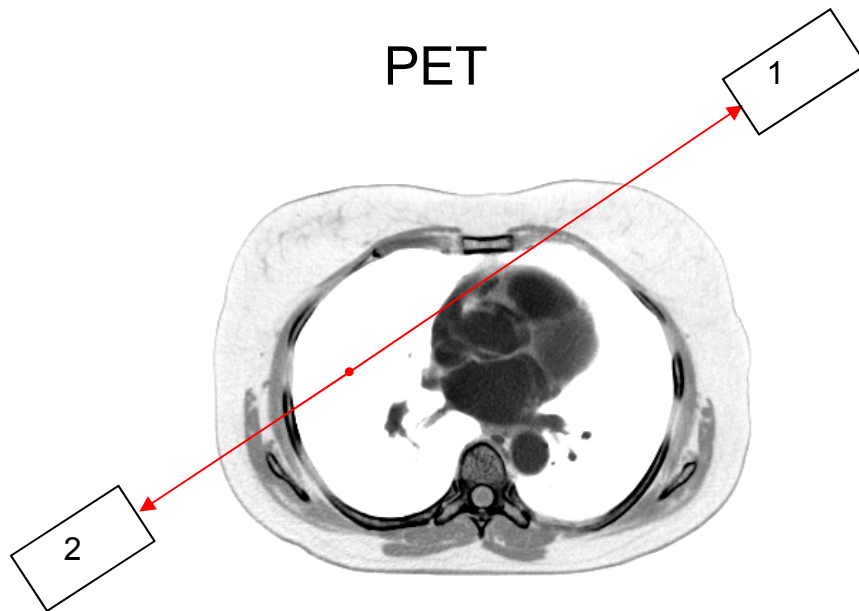
average the f values for every point: f_{avg}

Divide the non-corrected image by f_{avg} values

attenuation correction : PET

needs a μ -map for für keV

- Transmissionmeasurement
- CT



correction (analytical):

forward projection of the μ -map to the PET-projections

Calculate correction sinograms $acf = 1/f$

Multiplicate the measured PET-sino with the acf

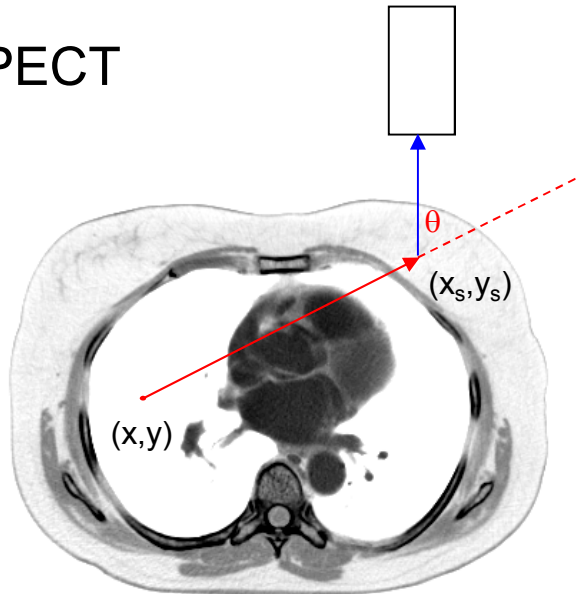
$$m_i^{ac} = m_i \cdot acf$$

$$a_j^{(k+1)} = a_j^{(k)} \cdot \frac{1}{\sum_i w_{ij}} \cdot \sum_i w_{ij} \cdot \frac{m_i^{ac}}{n_i^{(k)}}$$

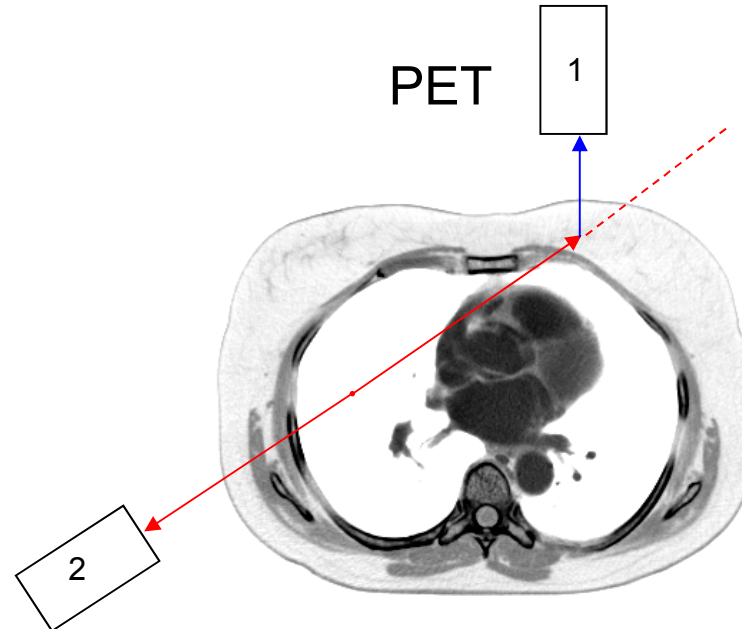
$$f = f_1 \cdot f_2 = \exp \left[- \int_{\text{Detektor1}}^{\text{Detektor2}} \mu(x', y') dL \right]$$

Compton scatter

SPECT



PET



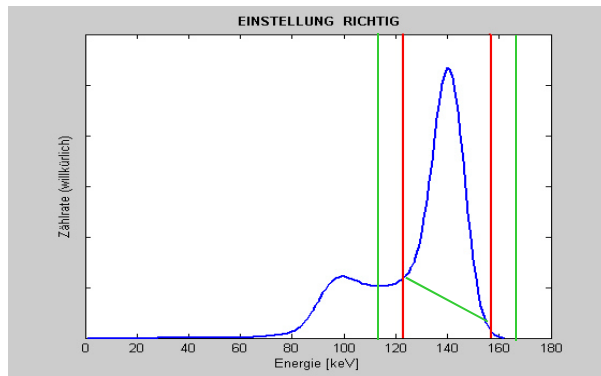
Probability p_s for scattering at (x_s, y_s) at angle θ

$$p_s \sim \frac{\mu(x_s, y_s)}{r^2} \cdot \exp\left(-\int_{(x,y)}^{(x_s, y_s)} \mu(x', y') dL\right) \cdot \exp\left(-\int_{(x_s, y_s)}^{\text{Detektor}} \mu(x', y') dL'\right) \cdot \frac{d\sigma}{d\Omega}(\theta)$$

analytic solution is not possible, but an approach for scatter order 1 in PET

scatter correction

Triple-Energy-Window



Measurement with three energy windows and interpolation of the scatter in the photopeak window.

$$N_{scatter} = \frac{1}{2} \left(\frac{N_{down}}{\Delta E_{down}} + \frac{N_{up}}{\Delta E_{up}} \right) \cdot \Delta E_{photo}$$

easy to implement

not accurate but useful

model-based correction in PET

(Ollinger et.al, Werling et.al.)

needs a μ -map

- Monte-Carlo based calculation
- Calculation of the scatter weights
- selected scatter centres
- Interpolation
- Subtraction

„good“ approach for scatter order 1

It does not work in SPECT (multiple scatter)

Universitätsklinikum Tübingen

**Kompetenz
mit Herz**

www.uniklinikum-tuebingen.de

Vielen Dank
für Ihre Aufmerksamkeit!

