

Lasers and Light Sources

Jürgen Köhler

Institute for
Photovoltaics

juergen.koehler@ipv.uni-stuttgart.de

Contact

Jürgen Köhler Institute for Photovoltaics

Room No.: 1.235

Phone: 685-67159

juergen.koehler@ipv.uni-stuttgart.de

Consultation hours: Friday 2:15pm to 3:30pm

Institut für Photovoltaik

a) Structure of *ipv*

Employees: ~ 30

Annual Turnover: ~ 2.5 Mill. €

PhD Students: ~ 11

Research Groups: 4

Student Works: 15 - 20 per Year

b) Main Focus of Research

Micro- and Optoelectronics

Sensor Technology and Photovoltaics

Semiconductor Technology and Processing

Semiconductor Physics

Solar Cells and Thin Film Technology



Table of Contents

1. The Human Eye
2. Light and Color
3. Photometry
4. Incoherent Light Sources
5. Light Emitting Diodes
6. Lasers
7. Laser Processing

1. The Human Eye

- 1.1 Anatomy of the human eye
- 1.2 Refracting power D of the human eye
- 1.3 Near point / far point
- 1.4 Focusing Problems
- 1.5 Rods, cones
- 1.6 Color vision
- 1.7 Color deficiency
- 1.8 Internet Links

2. Light and Color

- 2.1 Light sensitivity of the human eye
- 2.2 CIE Color matching functions
- 2.3 Color coordinates and color chromaticity coordinates
- 2.4 Chromaticity diagram
- 2.5 Color Mixing

3. Photometry

3.1 Energetic Quantities of Radiation and Irradiation

3.2 Visual Quantities of Light and Illumination

4. Incoherent Light Sources

4.1 Black body radiation

4.2 Gray body radiation

4.3 Selective body radiation of a semiconductor

4.4 Emissivity of non-black bodies

4.5 Incandescent lamps

4.6 Gas discharge light sources

5. Light Emitting Diodes

5.1 Working principle

5.2 The emitted spectrum of an LED

5.3 Materials for LEDs (and lasers)

5.4 Emission efficiency of LEDs

5.5 Organic Light Emitting Diodes (OLEDs)

©JHW



6. Lasers

6.1 Fundamental processes

6.2 Occupation probability in thermodynamic equilibrium

6.3 Microscopic analysis

6.4 Setup of a laser

6.5 Optical resonator

6.6 Line width

6.7 Laser types

6.8 Semiconductor lasers

7. Laser Processing

7.1 Laser doping

1. The Human Eye

The human eye [1.1]

- Detects color and luminosity Helligkeit
- Color = sensory perception Sinneswahrnehmung
- Color is directly related to human vision
- Color = biological term
- Radiation = physical term

1.1 Anatomy of the human eye

1.1.1 View from outside [1.2, 1.3]

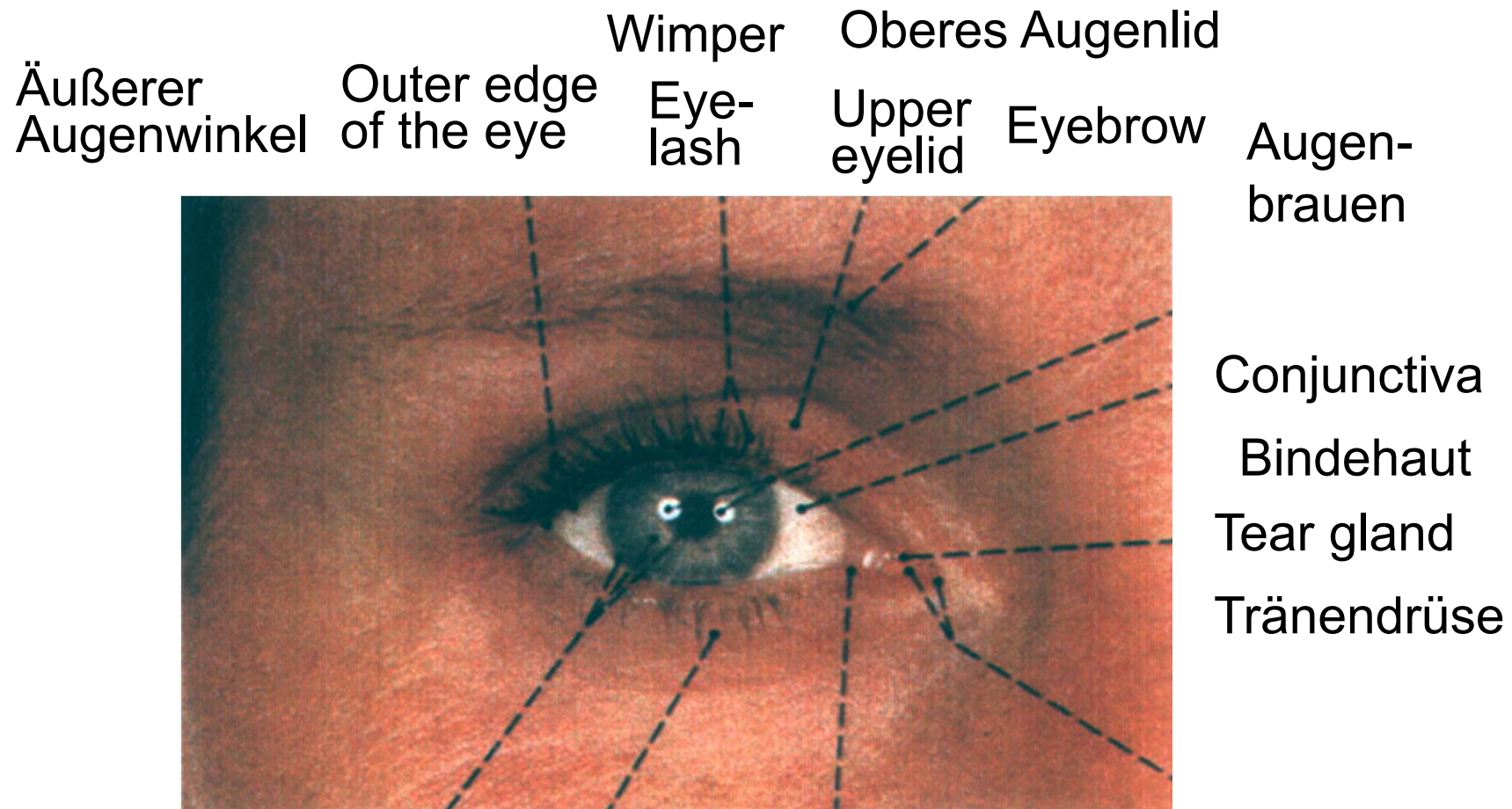
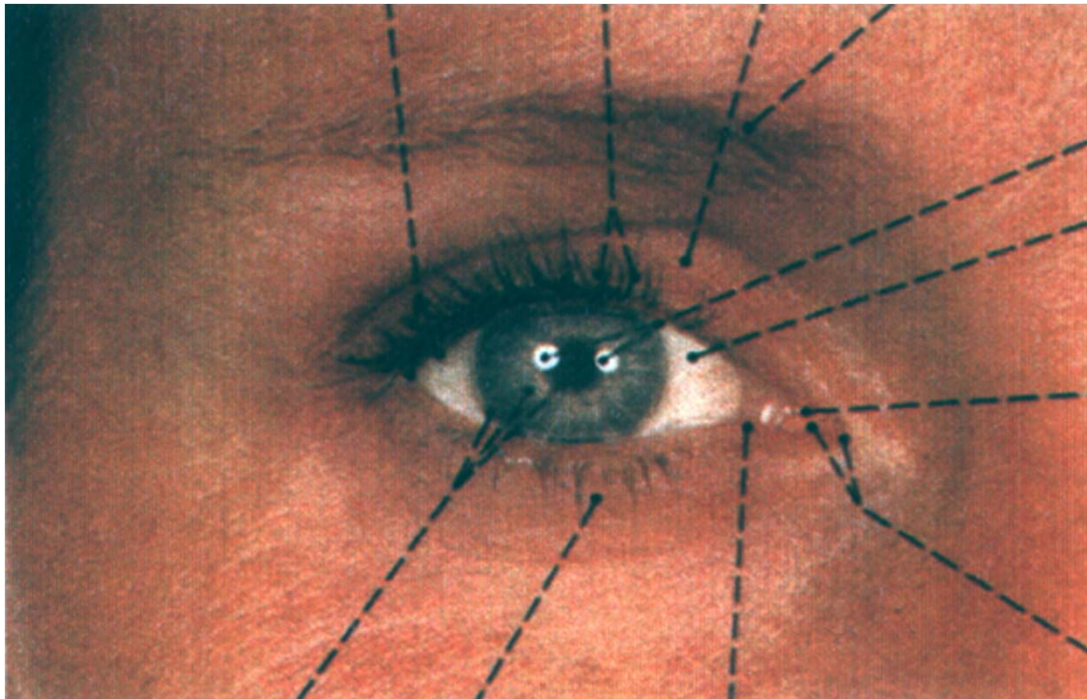


Figure 1.1 : Outer view of the human eye

©JHW



Innerer
Augenwinkel

Inner edge
of the eye

Iris and
pupil

Lower
eyelid

Iris und
Pupille

Unteres
Augenlid

Figure 1.1 : Outer view of the human eye

- Eyelids: protect eyes from the environment, injury and light. They maintain the smooth surface by spreading tears evenly over the eye.
- Pupil: the black, circular opening in the center of the iris. It opens and closes in order to regulate the amount of light entering the eyeball.

Bindehaut

Gewebe

- Conjunctiva: the thin, transparent *tissue* that covers the outer surface of the eye. It also secretes oil and *mucous* that *moisten* and *lubricate* the eye.

Schleim

schmieren befeuchten

- Iris: the colored part of the eye. It controls light level inside the eye similar to the aperture of a camera. The iris is embedded with tiny muscles that widen and narrow the pupil size.



Drüsen

- Tears: are formed by tiny *glands* that surround the eye. The tear film is comprised of three layers: oil, water, and *mucous*. The mucous layer serves as *anchor* for the tear film and helps it adhere to the eye. The middle layer is comprised of water. The upper oil layer seals the tear film and prevents evaporation

Anker, Haftschrift

- Tear production: The outermost oily layer is produced by glands which line the edge of the eyelids. The watery production is produced by a gland that lies just below the eyebrow. The mucous layer comes from microscopic goblet cells in the conjunctiva. The tear film travels to a *duct* at the inner edge of the eye and from there it drains directly into the nose. Each day we produce about 500 ml of tears.

Kanal, Leitung

Pupil:

- diameter: 1 mm to 8 mm
- delay time for contraction and relaxing: 0.25 s
- speed of change: several mm/s

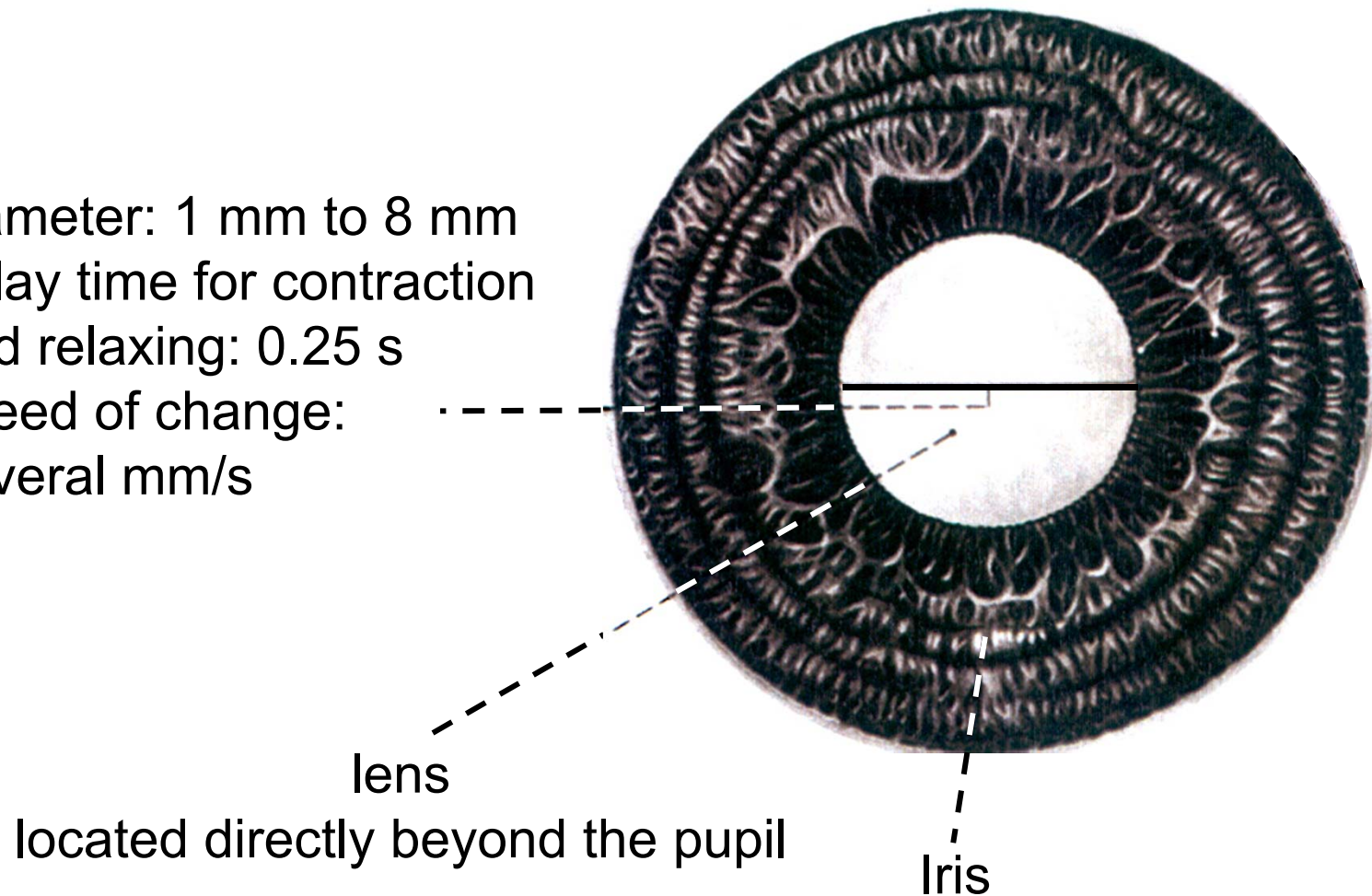


Figure 1.2: Iris and pupil

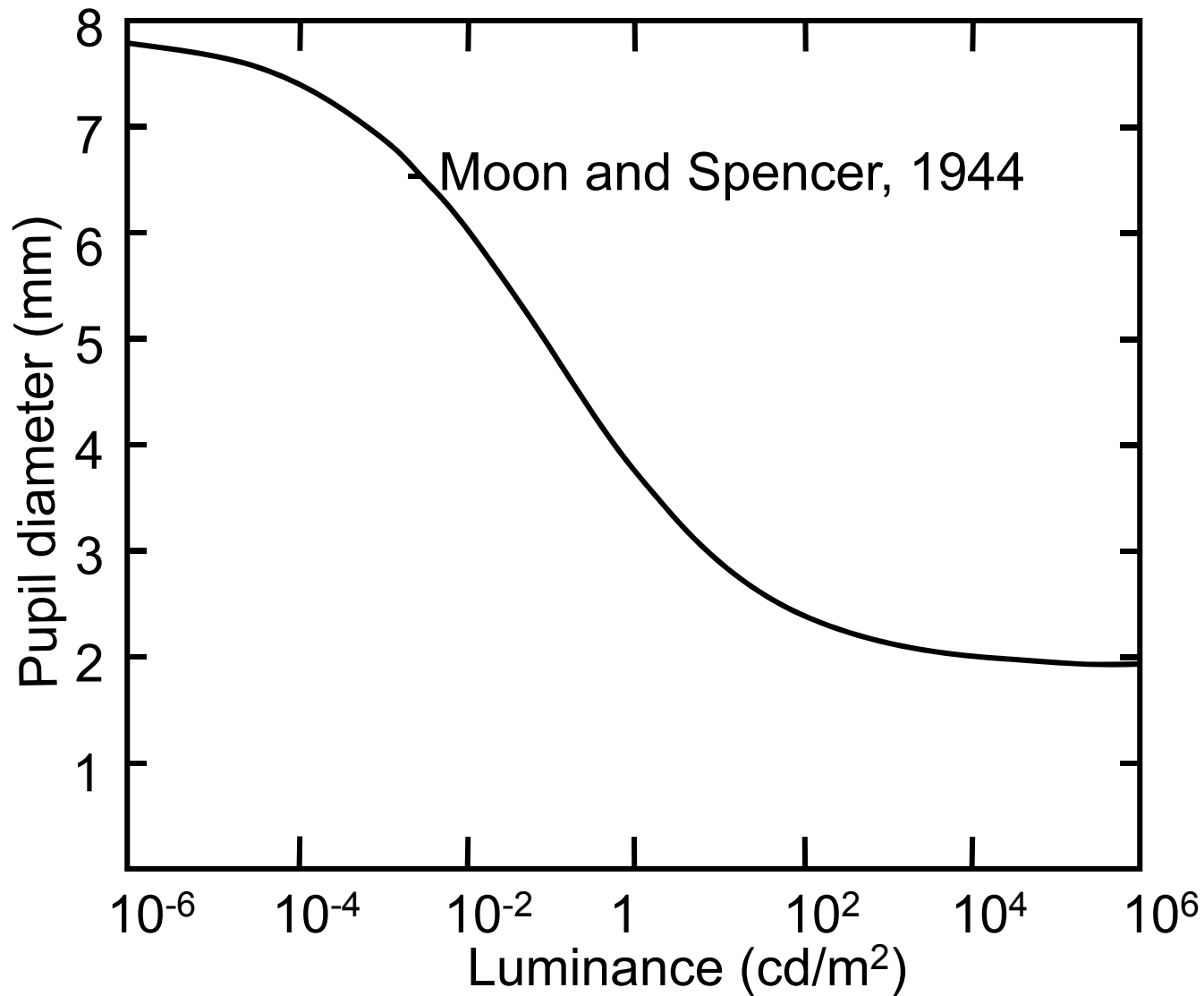


Figure 1.3: Pupil diameter depends on light intensity (luminance)

1.1.2 Inner structure of the human eye

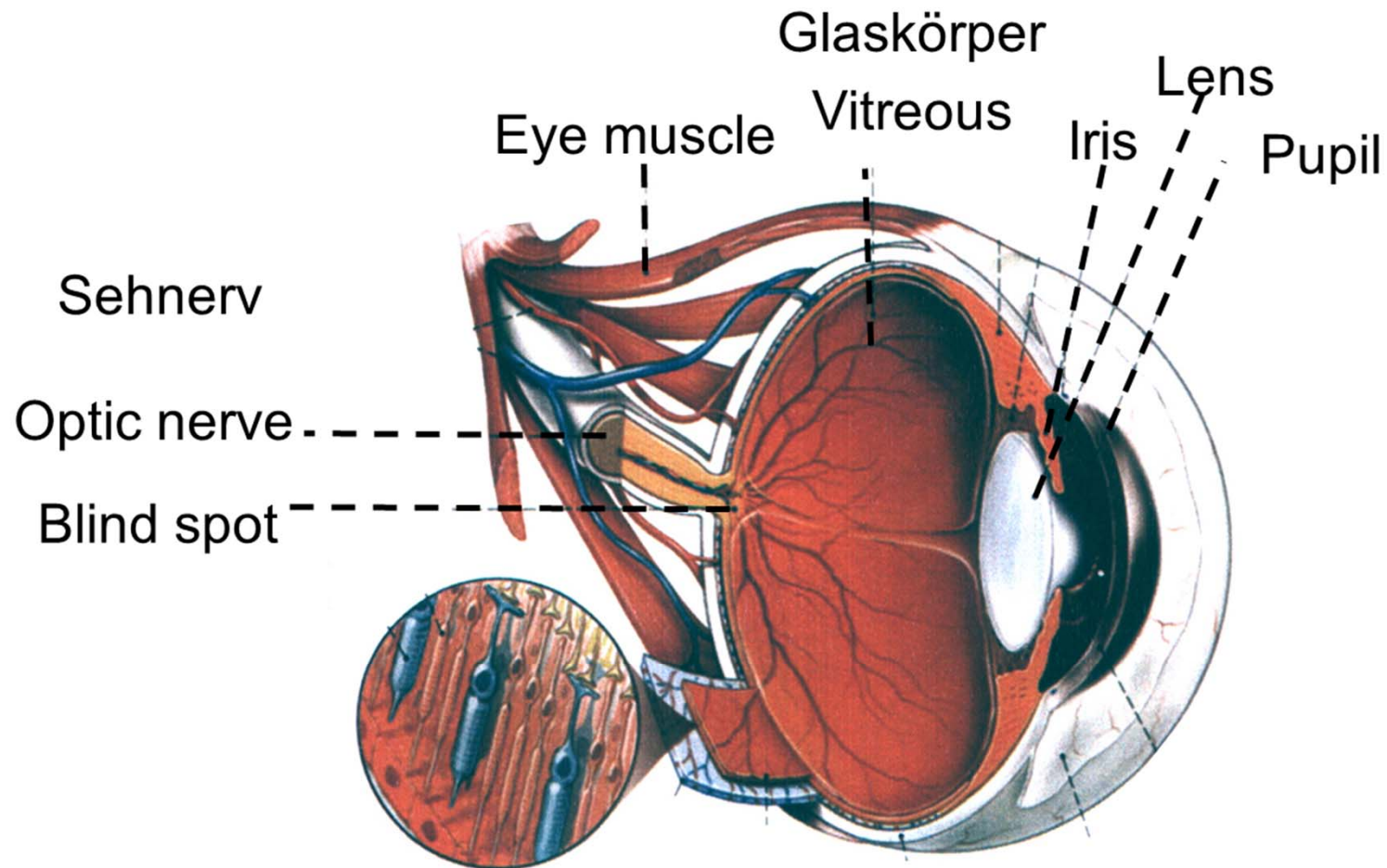


Figure 1.4: Cross section of the human eye

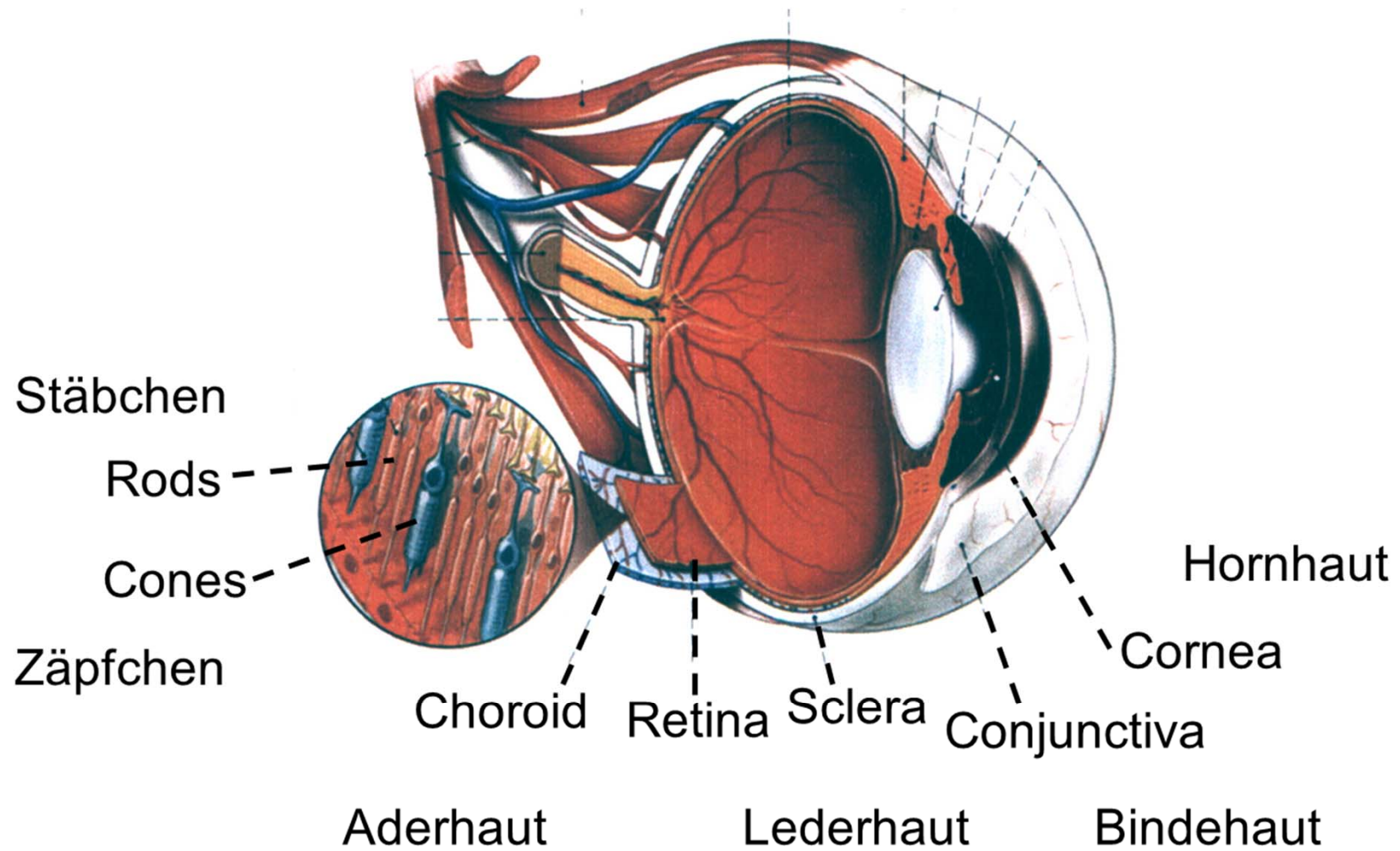


Figure 1.4: Cross section of the human eye

- Size: The human eye is approximately 22 mm deep and 24 mm wide.

Hornhaut

- *Cornea*: the transparent, dome-shaped window covering the front eye. It is a powerful refracting surface, providing 2/3 of the eye's focusing power. Index of refraction $n_r = 1.376$.

Lederhaut

- *Sclera*: commonly known as “the white of the eye,” is the tough, opaque tissue that serves as the eye's protective outer coat.

vordere Augenkammer

- *Anterior chamber*: contains the liquid aqueous humor. Index of refraction $n = 1.336$



Glaskörper

- *Vitreous*: The vitreous is a thick, transparent substance, called aqueous humor, that fills the center of the eye. It is mainly composed of water and comprises about 2/3 of the eye's volume, giving it form and shape.
- Lens: is located just behind the iris. Focuses light onto the retina. It has onion-like structure containing approximately 22,000 tiny layers, making the lens flexible. The flexibility results in a variable focal length. The flexibility of the lens decreases after age 40 and most noticeably after 45.

Aderhaut

Lederhaut

- *Choroid*: lies between retina and *sclera*. It is composed of layers of blood vessels that *nourish* the back of the eye.

versorgen



Netzhaut

- *Retina*: is a very thin layer of tissue of 120 microns in thickness, that lines the inner part of the eye. It is responsible for capturing the light rays that enter the eye. Much like the film's role in photography.

Sehnerv

- *Optic Nerve*: transmits electrical impulses from the retina to the brain. It connects to the back of the eye near to the macula. The visible portion of the optic nerve is called the optic disc. This portion of the retina is insensitive to light radiation and is therefore known as the

blinder Fleck

- *blind spot*.

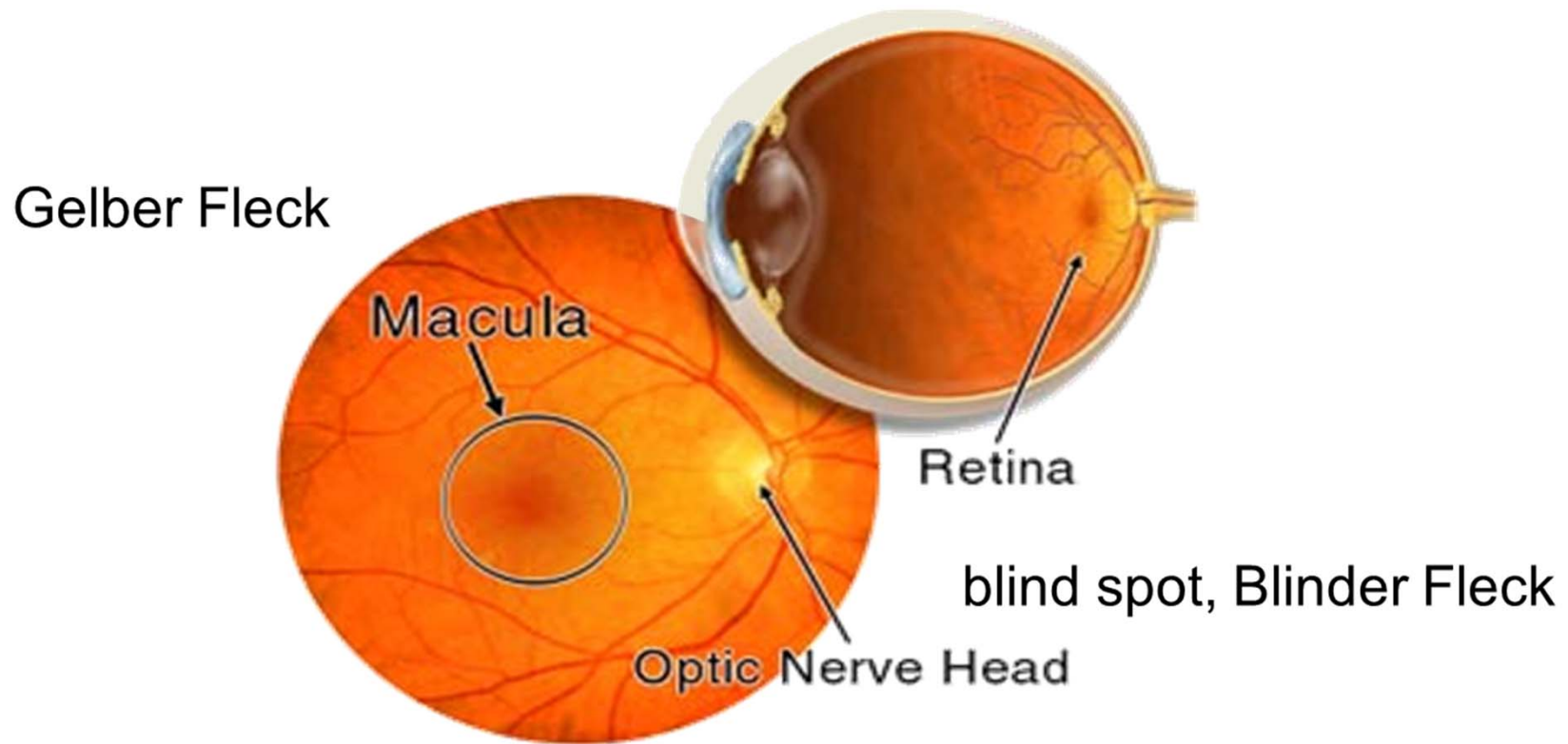


Figure 1.5: The Macula is the small, yellowish portion of the retina and is located roughly in its center, temporal to the optic nerve. It is a small and highly sensitive part of the retina responsible for detailed central vision.

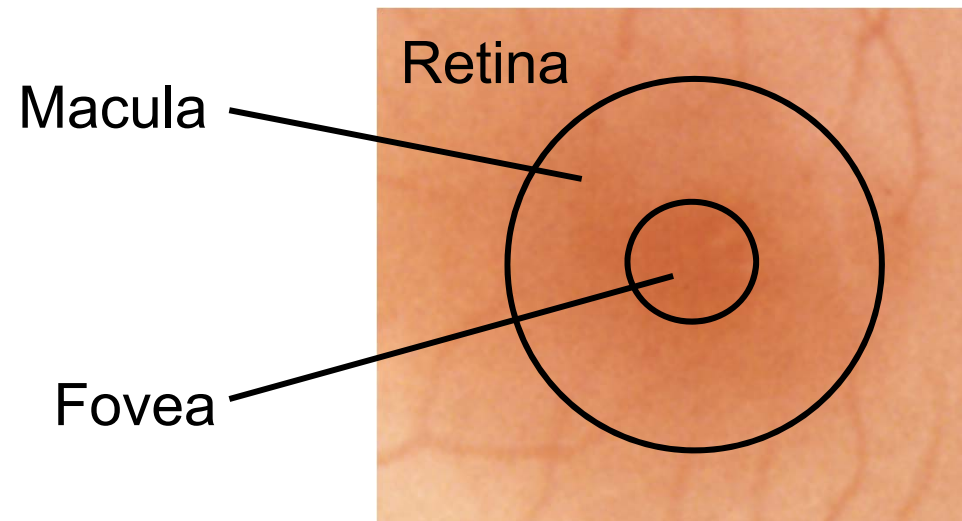


Figure 1.6: The fovea is the very center of the macula. The macula allows us to appreciate detail and perform tasks that require central vision such as reading.

1.2 Refracting power D of the human eye

Definition:

$$D = \frac{1}{f[m]}$$

; f = focal length, D = Refracting Power,
 D = Diopter

Examples [1.4]:

convex (focusing) lens:	$f = 1 \text{ m}$	$D = 1 \text{ D}$
concave (diverging) lens:	$f = -0.25 \text{ m}$	$D = -4 \text{ D}$

Cornea:	$D = 42 \text{ D}$
---------	--------------------

<u>Lens (relaxed, unaccommodated):</u>	<u>$D = 16 \text{ D}$</u>
--	--------------------------------------

Total:	$D = 58 \text{ D}$
--------	--------------------

Accommodated (young person): $+ 14 \text{ D}$ $D_{\max} = 72 \text{ D}$

1.3 Near point / far point

- Near point: upper limit of accommodation, nearest point which can be focused, distance a_N to the eye
- Far point: lower limit of accommodation, furthest point which can be focused, distance a_F to the eye (infinity, for not nearsighted people)

Width of accommodation: $A = D_N - D_F = a_N^{-1} - a_F^{-1}$

Table 1.1: age dependence of accommodation width

Age	a_N (cm)	a_F (cm)	$ A (\text{m}^{-1})$
10	8	∞	12.5
20	9	∞	11.1
30	12	∞	8.3
40	19	∞	5.3
50	50	∞	2.0
60	90	∞	1.1
80	100	∞	1.0

1.4 Focusing Problems

1.4.1 Astigmatism [1.5]

is a general inability of the eye to focus clearly at any distance because of uneven curvature of the cornea. Instead of being spherical, astigmatic corneas have more curvature in one meridian than in the other.

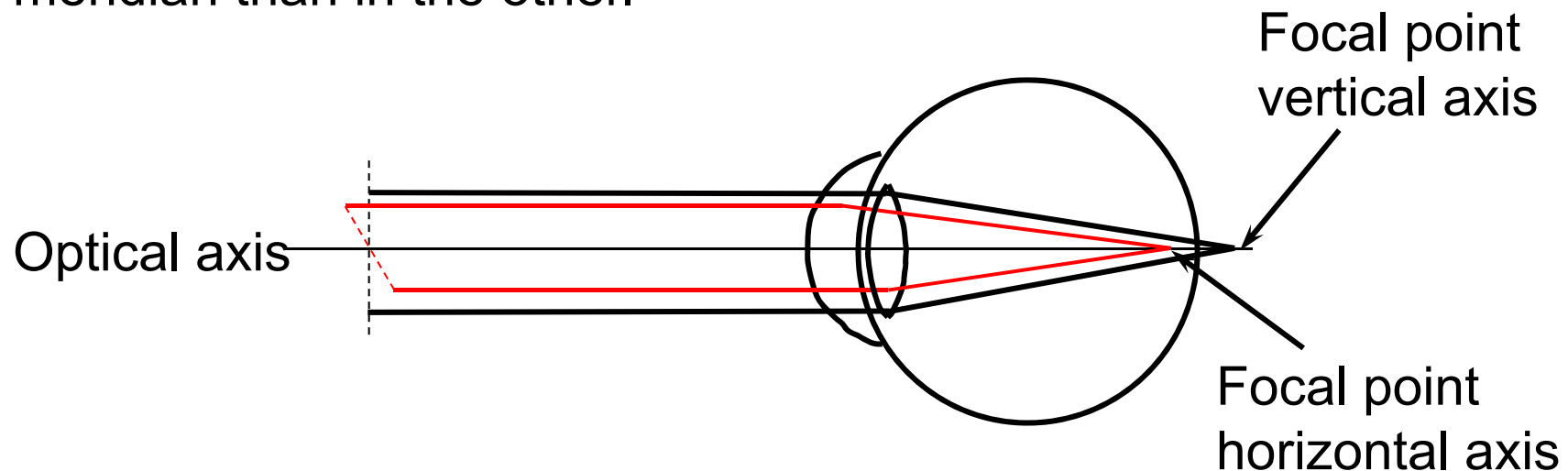


Figure 1.7: The astigmatic eye

1.4.2 Old age vision (*presbyopia*) [1.5] Alterssichtigkeit

Old age vision is a gradual loss of the ability to focus on near objects, generally if the near point is beyond 30 cm.

1.4.3 Farsightedness (*hyperopia*) [1.5] Weitsichtigkeit

is a focusing problem caused by an eyeball that is shorter than normal. Because of the reduced length, the cornea and lens do not have enough space in which to bring the light rays to a focal point before they reach the retina.

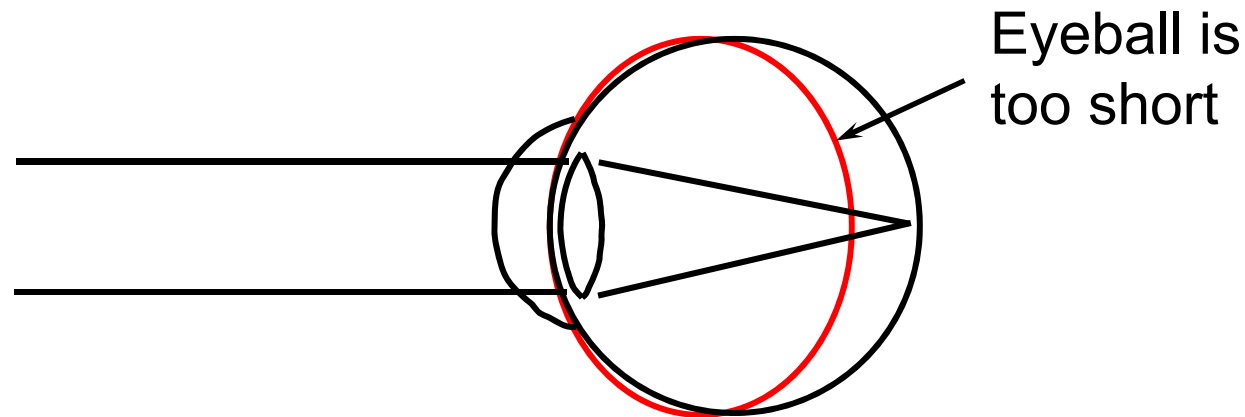


Figure 1.8: The hyperopic (farsighted) eye

This effect varies with age because of the diminishing flexibility of the lens. Young people may not notice any effects. But as the eyes begin to age, near objects become increasingly difficult to see.

1.4.4 Nearsightedness (*myopia*) [1.5] Kurzsichtigkeit

is the *most common focusing problem*, affecting 25% of the population. It results from an eyeball that is too long. Because of the extra length, light coming from distant objects converge to a focal point before it reaches the retina. Beyond this point the light begins to diverge and a blurred image is detected.

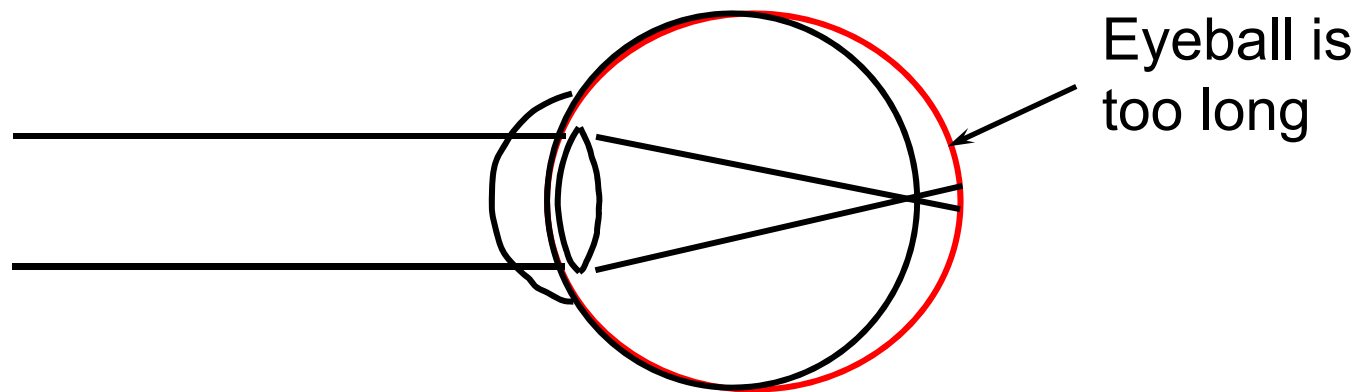


Figure 1.9: The myopic (nearsighted) eye

1.5 Rods, cones

Stäbchen, Zäpfchen

Rods and cones are the light sensitive detectors (photoreceptors) within the eye [1.5]. They are part of the retina, which contains

1.3×10^8 rods ($\varnothing = 2 \mu\text{m}$) and
 7×10^6 cones ($\varnothing = 4 \mu\text{m}$).

rods,
cones

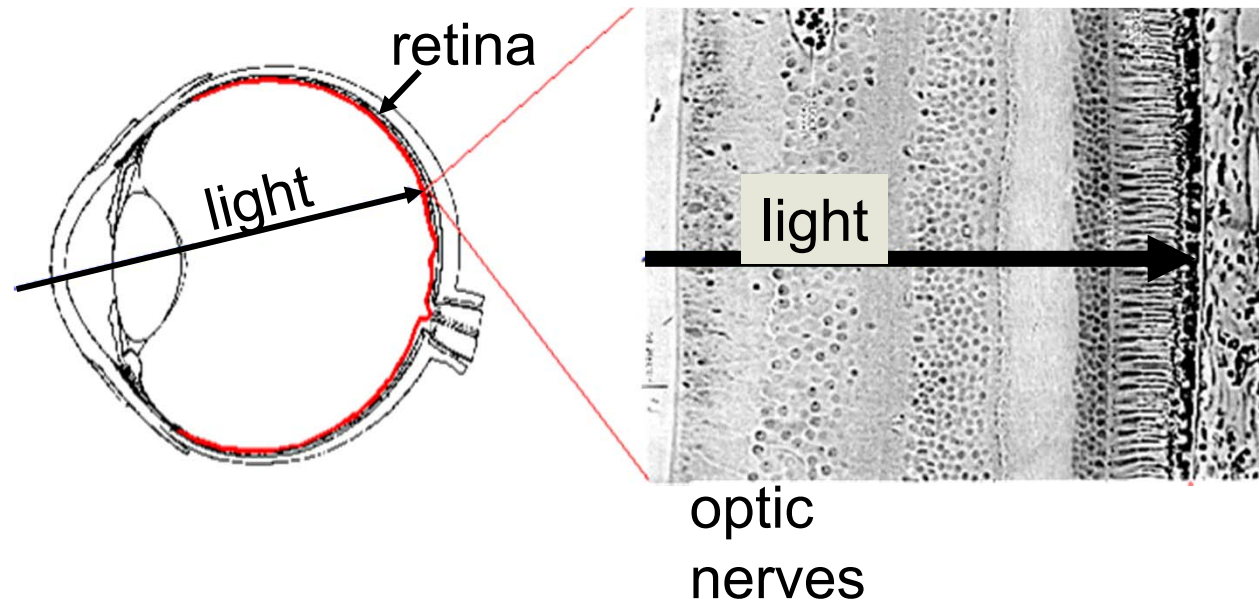


Figure 1.10: Location of rods and cones inside the retina

Table 1.2: Properties of rods and cones

cones	rods
most are located in the fovea	all are located outside the fovea
color vision during daylight conditions	night vision, only different gray levels, no colors are detected
low field of vision and high <i>visual acuity</i>	high field of vision and low <i>visual acuity</i> Sehschärfe

Table 1.2: Properties of rods and cones

cones	rods
low sensitivity $K_m = 683 \text{ lm/W}$	high sensitivity $K'_m = 1699 \text{ lm/W}$
maximum sensitivity $V(\lambda_{max}) = 1; \lambda_{max} = 555 \text{ nm}$	$V'(\lambda_{max}) = 1; \lambda_{max} = 507 \text{ nm}$

1.6 Color vision

To see any color, the retinal cone cells must be stimulated by light.

“Red sensitive” cones are mostly stimulated by light in the red to yellow range, “green sensitive” in the yellow to green range and “blue sensitive” cones in the blue to violet range.

According to their sensitivities to long (L), medium (M), and short (S) wavelengths the three different types of cones are referred to as “L”-cones, “M”-cones and “S”-cones.

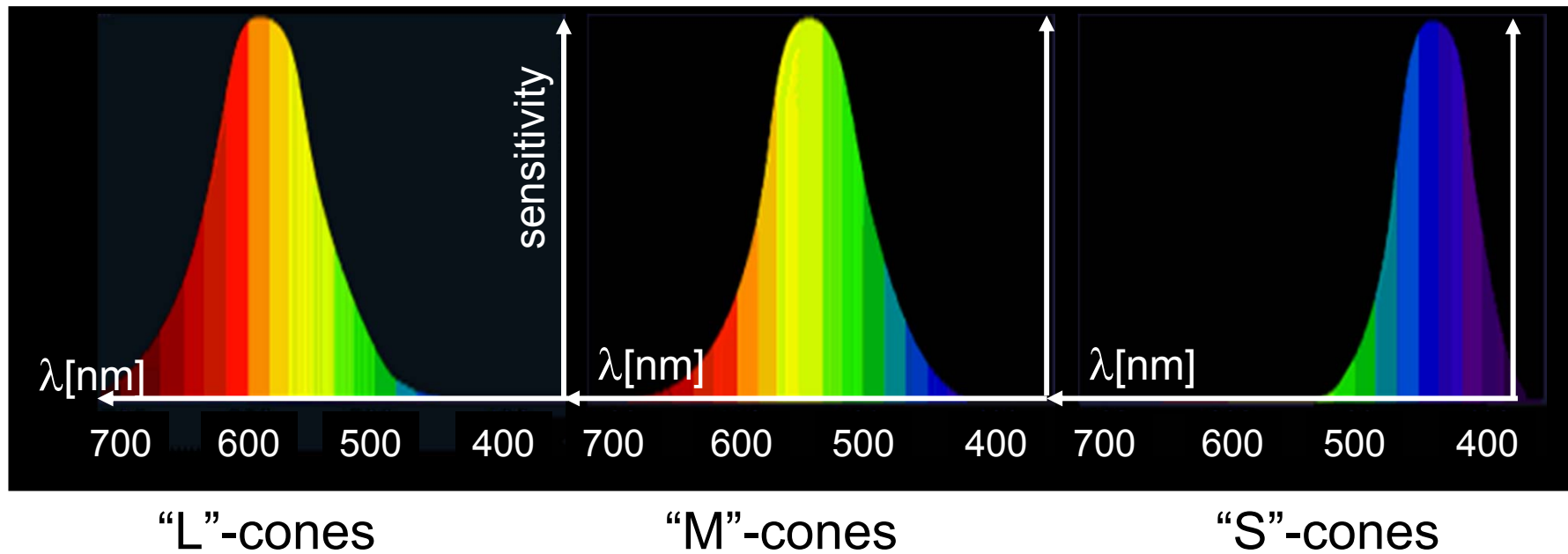


Figure 1.11: Spectral sensitivity of the three different types of cones

1.7 Color deficiency

Red-green deficiency is by far the most common form and it results in the inability to distinguish certain shades of red and green (case 1, and case 2). Those with a less common type have difficulty distinguishing blue and yellow (case 3). In very rare cases, color deficiency exists to an extent that no colors can be detected (colorblindness), only shades of black, white and gray [1.7].

If one type of cone receptors does not work at all, a *dichromatism* results:

1. L-cones do not work: protanopia (difficult to distinguish between blue/green and red/green)
2. M-cones do not work: deuteranopia (difficult to distinguish between red/purple and green/purple)
3. S-cones do not work: tritanopia (difficult to distinguish between yellow/green and blue/green)

1.8 Internet Links

[1.1]: <http://www.auge-online.de/>

Dr. Werdermann, Ochsenfurt: *Informationen rund um das Auge.*

[1.2]: <http://www.orlions.org/eyetour/eyetourhome.html>

Oregon Lions Sight & Hering Foundation: *Transparencies from "The Human Eye in Anatomical Transparencies"* published by Bausch & Lomb Press, 1943.

[1.3]: <http://www.stlukeseye.com/anatomy/Cornea.asp>

St. Luke's Cataract & Laser Institute: *Anatomy of the human eye.*

[1.4]: <http://retina.anatomy.upenn.edu/~lance/eye/lens.html>

University of Pennsylvania Medical Center: *The Physical Structures of the Eye*

[1.4]: <http://www.gimbeleyecentre.com/refracti/focusing.htm>

Gimbel Eye Centre; Calgary, Edmonton; Alberta: *Focusing problems and their origin.*

[1.5]: <http://hyperphysics.phy-astr.gsu.edu/hbase/vision/rodcone.html>

Hyperphysics, © 2000 C. R. Nave, Georgia State University: *Light and Vision.*

[1.6]: <http://www.toledo-bend.com/colorblind/aboutCB.html>

Terrace L. Waggoner, O.D., Staff Naval Hospital: *About Color Blindness (Color Vision Deficiency).*

2. Light and Color

Light = *Visible* electromagnetic radiation
of wavelength $380 \text{ nm} < \lambda < 780 \text{ nm}$.

Ultraviolet radiation (UV): $\lambda < 380 \text{ nm}$
Infrared radiation (IR): $\lambda > 780 \text{ nm}$

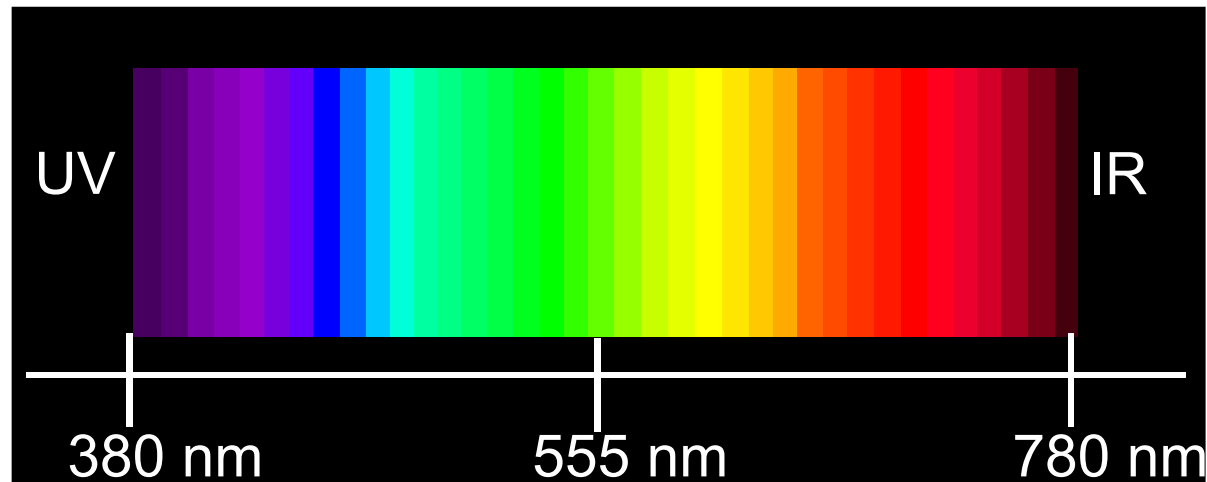


Figure 2.1: Human color vision

2.1 Light sensitivity of the human eye

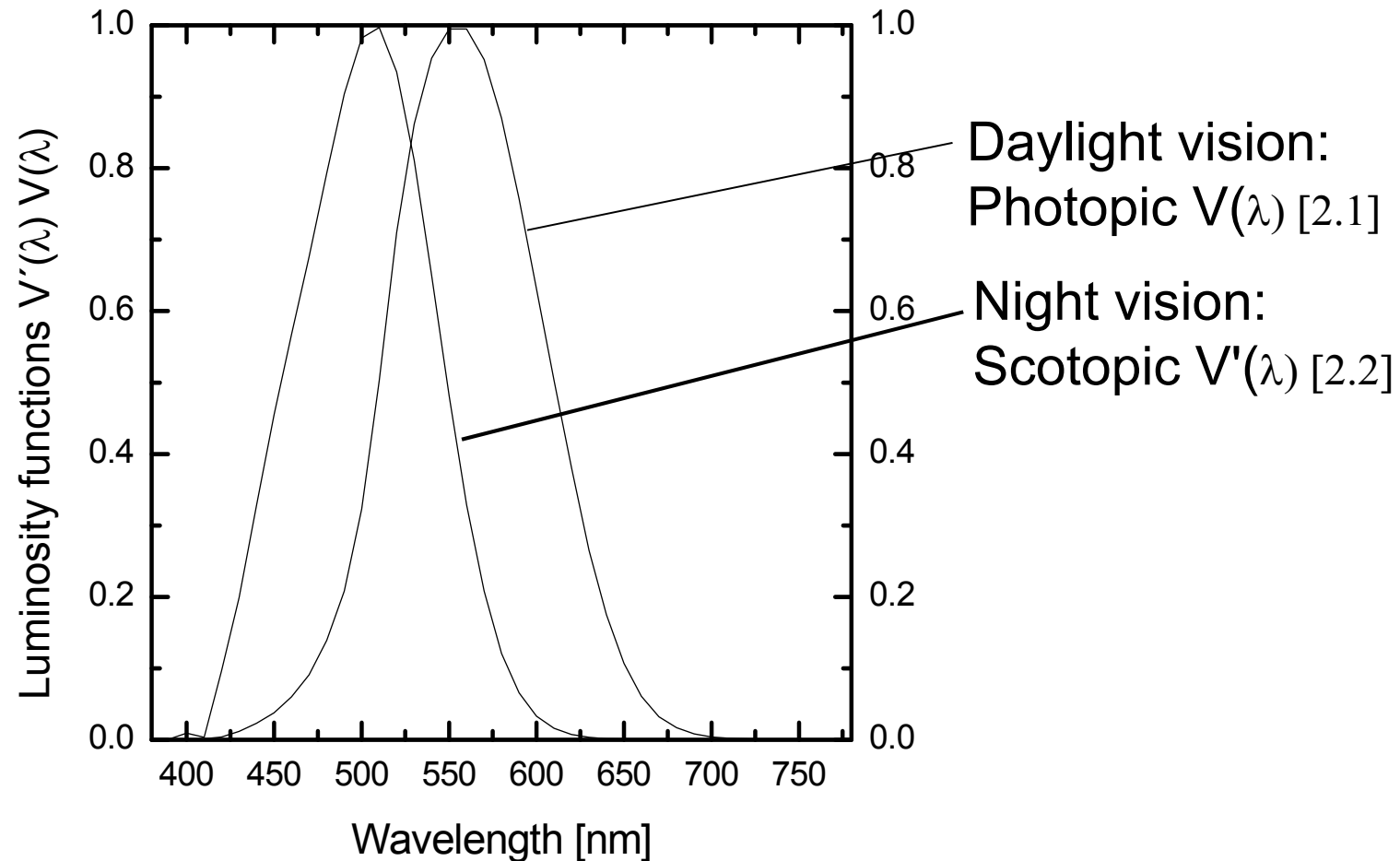


Figure 2.2: The sensitivity of the eye determines the luminosity functions.

Sensitivity of eyes are slightly different between different humans. Therefore luminosity functions were deduced from averaged sensitivities of a large number of persons.

Maximum sensitivity at wavelength $\lambda = 555 \text{ nm}$

Normbeobachter

Scientific term: *average eye* or *standard observer*

- (Photopic) function $V(\lambda)$ relates to *daylight vision*: Cones are the relevant photoreceptors [2.1].
- (Scotopic) function $V'(\lambda)$ relates to *night vision*: Rods are the relevant photoreceptors [2.2].

- Color vision only possible under daylight conditions, luminance $L_v > 30 \text{ cd/m}^2$.
- L-, M-, S-cones enable color vision.

Purkinje-effect:

Equal irradiation intensities of blue light ($\lambda = 480 \text{ nm}$) and yellow light ($\lambda = 580 \text{ nm}$) result in:

- Brighter blue under low intensities (night vision)
- Brighter yellow under high intensities (daylight vision)

2.2 CIE Color matching functions

Normspektralwerte

Commission Internationale de l'Éclairage [2.3]

Colorimetry does not use the spectral sensitivity functions of the three different types of cones [2.4]. Instead of these functions, the so called *CIE color matching functions* [2.5] are commonly used, which are deduced via a linear transformation from those.

Advantage: The CIE color matching functions (CMF) are always positive and all colors can be represented by a linear combination of them.

$\bar{x} \hat{=}$ red – CMF

Rotwert

$\bar{y} \hat{=}$ green – CMF

Grünwert

$\bar{z} \hat{=}$ blue – CMF

Blauwert



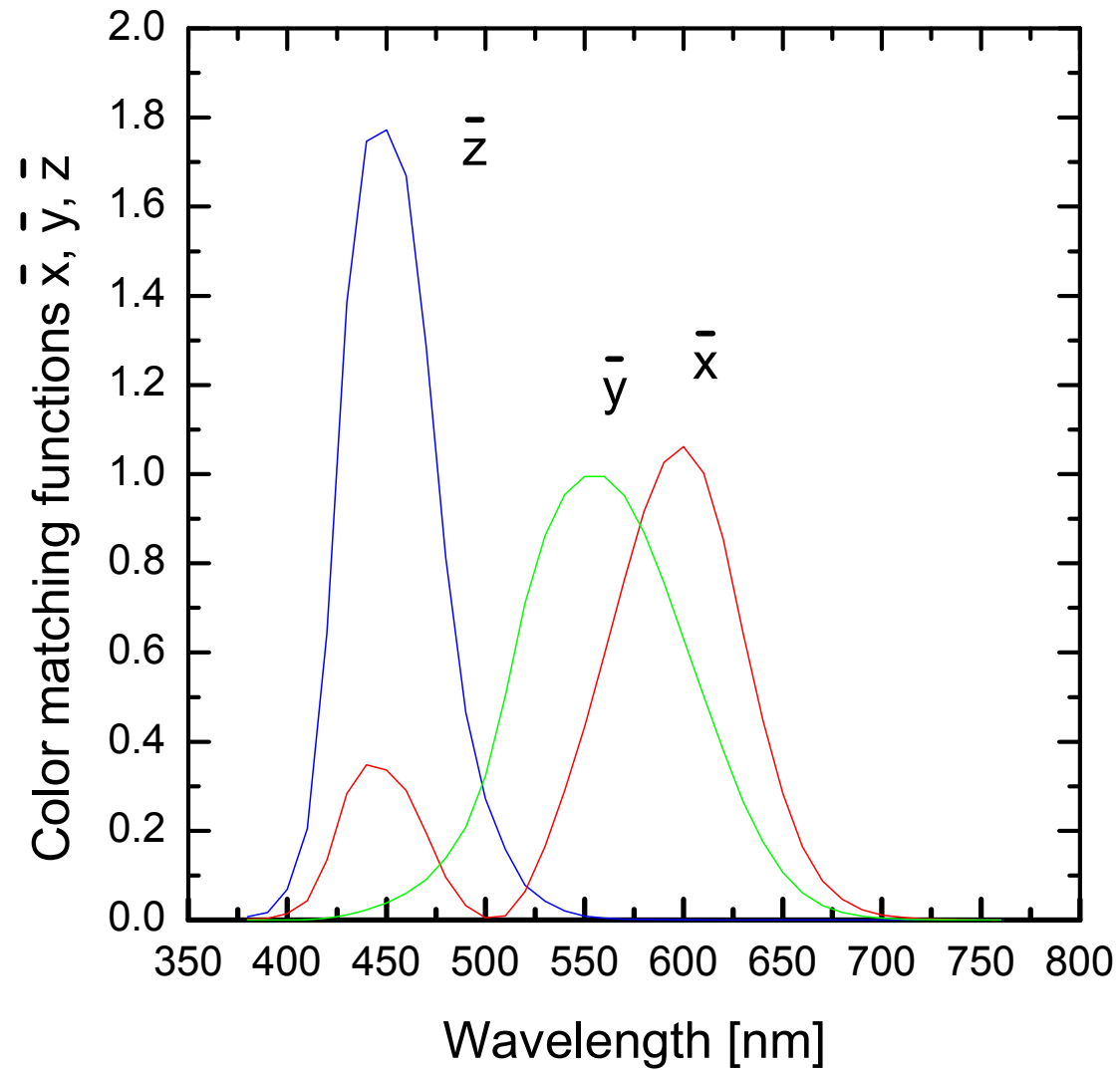


Figure 2.3: CIE color matching functions

©JHW



Function $\bar{y}(\lambda)$ is identical to the photopic $V(\lambda)$ function

$$V(\lambda) = \bar{y}(\lambda)$$

and, by definition

$$V_{\max}(\lambda) = \bar{y}_{\max}(\lambda) = 1.$$

Functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ are tabulated by 40 values, each representing a wavelength interval $\Delta\lambda$ of 10 nm, standardized according to

$$\sum \bar{x}(\lambda_i) = \sum \bar{y}(\lambda_i) = \sum \bar{z}(\lambda_i).$$

Table 2.2: Tabulated CIE color matching functions

Wavelength [nm]	$\bar{x}$	$\bar{y} = V(\lambda)$	$\bar{z}$
380	1,368000000E-03	3,900000000E-05	6,450001000E-03
390	4,243000000E-03	1,200000000E-04	2,005001000E-02
400	1,431000000E-02	3,960000000E-04	6,785001000E-02
410	4,351000000E-02	1,210000000E-03	2,074000000E-01
420	1,343800000E-01	4,000000000E-03	6,456000000E-01
430	2,839000000E-01	1,160000000E-02	1,385600000E+00
440	3,482800000E-01	2,300000000E-02	1,747060000E+00
450	3,362000000E-01	3,800000000E-02	1,772110000E+00
460	2,908000000E-01	6,000000000E-02	1,669200000E+00
470	1,953600000E-01	9,098000000E-02	1,287640000E+00
480	9,564000000E-02	1,390200000E-01	8,129501000E-01
490	3,201000000E-02	2,080200000E-01	4,651800000E-01
500	4,900000000E-03	3,230000000E-01	2,720000000E-01
510	9,300000000E-03	5,030000000E-01	1,582000000E-01
520	6,327000000E-02	7,100000000E-01	7,824999000E-02
530	1,655000000E-01	8,620000000E-01	4,216000000E-02
540	2,904000000E-01	9,540000000E-01	2,030000000E-02
550	4,334499000E-01	9,949501000E-01	8,749999000E-03
560	5,945000000E-01	9,950000000E-01	3,900000000E-03

Table 2.2: Tabulated CIE color matching functions

Wavelength [nm]	$\bar{x}$	$\bar{y} = V(\lambda)$	$\bar{z}$
570	7,621000000E-01	9,520000000E-01	2,100000000E-03
580	9,163000000E-01	8,700000000E-01	1,650001000E-03
590	1,026300000E+00	7,570000000E-01	1,100000000E-03
600	1,062200000E+00	6,310000000E-01	8,000000000E-04
610	1,002600000E+00	5,030000000E-01	3,400000000E-04
620	8,544499000E-01	3,810000000E-01	1,900000000E-04
630	6,424000000E-01	2,650000000E-01	4,999990000E-05
640	4,479000000E-01	1,750000000E-01	2,000000000E-05
650	2,835000000E-01	1,070000000E-01	0,000000000E+00
660	1,649000000E-01	6,100000000E-02	0,000000000E+00
670	8,740000000E-02	3,200000000E-02	0,000000000E+00
680	4,677000000E-02	1,700000000E-02	0,000000000E+00
690	2,270000000E-02	8,210000000E-03	0,000000000E+00
700	1,135916000E-02	4,102000000E-03	0,000000000E+00
710	5,790346000E-03	2,091000000E-03	0,000000000E+00
720	2,899327000E-03	1,047000000E-03	0,000000000E+00
730	1,439971000E-03	5,200000000E-04	0,000000000E+00
740	6,900786000E-04	2,492000000E-04	0,000000000E+00
750	3,323011000E-04	1,200000000E-04	0,000000000E+00
760	1,661505000E-04	6,000000000E-05	0,000000000E+00
770	8,307527000E-05	3,000000000E-05	0,000000000E+00
780	4,150994000E-05	1,499000000E-05	0,000000000E+00

2.3 Color coordinates X, Y, Z and color chromaticity coordinates x, y, z

2.3.1 Color coordinates X, Y, Z Normfarbwerte

Light with a spectral distribution $L(\lambda)$ enters the eye. The L-, M-, and S-cones in the retina convert the incoming light to signals X, Y, Z , by weighting $L(\lambda)$ with the CMFs [2.4, 2.5] according to

$$X = \sum L_{\lambda}(\lambda) \bar{x}(\lambda)$$

Red-value

$$Y = \sum L_{\lambda}(\lambda) \bar{y}(\lambda)$$

Green-value

$$Z = \sum L_{\lambda}(\lambda) \bar{z}(\lambda)$$

Blue-value

2.1

It is not possible to reconstruct the original distribution of $L(\lambda)$ from the values X , Y , Z , because equations 2.1 result in a loss of information.

This means that different distributions of $L(\lambda)$ can result in the same values of X , Y , Z , with the consequence that they are recognized by the eye as the same colors.

2.3.2 Color chromaticity coordinates x, y, z

Normfarbwert-
anteile

The sum S of the signals or color coordinates X, Y, Z

$$S = X + Y + Z$$

contains

$$x = X / S$$

relative parts of red color,

$$y = Y / S$$

relative parts of green color,

$$z = Z / S$$

relative parts of blue color,

with

$$x + y + z = 1.$$

Therefore it is sufficient to characterize each color by its chromaticity coordinates x and y , because $z = 1 - x - y$.

The normalization to x, y, z results

Helligkeit

- in loss of intensity information (*luminosity*) under which the color appears to the eye (disadvantage),
- but enables representation of each color as a vector with only two components, which can be displayed as a chromaticity point or dot in a two-dimensional plot (advantage) [2.6].

The luminosity of the color is represented by the color coordinate Y , which is by definition its luminosity

$$V(\lambda) = \bar{y}(\lambda)$$



$$Y = \sum L_{\lambda}(\lambda) \bar{y}(\lambda) = \sum L_{\lambda}(\lambda) V(\lambda)$$

The unambiguous characterization of a color requires to give

x, y and Y .



2.4 Chromaticity diagram

Normfarbtafel

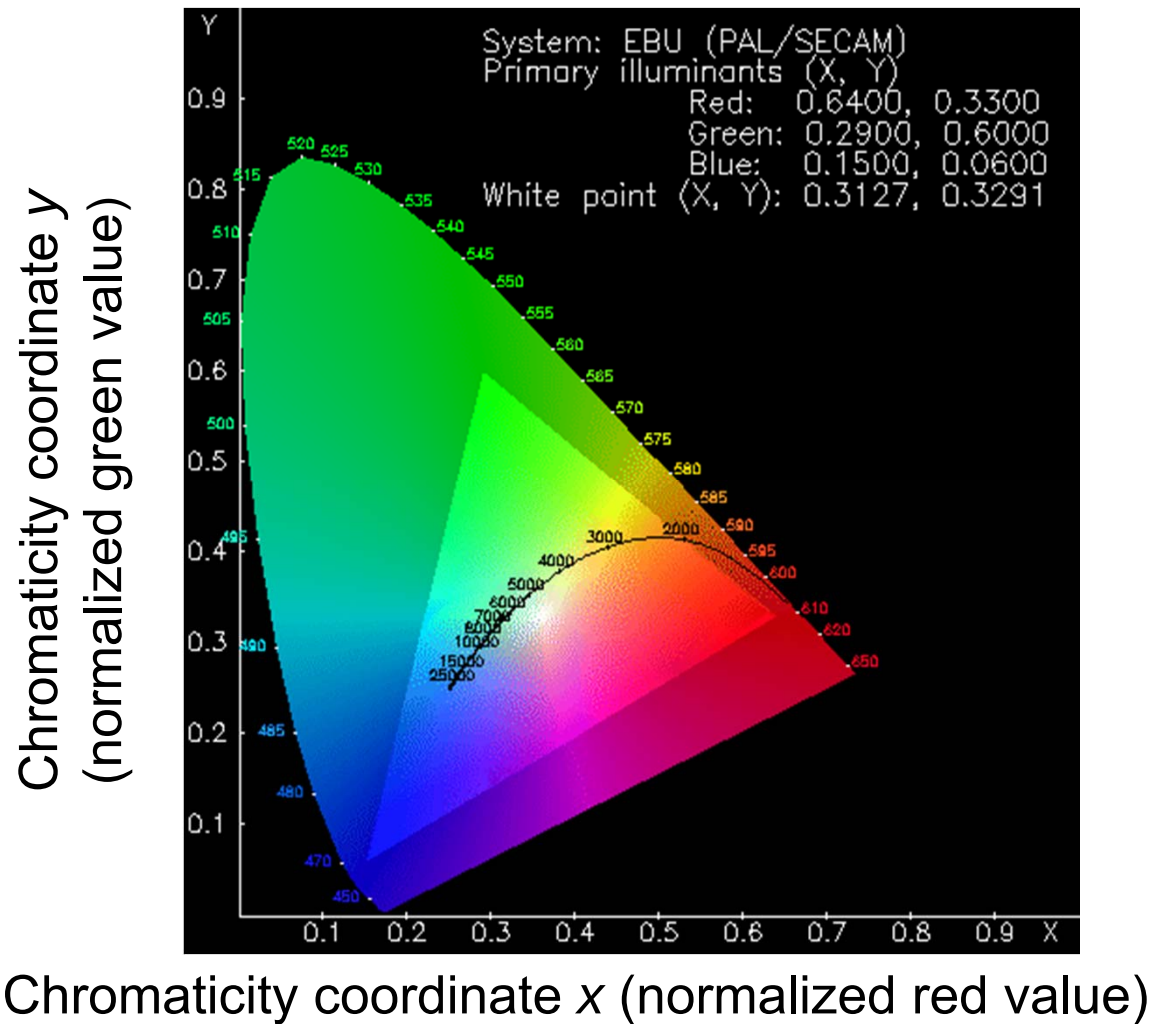


Figure 2.4: Chromaticity diagram

©JHW



By definition it holds

$$x + y + z = 1,$$

with $x, y, z \geq 0$.



$$x + y \leq 1$$



All colors are left from the diagonal, straight line $x + y = 1$ which connects $(1,0)$ with $(0,1)$.

2.4.1 Monochromatic light

= light with specific wavelength

Here $L_{\lambda}(\lambda) = \delta(\lambda - \lambda_0)$

We calculate the color coordinates X_{spec} , Y_{spec} , Z_{spec} of monochromatic colors as

$$X_{\text{spec}} = \sum \delta(\lambda - \lambda_0) \bar{x}(\lambda) = \bar{x}(\lambda_0)$$

$$Y_{\text{spec}} = \sum \delta(\lambda - \lambda_0) \bar{y}(\lambda) = \bar{y}(\lambda_0)$$

$$Z_{\text{spec}} = \sum \delta(\lambda - \lambda_0) \bar{z}(\lambda) = \bar{z}(\lambda_0)$$

And their chromaticity coordinates as

$$x_{\text{spec}} = \frac{X}{X + Y + Z} = \frac{\bar{x}(\lambda_0)}{\bar{x}(\lambda_0) + \bar{y}(\lambda_0) + \bar{z}(\lambda_0)}$$

$$y_{\text{spec}} = \frac{Y}{X + Y + Z} = \frac{\bar{y}(\lambda_0)}{\bar{x}(\lambda_0) + \bar{y}(\lambda_0) + \bar{z}(\lambda_0)}$$

where $x_{\text{spec}} + y_{\text{spec}} + z_{\text{spec}} = 1$.

- Each point on the border of the chromaticity diagram represents unambiguously one wavelength.
- Each point inside the diagram represents a whole spectral distribution of wavelengths.

2.4.2 Purple line Purpurgerade

The chromaticity curve of monochromatic colors between 380 nm and 780 nm has permanently positive (to the right side) curvature. The end points at 380 nm and 780 nm are the curve's general limits as a consequence of the luminosity function $V(\lambda)$.

The interconnection between the chromaticity points of 380 nm and 780 nm is called the *purple line*.

The purple line together with the chromaticity curve of monochromatic light represent a closed curve which encloses all possible colors.

The color coordinates x , y for any color on the purple line are

$$(x, y) = A (x_{380}, y_{380}) + B (x_{780}, y_{780})$$

A and B are the intensities (luminosities) of the mixed monochromatic colors at 380 nm and 780 nm.

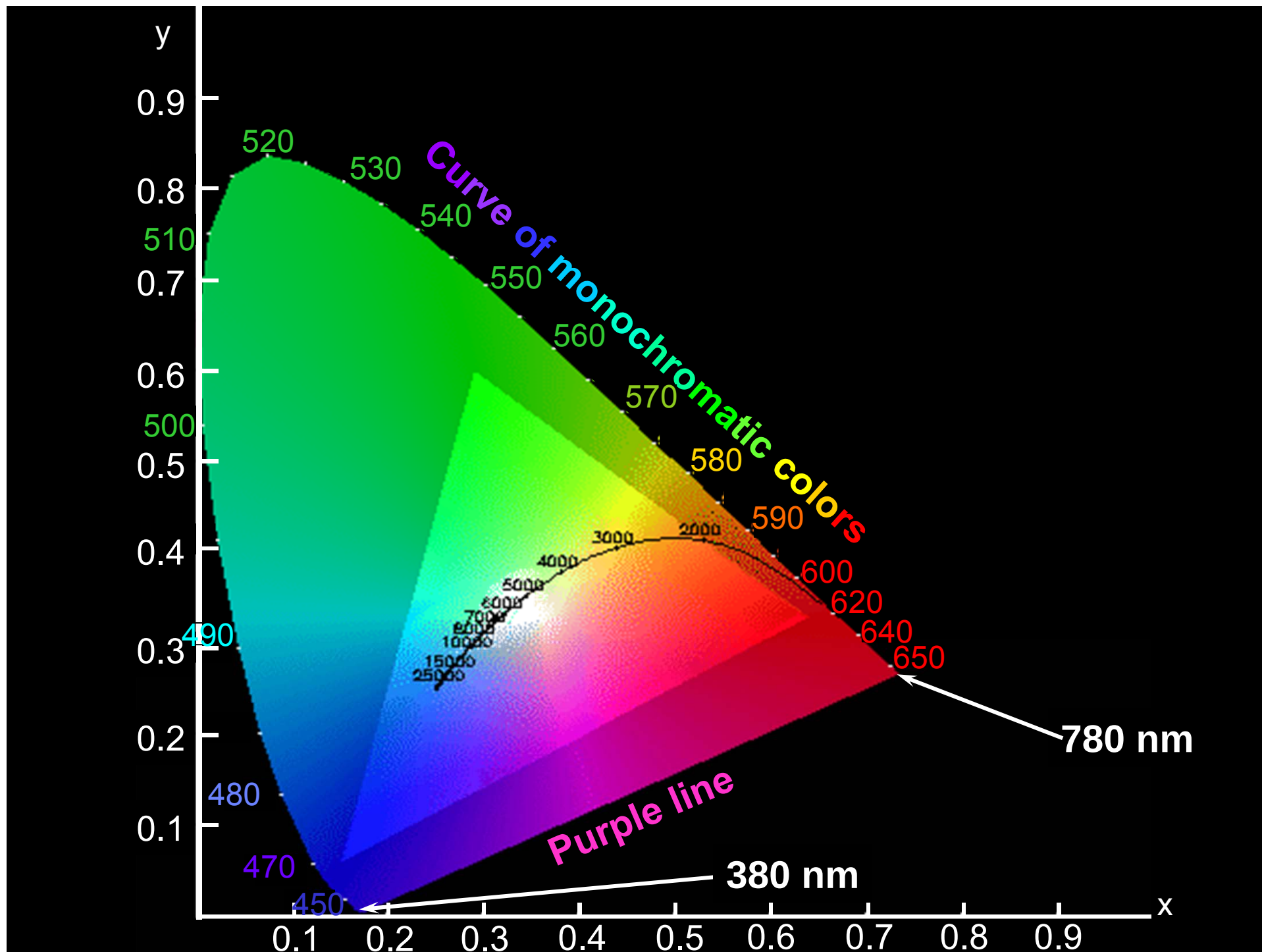


Table 2.3: Tabulated CIE chromaticity coordinates of monochromatic colors

Wavelength (nm)	x_{Spec}	y_{Spec}	Wavelength (nm)	x_{Spec}	y_{Spec}
380	0,17473118	0,00537634	590	0,57515323	0,42423251
390	0,17367169	0,00475813	600	0,62703958	0,37248908
400	0,17331478	0,00479283	610	0,66576078	0,33401209
410	0,1725607	0,00480989	620	0,69148926	0,30835471
420	0,17143146	0,00510874	630	0,70791276	0,29203132
430	0,16887806	0,00690245	640	0,71901982	0,28093964
440	0,16441652	0,0108568	650	0,72599959	0,27400041
450	0,15664045	0,01770422	660	0,72943988	0,27056012
460	0,14396042	0,02970161	670	0,7320142	0,2679858
470	0,12414176	0,05780232	680	0,73333333	0,26666667
480	0,09128887	0,13268175	690	0,73454367	0,26545633
490	0,0453957	0,29494808	700	0,73473282	0,26526718
500	0,00816386	0,5384281	710	0,73465704	0,26534296
510	0,01387467	0,75018275	720	0,73478261	0,26521739
520	0,07430784	0,83379911	730	0,73643411	0,26356589
530	0,15472325	0,8058598	740	0,73134328	0,26865672
540	0,22962038	0,75433204	750	0,74074074	0,25925926
550	0,30156822	0,69234413	760	0,74074074	0,25925926
560	0,3731082	0,62444456	770	0,74074074	0,25925926
570	0,44419158	0,55458741	780	0,74074074	0,25925926
580	0,51248419	0,48659037			

2.5 Color Mixing

Two colors A and B are mixed with different luminosities resulting in a mixed color C.

The mixed color C is located on the interconnecting line between the chromaticity points of colors A and B in the chromaticity diagram.

a) Two color mixing

$$(x_C, y_C) = A(x_A, y_A) + B(x_B, y_B)$$

From equation 2.1 it follows:

$$A = S_A / (S_A + S_B), B = S_B / (S_A + S_B),$$

with $S_A = X_A + Y_A + Z_A$,

$$S_B = X_B + Y_B + Z_B$$

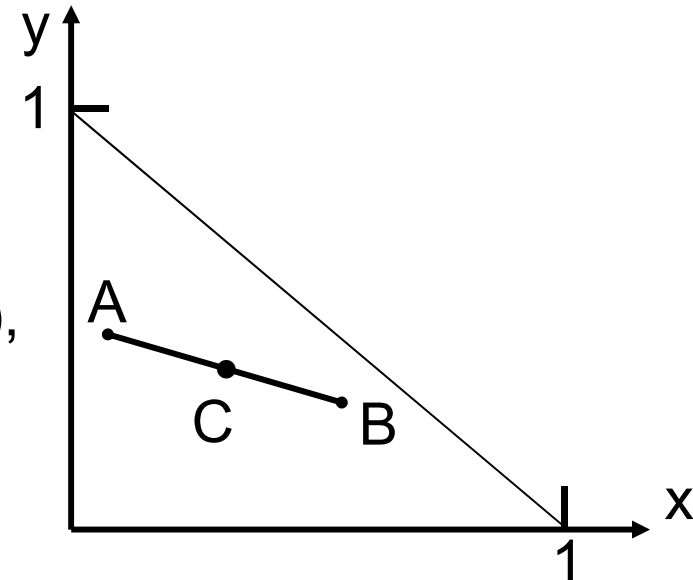


Figure 2.5: Mixing of Color C using colors A and B

b) Three color mixing [2.7]

Three colors A, B and C are mixed resulting in a mixed color D.

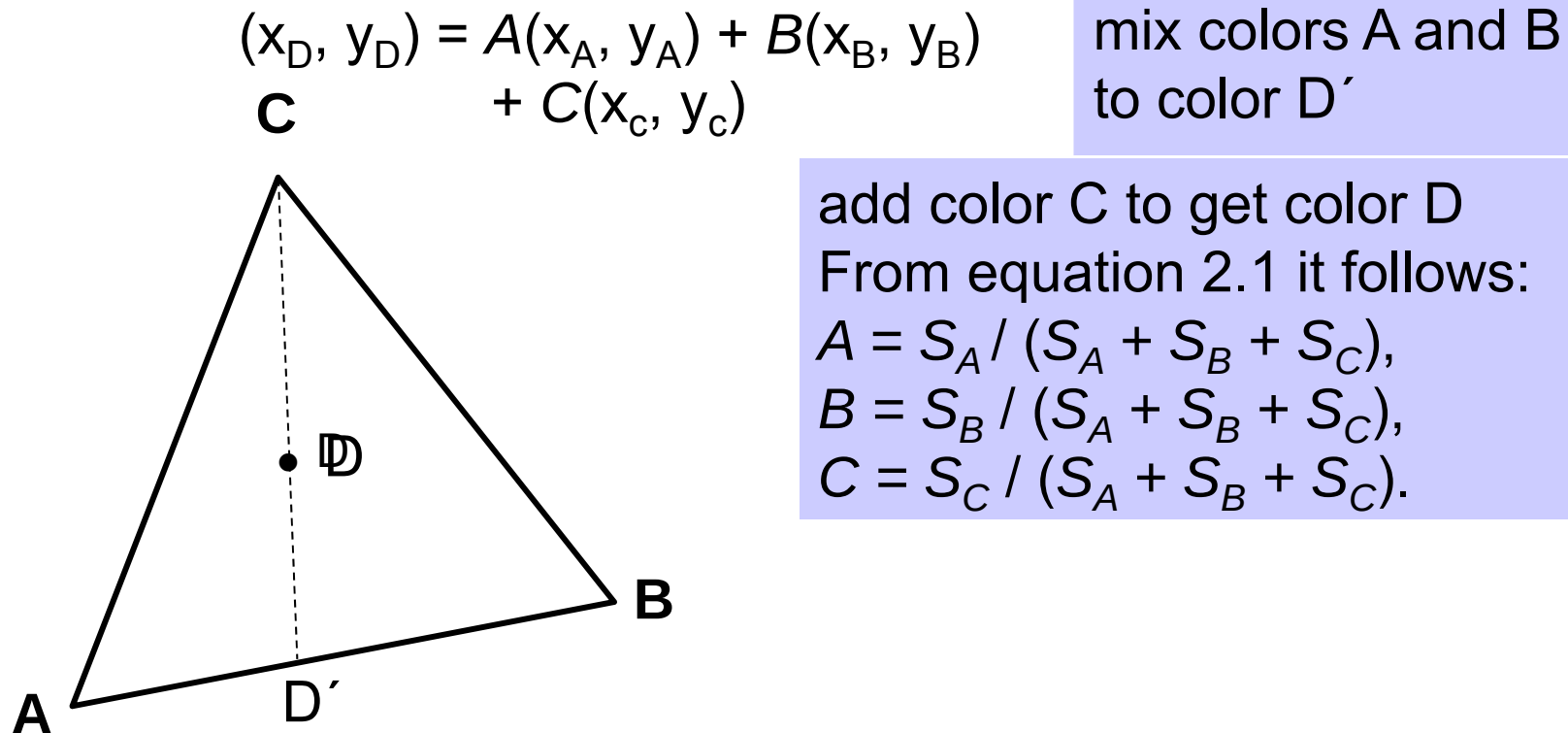


Figure 2.6: Mixing of Color D
using colors A, B and C

Every color within the black framed triangle in Fig. 2.7 is generated by mixing the monochromatic colors

Red $\lambda = 620 \text{ nm}$

Green $\lambda = 520 \text{ nm}$

Blue $\lambda = 470 \text{ nm}$

with luminosities R, G, B .

$$R + G + B = 1$$

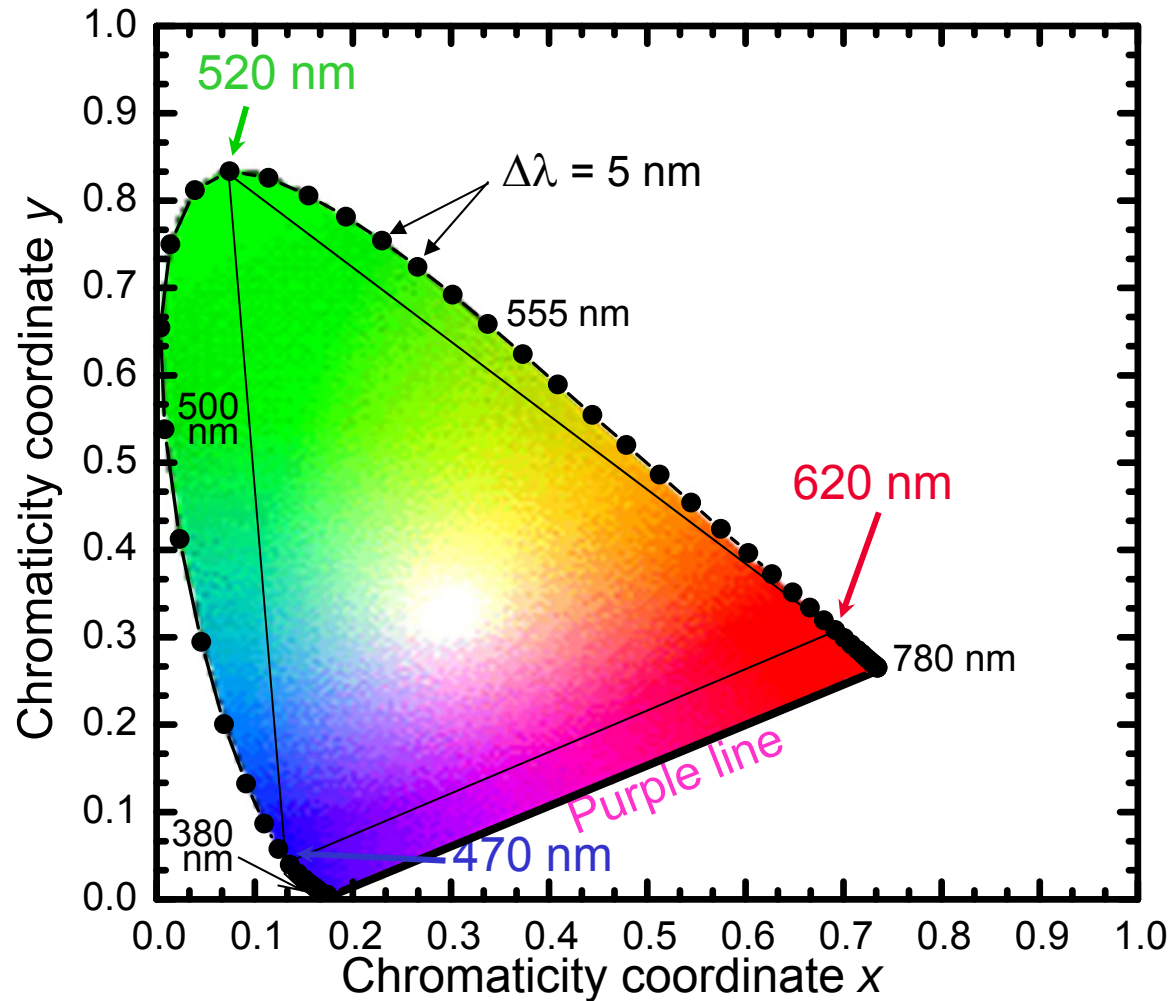


Figure 2.7: Chromaticity diagram and colors which can be mixed using three monochromatic colors red (620 nm), green (520 nm) and blue (470 nm).

2.5.1 Additive color mixing

- Principle works as described in the previous section [2.8].
- Color TV: Instead of using monochromatic colors, the colors of Table 2.3 are used for mixing.

Table 2.3: chromaticity coordinates (x,y) and luminosity Y of TV colors according to DIN 6169.

Color		x	y	Y
Red	R	0.6400	0.3300	22.19
Green	G	0.2900	0.6000	70.68
Blue	B	0.1500	0.0600	7.13

Table 2.4: chromaticity coordinates (x,y) and luminosity Y of mixed TV colors Cyan, Magenta, Yellow and White according to DIN 6169.

Color		x	y	Y
Cyan	C	0.2197	0.3288	77.81
Magenta	M	0.3270	0.1576	29.32
Yellow	Y	0.4172	0.5019	92.87
White	W	0.3127	0.3291	100

With:

Cyan (C)	=	Green (G)	+	Blue (B)
Magenta (M)	=	Red (R)	+	Blue (B)
Yellow (Y)	=	Red (R)	+	Green (G)

Generation of White (W) using three color mixing:

White (W)	=	Red (R)	+	Green (G)	+	Blue (B)
		+		+		+
White (W)	=	Cyan (C)	+	Magenta (M)	+	Yellow (Y)
		White (W)		White (W)		White (W)

Generation of White (W) using two color mixing:

Cyan (C), Magenta (M), Yellow (Y) are complementary colors to Red (R) , Green (G), Blue (B).

Combinations of each pair of complementary colors results in White:

White (W)	=	Red (R)	+	Cyan (C)	or
White (W)	=	Green (G)	+	Magenta (M)	or
White (W)	=	Blue (B)	+	Yellow (Y)	

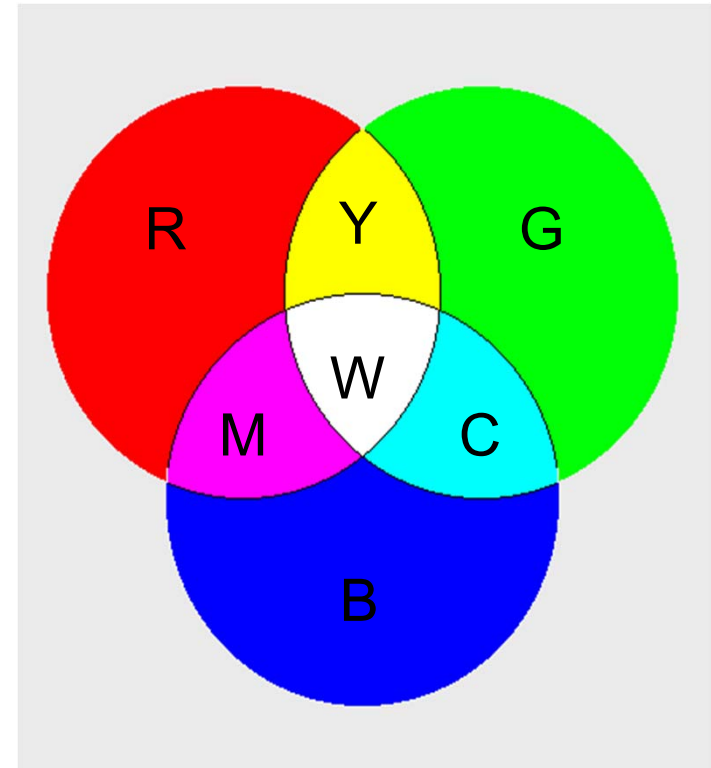
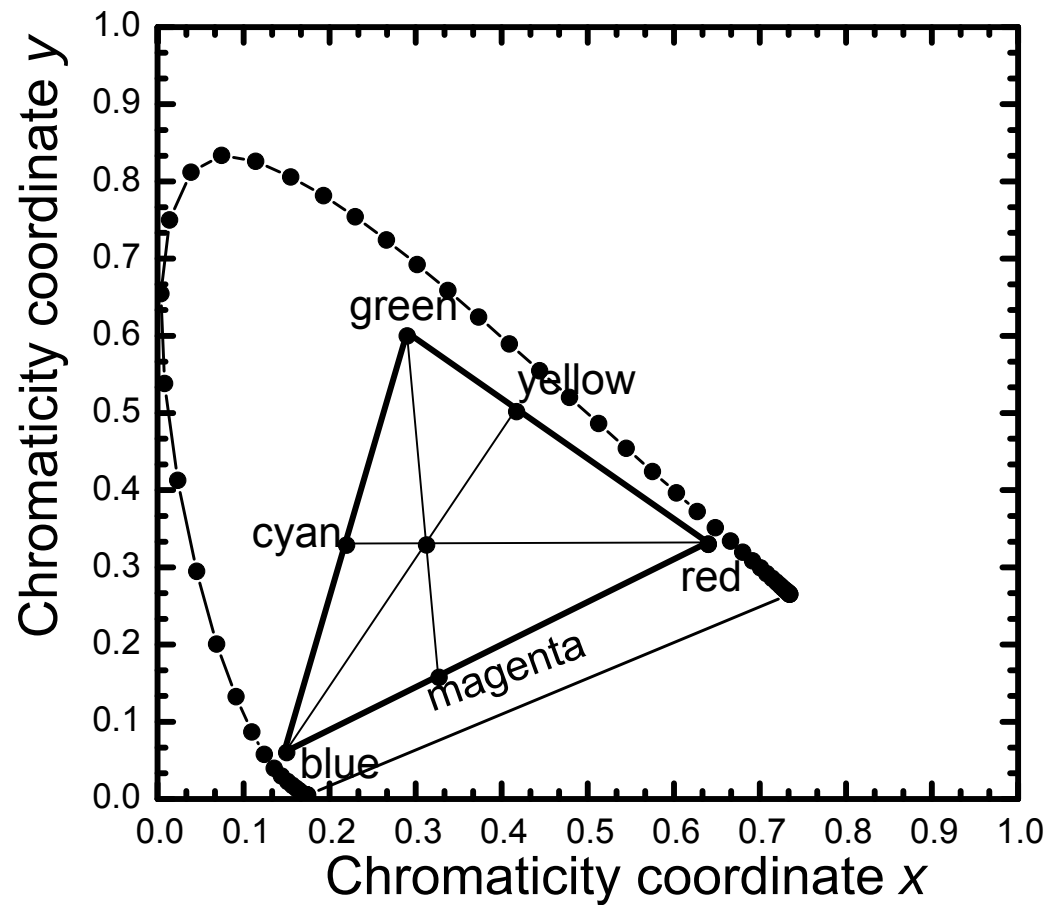


Figure 2.8: Prime colors of color TV, red, green and blue and mixed colors yellow, cyan, magenta, white.

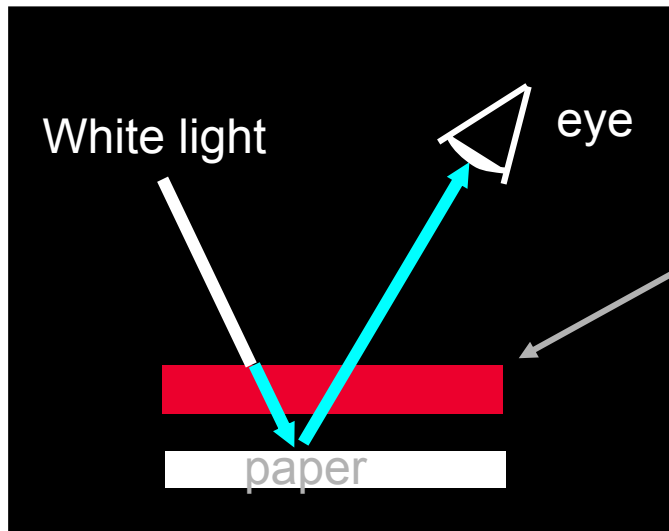
2.5.1 Subtractive color mixing

Examples:

- Painting
- Color printing
- Slide projection

Principle [2.8]:

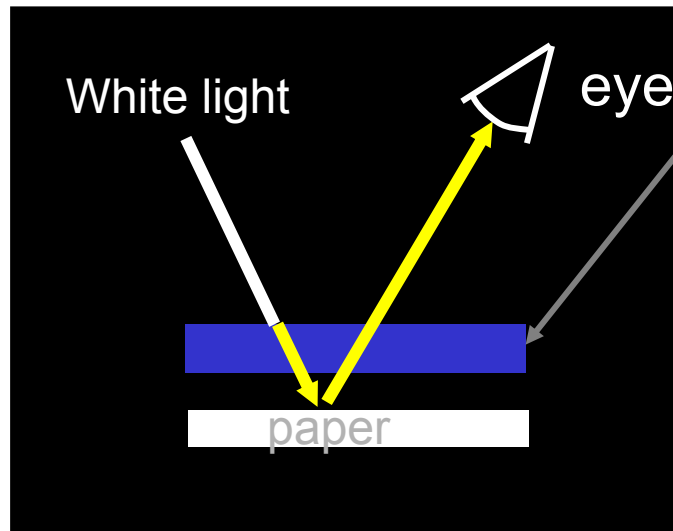
- White light enters the color medium.
- One part of the spectrum is absorbed by the color medium.
- The remaining part is reflected by the white paper
- and filtered again by the color medium before it enters the eye.



Color filter Cyan absorbs red part of white light:

$$W - R = G + B = C \text{ (Cyan)}$$

Figure 2.9: A filter on white paper which *absorbs red light* appears cyan.

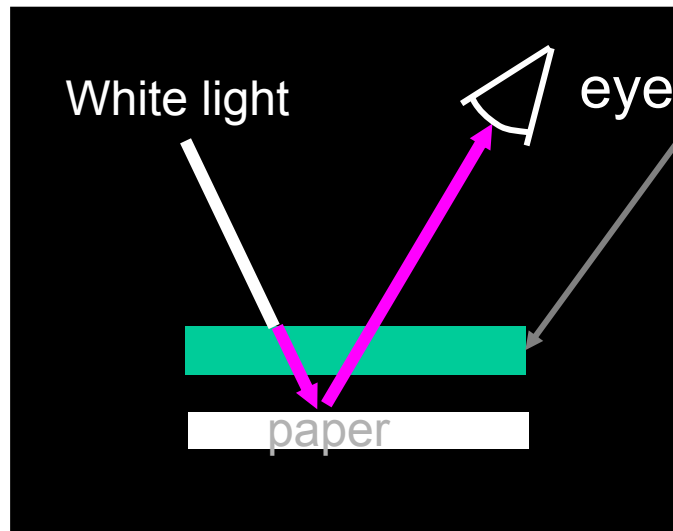


Color filter Yellow *absorbs blue part of white light*:

$$W - B = R + G = Y \text{ (Yellow)}$$

Figure 2.10:

A filter on white paper, which *absorbs blue light*, appears yellow

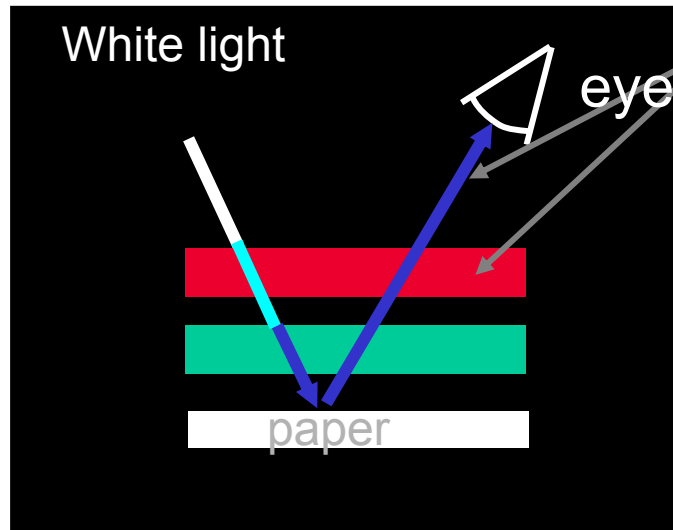


Color filter Magenta *absorbs green part of white light*:

$$W - G = R + B = M \text{ (Magenta)}$$

Figure 2.11:

A filter on white paper, which *absorbs green light*, appears magenta.

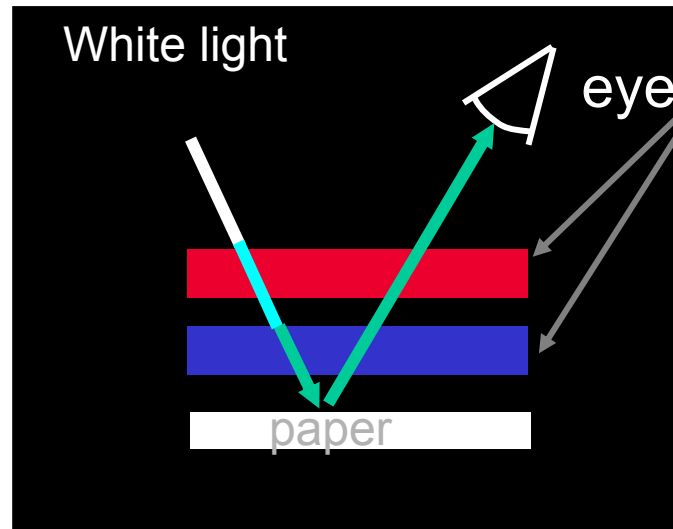


Color filters Cyan and Magenta *absorb* red and green part of white light:

$$W - R - G = B \text{ (Blue)}$$

Figure 2.12:

Filters on white paper, which *absorb* red and green light, appear blue

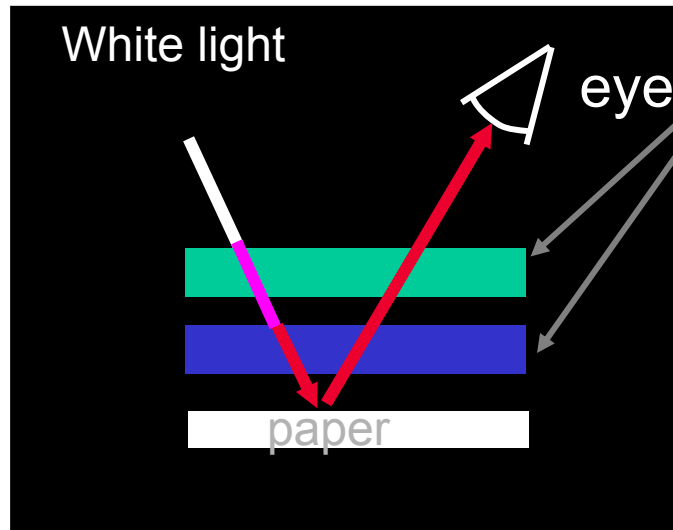


Color filters Cyan and Yellow *absorb* red and blue part of white light:

$$W - R - B = G \text{ (Green)}$$

Figure 2.13:

Filters on white paper, which *absorb* red and blue light, appear green

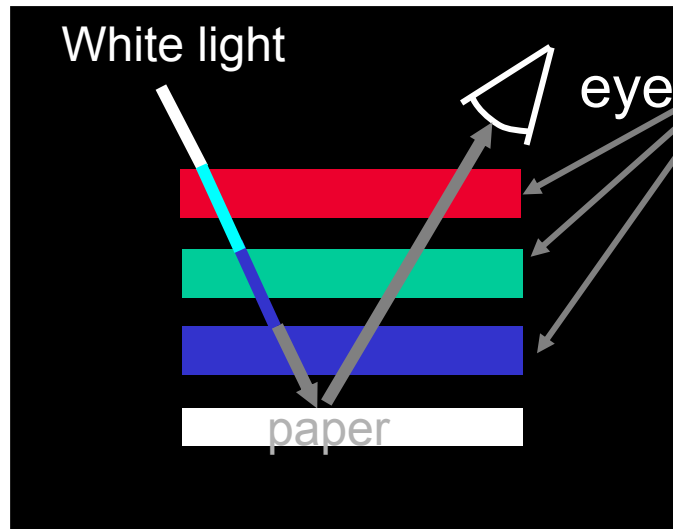


Color filters Magenta and Yellow *absorb green and blue* part of white light:

$$W - G - B = R \text{ (Red)}$$

Figure 2.14:

Filters on white paper, which *absorb green and blue* light, appear red



Color filters Cyan, Magenta and Yellow *absorb red, green and blue* part of white light:

$$W - R - G - B = \text{Black}$$

Figure 2.15:

Filters on white paper, which absorb red, green and blue light, appear black

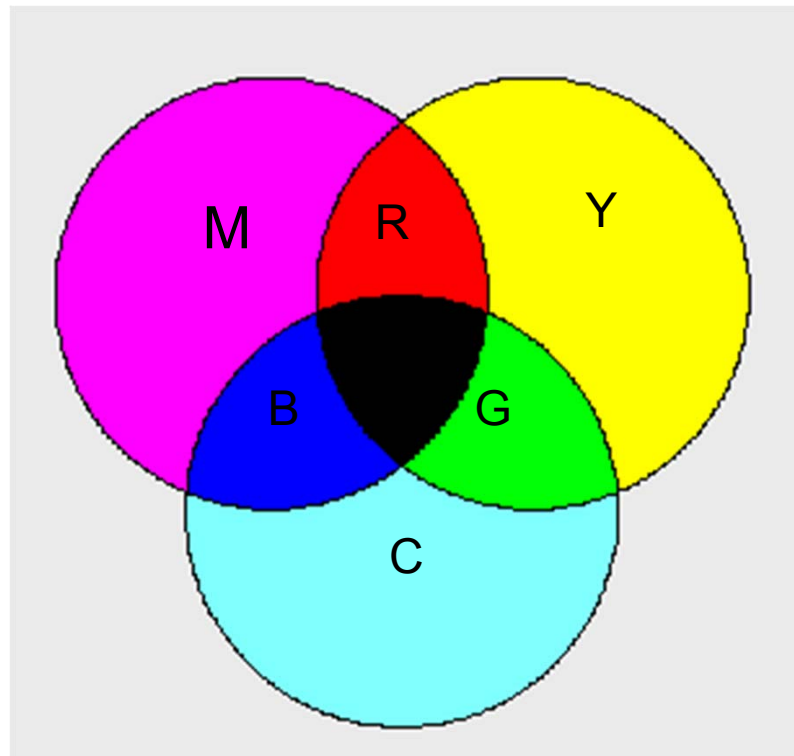


Figure 2.16: Subtractive color mixing. The colors red (R), green (G) and blue (B) can be generated by printing two of the colors cyan (C), magenta (M) and yellow (Y) on white paper. All light is absorbed if all colors C, M, Y are printed and the area appears black.

2.6 Literature, Internet Links

[2.1]: <http://www-cvrl.ucsd.edu/database/text/lum/vljv.htm>: Color and vision resarch laboratories, London, Tübingen: *Photopic luminosity function*.

[2.2]: <http://www-cvrl.ucsd.edu/database/text/lum/scvl.htm>: Color and vision resarch laboratories, London, Tübingen: *Scotopic luminosity function*.

[2.3]: <http://www.cie.co.at/cie/>: Homepage of the INTERNATIONAL COMMISSION ON ILLUMINATION

[2.4]: Niedrig, H. (Hrsg.), *Bergmann Schäfer Lehrbuch der Experimentalphysik, Band 3*, 9. Aufl., (Walter de Gruyter, Berlin New York 1993), S. 727.

[2.5]: <http://hyperphysics.phy-astr.gsu.edu/hbase/vision/cieprim.html#c2>: C. R. Nave Department of Physics and Astronomy Georgia State University Atlanta: *Color matching functions*.

[2.6]: <http://hyperphysics.phy-astr.gsu.edu/hbase/vision/ciecal.html#c1>: C. R. Nave Department of Physics and Astronomy Georgia State University Atlanta: *CIE Chromaticity Coordinates*.

[2.7]: http://www.cs.rit.edu/~ncs/color/a_chroma.html: Eugene Vishnevsky: *Color Applets*

[2.8]: <http://home.att.net/~RTRUSCIO/COLORSYS.htm>: Robert Truscio: *Color mixing*.

3. Photometry

- Objective Photometry (Radiometry) [3.1]:
Detection of photometric quantities without consideration of the eye's sensitivity results in *energetic* quantities, which characterize the physical properties of *radiation* and *irradiation*, indicated by the index e .
- Subjective Photometry [3.2]:
Detection of photometric quantities with consideration of the eye's sensitivity results in *visual* quantities, which characterize the properties of *light* and *illumination*, indicated by the index v .

3.1 Energetic Quantities of Radiation and Irradiation

3.1.1 Radiant flux, radiation power Φ_e

Strahlungsfluß, Strahlungsleistung

$$\Phi_e = \frac{dQ_e}{dt} \quad \left[\frac{J}{s} \right] = [W]$$

Total radiation energy Q_e per time interval dt emitted by the radiation source [3.3].

3.1.2 Radiant exitance M_e

Spezifische Ausstrahlung

$$M_e = \frac{d\Phi_e}{dA} \quad \left[\frac{W}{cm^2} \right]$$

Radiant flux Φ_e emitted per surface element dA of the radiation source [3.3].

Example: Sun $\sim 64 \text{ MW/m}^2$

3.1.3 Radiant intensity I_e

Strahlstärke

$$I_e = \frac{d\Phi_e}{d\Omega} \left[\frac{W}{sr} \right]$$

Radiant flux Φ_e emitted by the radiation source in solid angle $d\Omega$ [3.3].

The same radiant flux results in a higher radiant intensity, if the radiation is emitted in a smaller solid angle.

3.1.4 Radiance L_e

Strahldichte

$$L_e = \frac{dI_e}{dA} \left[\frac{W}{cm^2 sr} \right] \quad \text{Emitted radiant intensity per surface element } dA \text{ [3.3].}$$

The radiance of a radiation source of constant radiant intensity I_e increases if the radiating area A gets smaller.

3.1.5 Irradiance E_e

Bestrahlungsstärke

$$E_e = \frac{d\Phi_e}{dA} \quad \left[\frac{W}{m^2} \right] \quad \text{Radiant flux } \Phi_e \text{ per irradiated area } dA \text{ [3.3].}$$

Example: The irradiance of direct sun light on earth is approximately: $L_e \approx 1 \text{ kW/m}^2$.

3.1.6 Spectral quantities

Above quantities contain no information about the spectral distribution of radiation. Spectral quantities (quantity per wavelength interval $d\lambda$) are indicated by the index λ .

Table 3.1: integral and spectral quantities of radiation and irradiation

integral		spectral	
$\Phi_e = \int_{\lambda_1}^{\lambda_2} \Phi_{e\lambda} d\lambda$	$[W]$	power, flux	$\Phi_{e\lambda} = \frac{d\Phi_e}{d\lambda}$ $[W / nm]$
$M_e = \int_{\lambda_1}^{\lambda_2} M_{e\lambda} d\lambda$	$\left[\frac{W}{cm^2} \right]$	radiant exitance	$M_{e\lambda} = \frac{dM_e}{d\lambda}$ $\left[\frac{W}{cm^2 nm} \right]$
$I_e = \int_{\lambda_1}^{\lambda_2} I_{e\lambda} d\lambda$	$\left[\frac{W}{sr} \right]$	radiant intensity	$I_{e\lambda} = \frac{dI_e}{d\lambda}$ $\left[\frac{W}{sr nm} \right]$

Table 3.1: integral and spectral quantities of radiation and irradiation

integral		spectral	
$L_e = \int_{\lambda_1}^{\lambda_2} L_{e\lambda} d\lambda$	$\left[\frac{W}{cm^2 sr} \right]$	radiance	$L_{e\lambda} = \frac{dL_e}{d\lambda} \left[\frac{W}{cm^2 sr nm} \right]$
$E_e = \int_{\lambda_1}^{\lambda_2} E_{e\lambda} d\lambda$	$\left[\frac{W}{m^2} \right]$	irradiance	$E_{e\lambda} = \frac{dE_e}{d\lambda} \left[\frac{W}{m^2 nm} \right]$

3.2 Visual Quantities of Light and Illumination

The sensitivity of the human eye determines the visual quantities X_v , which characterize light and illumination. To this end we evaluate the spectral energetic quantities $X_{e\lambda}$ according to:

$$X_v = K_m \int_{380nm}^{780nm} X_{e\lambda} V(\lambda) d\lambda$$

With the constant $K_m = 683 \text{ lm/W}$, the spectral energetic quantity $X_{e\lambda}$ and the corresponding visual quantity X_v , and the unit for light measuring Lumen [lm].

Using the above formula we can calculate from each of the energetic quantities, Φ_e , M_e , I_e , L_e , E_e the corresponding visual quantity, Φ_v , M_v , I_v , L_v , E_v .

3.2.1 Luminous flux Φ_v [lm]

Lichtfluß

$$\Phi_v = K_m \int_{380nm}^{780nm} \Phi_{e\lambda} V(\lambda) d\lambda$$

Unit: Lumen [lm]

$$K_m = 683 \text{ lm/W}$$

Radiant flux detected by the eye

If a light source emits optical radiation (light) with 1W power at a wavelength of 555 nm, where the eyes sensitivity equals 1, then the luminous flux of the light source is 683 lm (Lumen) [3.4].

3.2.2 Luminous exitance M_v [lm/m²]

Spezifische Lichtausstrahlung

$$M_v = K_m \int_{380nm}^{780nm} M_{e\lambda} V(\lambda) d\lambda$$

The luminous exitance defines the amount of (visible) light, which is produced by each surface element of the light source [3.4].

3.2.3 Luminous intensity I_v [cd = lm/sr]

Lichtstärke

$$I_v = K_m \int_{380nm}^{780nm} I_{e\lambda} V(\lambda) d\lambda$$

Unit: Candela [cd]

Luminous flux Φ_v emitted by the light source in solid angle $d\Omega$.



The same luminous flux results in a higher luminous intensity, if the light is emitted in a smaller solid angle [3.4].

Example: Light source emits 1 lumen in 0.1 sr in a certain direction. The luminous intensity of the light source under this direction is 10 candela

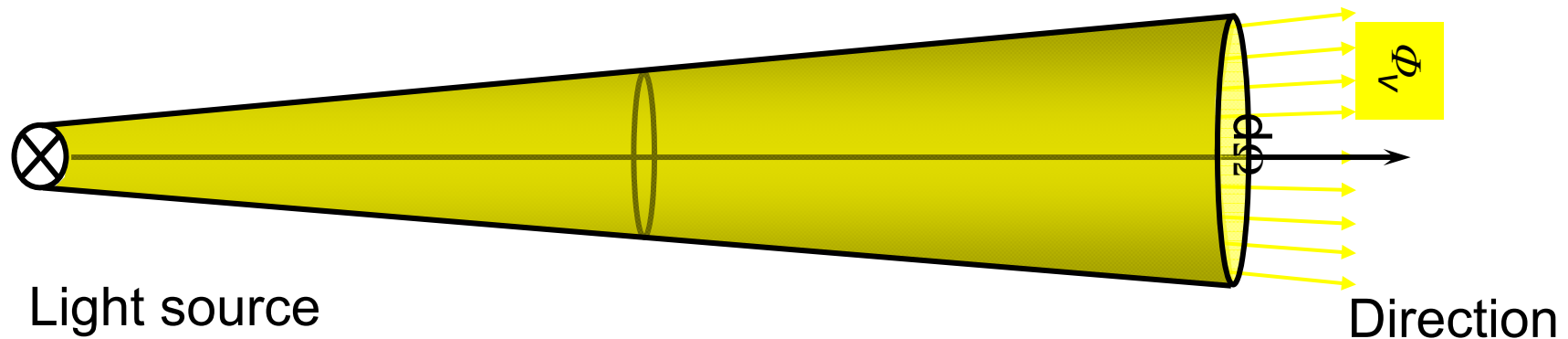


Figure 3.1: Light source emits light of luminous flux ϕ_v in solid angle $d\Omega$.

3.2.4 Luminance L_v [sb = lm/(m² sr)]

Leuchtdichte

Unit: Stilb [sb]

$$L_v = K_m \int_{380nm}^{780nm} L_{e\lambda} V(\lambda) d\lambda$$

Emitted luminous intensity per surface element dA of the light source.

Luminance L is the impression of brightness that an observer has of a light source or an illuminated surface. The eye is very good at distinguishing directly between different luminance values [3.4].

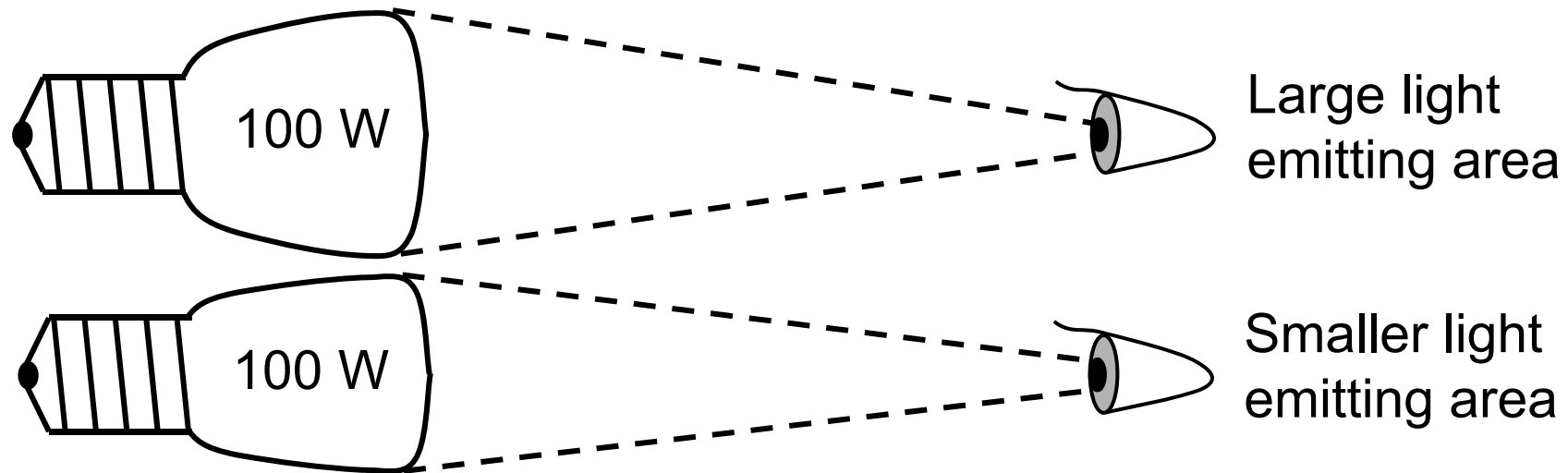


Figure 3.2: A light source of constant luminous intensity I_e appears brighter the smaller the light emitting area A is.

Table 3.2: Luminance L concerning different examples

Example	Luminance L [cd/m ²]
Xenon short arc lamp	$2 \times 10^5 - 5 \times 10^9$
Sun	1.6×10^8
Tungsten incandescent lamp	$2 \times 10^6 - 2.6 \times 10^7$
Compact fluorescent lamp	$2 \times 10^5 - 7 \times 10^5$
Fluorescent lamp	5,000 - 30,000
Sunlit clouds	10,000
Candle	7,500
Blue sky	5,000
White paper at 500 lx	100

3.2.5 Illuminance E_v [$\text{lx} = \text{lm}/\text{m}^2$]

Beleuchtungsstärke

Unit: Lux [lx]

$$E_v = K_m \int_{380\text{nm}}^{780\text{nm}} E_{e\lambda} V(\lambda) d\lambda$$

Lux is the quantity, which characterizes the quality of lighting systems [3.4].

Example: If the lighting system in your office illuminates your work place sufficiently the illuminance should exceed 500 lx.

The minimum illuminance at workplaces are defined in DIN 5035, Part 2 and the relevant workplace guidelines. Compliance with these standards and guidelines, however, is no guarantee für a good lighting system in every case.

Table 3.3: Illuminance E concerning different examples

Example	Illuminance E [lx]
Operating theatre lighting	20,000 - 120,000
Sunny summer day	60,000 - 100,000
Overcast summer day	20,000
Overcast winter day	3000
Well-lit workplace	500 - 750
Pedestrian precinct	5 - 100
Night time, full moon	0.25
Night time, new moon	0.01

3.2.6 Luminous efficacy η_v [lm/W]

Lichtausbeute

$$\eta_v = K_m \int_{380nm}^{780nm} \eta_{e\lambda} V(\lambda) d\lambda \quad \text{Unit: lumen per watt [lm/W]}$$

Luminous efficacy η_v indicates the efficiency with which the consumed (mostly electrical) power is converted into light. The spectral electrical efficiency $\eta_{e\lambda}$ is the amount of electrical energy which is converted into electromagnetic radiation of wavelength λ within the wavelength interval $d\lambda$ [3.4].

The maximum efficacy that can be obtained is 683 lm/W if the light source emits monochrome light at a wavelength of 555 nm and converts all electrical energy into light.

For a “white” light source the maximum efficacy is only 199 lm/W if the light source emits with constant power between 380 nm and 780 nm, because the eye’s sensitivity decreases below and above 555 nm.

Table 3.4: Luminous efficacy of light sources

Example	luminous efficacy η_v [lm/W]
Theoretical limit for monochrome green light (555 nm)	683
Theoretical limit for white light (380 nm - 780 nm)	199
Low-pressure sodium lamp	190
High efficiency fluorescent lamp	107
Incandescent tungsten halogen lamp	20 - 40
Incandescent tungsten lamp	5 - 20
Paraffin lamp	0.3
Candle	0.1

3.3 Literature, Internet Links

[3.1]: <http://www.cg.tuwien.ac.at/research/theses/matkovic/node12.html#SECTION00512000000000000000>: Krešimir Matković, Technische Universität Wien: *Radiometry*

[3.2]: <http://www.cg.tuwien.ac.at/research/theses/matkovic/node13.html>: Krešimir Matković, Technische Universität Wien: *Photometry*

[3.3]: Niedrig, H. (Hrsg.), *Bergmann Schäfer Lehrbuch der Experimentalphysik, Band 3*, 9. Aufl., (Walter de Gruyter, Berlin New York 1993), S. 611 - 614.

[3.4]: http://www.osram.com/service_corner/glossary/index.html: *Encyclopedia of Light*

4. Incoherent Light Sources

Optical radiation and light generated by

Interaction of different energy levels in

- atoms,
- molecules,
- crystalline or amorphous materials.

In most cases *energy levels* relate to *electrons*.

Electrons *in atoms*, occupy different *discrete* energy levels.

Lowest energy level → ground state E_0 ,

Higher energy levels → excited states $E_1, E_2, \dots$

Electron does not stay long time in excited state E_1 , E_2 , ...

Electron returns *spontaneously* into ground state E_0 , or lower excited state.

During transition photon of energy $h\nu = E_1 - E_0$ is emitted.

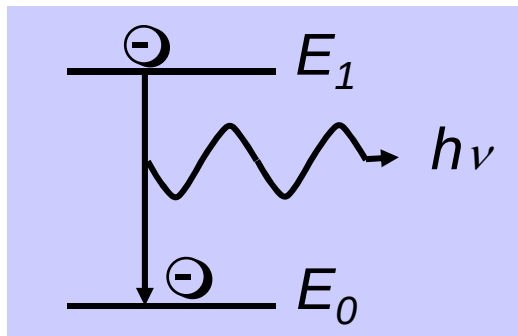


Figure 4.1: Excited electron in state E_1 returns to the ground state E_0 and emits photon of energy $h\nu = E_1 - E_0$.

Excitation of electrons into excited states

- heating (electric current),
- injection (light emitting diode, semiconductor laser),
- absorption of photons (luminescent phosphors),
- collision of electrons and atoms (gas discharges).

In case of *incoherent light sources* the electrons

- *oscillate independently* of each other, if electromagnetic radiation is regarded to consist of *electromagnetic waves*, or
- *recombine* and emit photons, if electromagnetic radiation is regarded to consist of *particles*.

In case of *coherent light sources* electrons

- interact with each other via emitted radiation or emitted photons,
- with the consequence that phase and direction of the photons are strongly correlated.
- Result is *laser radiation*.

4.1 Black body radiation

Schwarzkörperstrahlung

glühende Oberfläche

- The incandescent surface of a solid emits radiation of continuous spectral distribution.

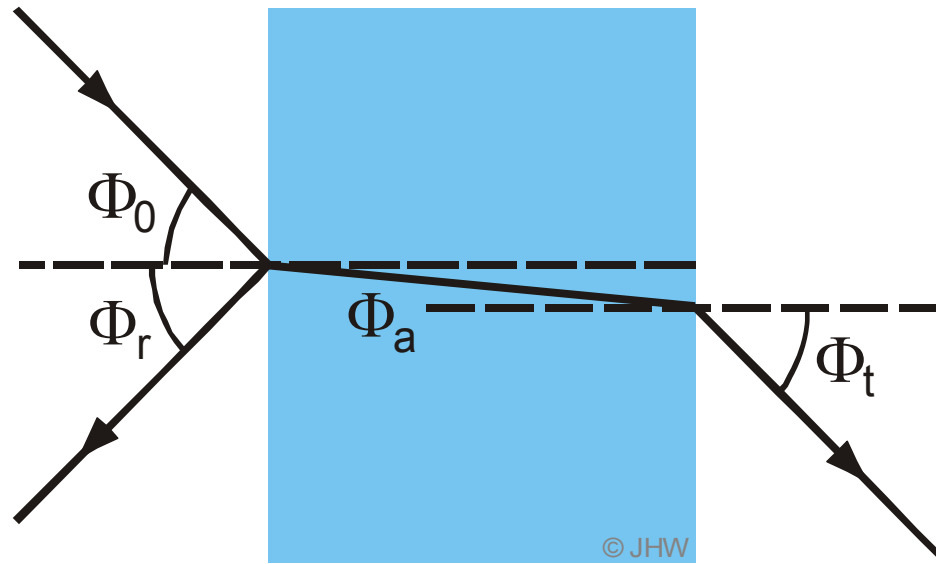
Glühlampen

- Incandescent lamps are typical representatives.
- A black body is a special case.

Schwarzer oder Planck'scher Strahler



4.1.1 Reflectance, absorbance, transmittance



$$r_{\Phi} = \frac{\Phi_r}{\Phi_0} \quad \text{reflectance, refl. coefficient}$$

$$a_{\Phi} = \frac{\Phi_a}{\Phi_0} \quad \text{absorbance, abs. coefficient}$$

$$t_{\Phi} = \frac{\Phi_t}{\Phi_0} \quad \text{transmittance, trans. coefficient}$$

Fig. 4.2: Reflected, absorbed, and transmitted radiation

$$r_{\Phi} + a_{\Phi} + t_{\Phi} = 1$$

r_{Φ} , a_{Φ} , t_{Φ} depend on frequency ν , polarization, angle of incidence, temperature T .

Absorbance a_Φ and absorption constant α

Both quantities are connected by the equation

$$\frac{\Phi_a(\lambda)}{\Phi_0(\lambda)} = a_\Phi(\lambda) = \int_0^w \alpha(\lambda) e^{-\alpha(\lambda)x} dx = 1 - e^{-\alpha(\lambda)w}$$

where w is the thickness of the sample.

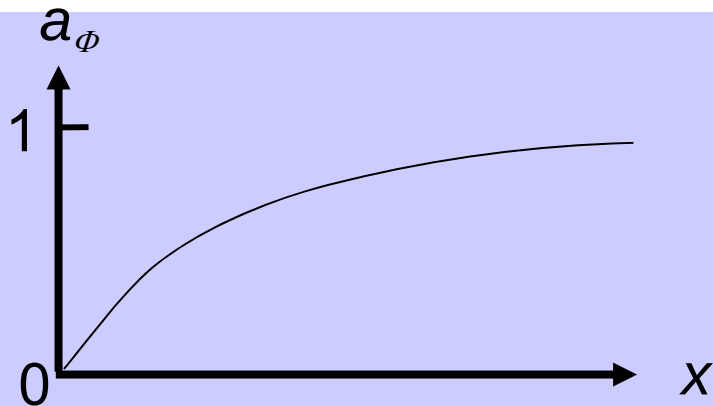


Figure 4.3:
Absorbance a_Φ in
medium with ab-
sorption constant α .

4.1.2 Definition of a black body

What is a **black** body?

To our eyes a body appears as **black**, if it absorbs all radiation *in the visible regime, i.e. all light!* Thus it holds that $a_{\Phi}(h\nu) = 1$ in this regime of the electromagnetic spectrum.

For an ideal black body, $a_{\Phi}(h\nu) = 1$ holds for all frequencies. Ideal black bodies do not exist; some systems are close to the ideal one:

- very thick non-reflecting bodies
- a tiny hole in a black shoe box
- the old stove of your great grand parents



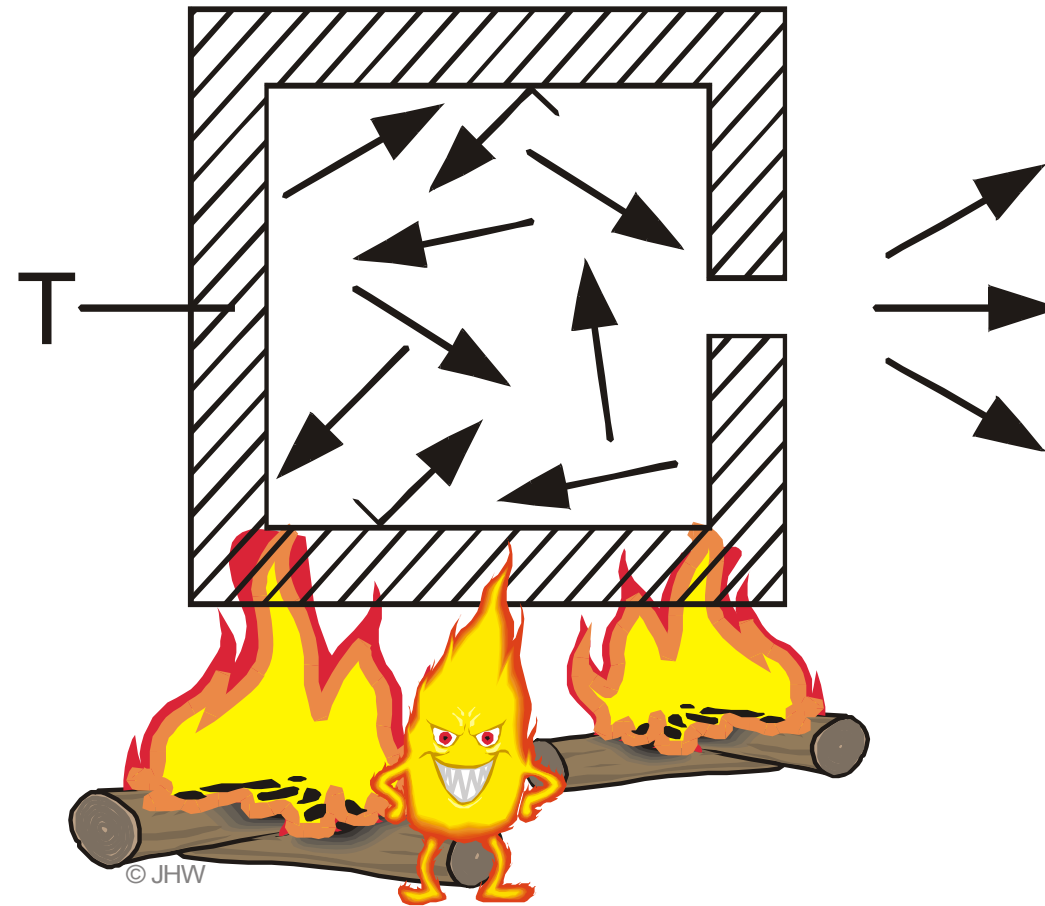


Fig. 4.4: Absorption and emission of radiation by the walls of temperature T result in a thermal equilibrium between the radiation field and the walls of a furnace, oven, or stove.

From our personal experience we know:

- A black body (black jeans) *absorbs* more radiation than a non-black body (blue jeans).
- However, a black body *emits* also more radiation than a non-black body!

*The stronger a body absorbs radiation,
the stronger it must emit radiation.*

- This rule is simply the consequence of the requirement for equal rates of absorbed and emitted energy at constant temperature.
- Unequal rates would result in a temperature change.
- A strongly absorbing body must get rid of the energy. As a consequence it must also have a strong emission (or explode or radiation overflow....).



4.1.3 Kirchhoff's radiation law

relates the spectral emitted power $L_{e\lambda}(\lambda)$ from a body to the absorbance a_Φ as

$$L_{e\lambda}(\lambda) = a_\Phi(\lambda) L_{e\lambda}^{black}(\lambda) \left[\frac{W}{m^2 sr \mu m} \right].$$

The radiated power $L_{e\lambda}(\lambda)$ per wavelength interval (μm) and steradian (sr) that is emitted per surface element (m^2) of a body depends on the absorbance $a_\Phi(\lambda)$.

Note: The absorbance a_Φ depends on the nature of the surface (color, texture, roughness etc.) and on wavelength λ . However, we can always calculate $L_{e\lambda}(\lambda)$ because $L_{e\lambda}^{black}(\lambda)$ is an universal function!

The universal function $L_{e\lambda}^{black}$ is the emitted power spectrum of a black body!

4.1.4 Planck's radiation law

gives the power spectrum of a black body [4.1]

$$L_{e\lambda}^{black} = \frac{d\Phi}{dA d\Omega d\lambda} = \frac{2hc_0^2}{\lambda^5} \frac{1}{e^{hc_0/(\lambda kT)} - 1} \left[\frac{W}{m^2 sr \mu m} \right]$$

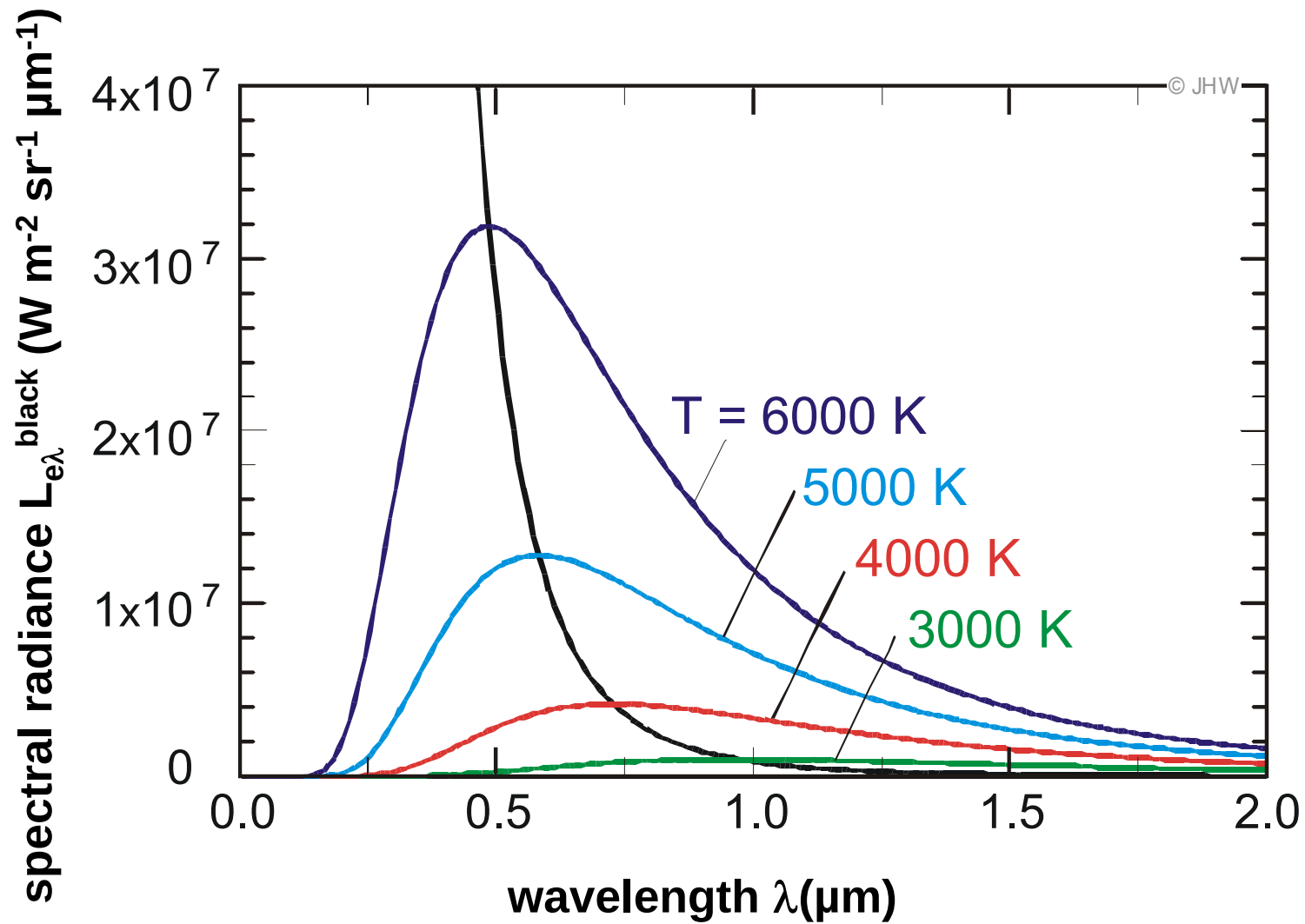


Fig. 4.5: Spectrum of a black body.

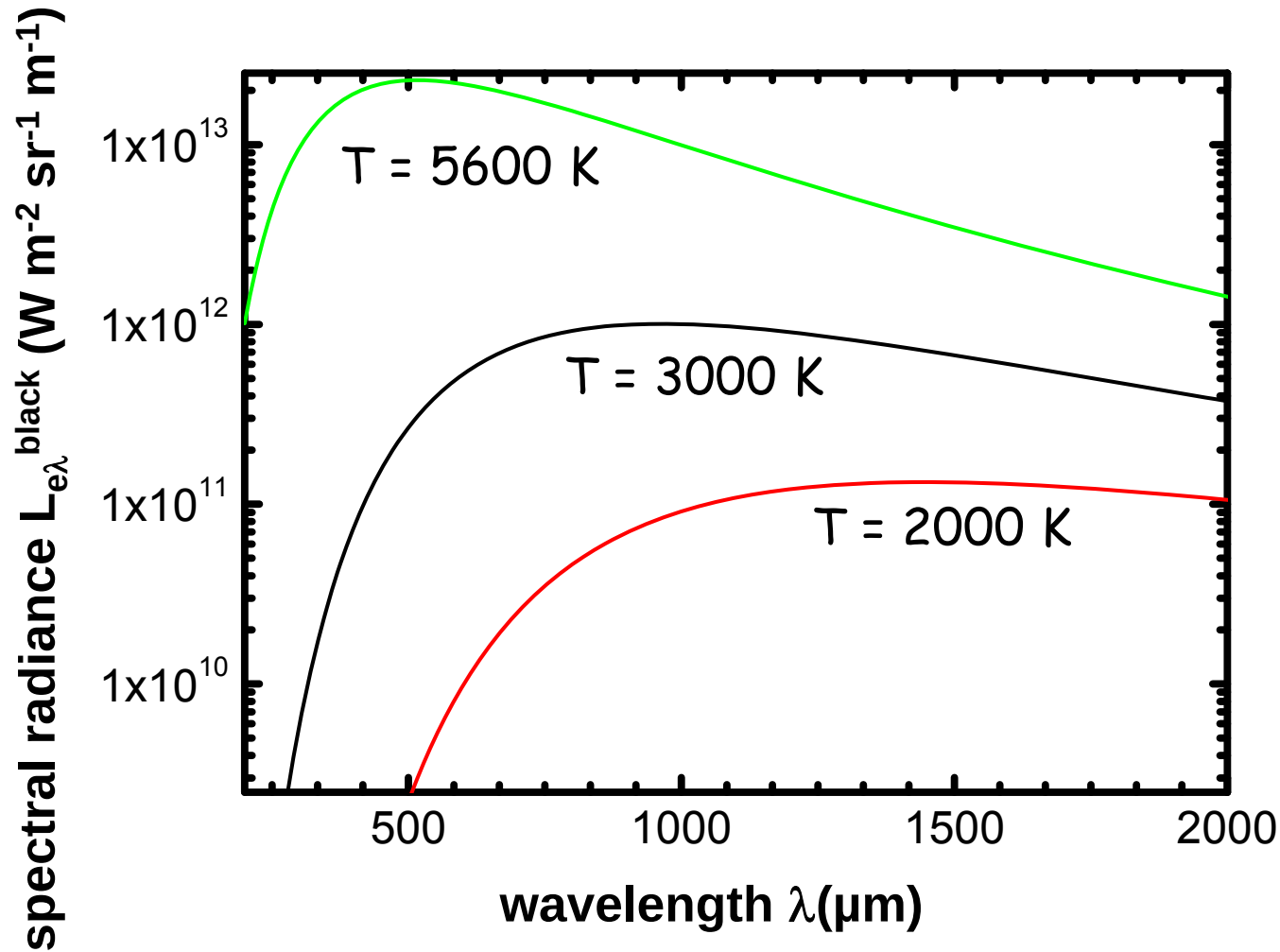
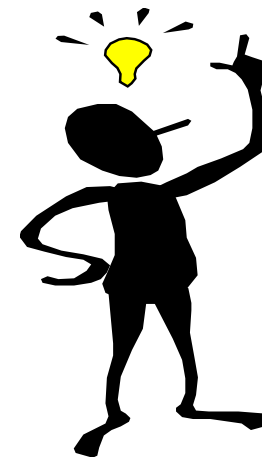


Figure 4.6: Logarithmic plot of spectral distribution of black body radiation for three different temperatures T .

- The hotter the body becomes, the more the spectrum shifts to shorter wavelengths.
- Remember having looked into the door of an oven? Everything appears black if the oven is cold.
- Upon heating, we see a *red* color. The hotter the oven gets, the more *yellow* the color becomes. In an oven, we do usually not see blue color. It does not become that hot!
- The radiation law of Planck is only a mathematical description of the color of a burning fire, heated oven!



4.1.5 Wien's displacement law

Wien'sches
Verschiebungsgesetz

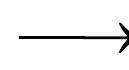
a) wavelength of maximum's position [4.2]

$$\lambda_{\max} T = 2.8978 \times 10^3 \mu\text{mK}$$
$$\lambda_{\max} \propto \frac{1}{T}$$

Examples:

Sun temperature

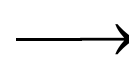
$$T_{\text{sun}} \cong 6000 \text{ K}$$



$$\lambda_{\max} \cong 500 \text{ nm}$$

Earth temperature

$$T_{\text{earth}} \cong 300 \text{ K}$$



$$\lambda_{\max} \cong 10 \mu\text{m}$$

Having a high “color temperature” for a light source is important for taking color pictures!



b) height of the maximum (for $\Omega = 2\pi$ sr)

$$M_{e\lambda}^{black}(\lambda_{\max}) = 2.6 \times 10^{-17} \frac{W}{m^2 \mu m K^5} T^5$$

$M_{e\lambda} \equiv$ Radiant Exitance

Definition see next slide

4.1.6 Angular dependence

The radiance depends on the angle θ under which the surface of the hole appears.

A blackbody radiates according to a cosine law, which means that the area of the surface appears to be zero if the angle θ equals 90 degrees.

$$L_{e\lambda}^{black}(\theta) = L_{e\lambda}^{black}(0) \cos \theta$$

Taking this into account the total radiance emitted into a hemisphere (solid angle = 2π) is calculated by integration, which yields the radiant exitance M_e .

$$M_{e\lambda}^{black}(\lambda, T) = \frac{dM_e^{black}}{d\lambda} = \frac{2\pi hc_0^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \left[\frac{W}{m^2 \mu m} \right]$$

4.1.7 Stefan-Boltzmann's law

By integrating equation

$$M_{e\lambda}^{black}(\lambda, T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

from $\lambda = 0$ to $\lambda = \infty$, the total radiation per surface area M_e^{black} (radiant exitance) of a black body is derived as [4.3].

$$M_e^{black}(T) = \sigma T^4$$

With $\sigma = 5.669 \times 10^{-8} \text{ W}/(\text{m}^2 \text{ K}^4)$

Example: The Sun with a surface temperature of $T = 5800 \text{ K}$ emits into a solid angle of 2π

64 MW/m²!!

16 m² Sun surface make up one nuclear power
plant
of 1 GW power!!!

The earth ($T = 300\text{K}$) emits 500 W per m² surface area.

4.2 Gray body radiation

For a *black* body, the absorbance is $a_{\Phi} = 1$ for all λ .

For a *gray* body, it holds $a_{\Phi} < 1$, and a_{Φ} is independent of λ !

$$a_{\Phi}(\lambda) = \text{constant} < 1$$

Therefore, the emitted power (Kirchhoff's law!) is the same as for a black body but reduced by a constant factor (a_{Φ}) for all wavelengths.

4.3 Selective body radiation of a semiconductor

4.3.1 Selective body radiation

Absorbance $a_\phi < 1$ depends on λ .

As a consequence (Kirchoff's law!!), the emitted power is not only reduced by a constant factor (as for the gray body), but depends also on wavelength λ .

4.3.2 Radiation from a semiconductor

a) the absorbance of a semiconductor of gap E_g

Within the simplest model, a semiconductor absorbs no light for photon energies $h\nu < E_g$, and all light for energies $h\nu \geq E_g$.

As a consequence, the absorbance a_ϕ (the ratio of absorbed and incident radiation) is a step function.

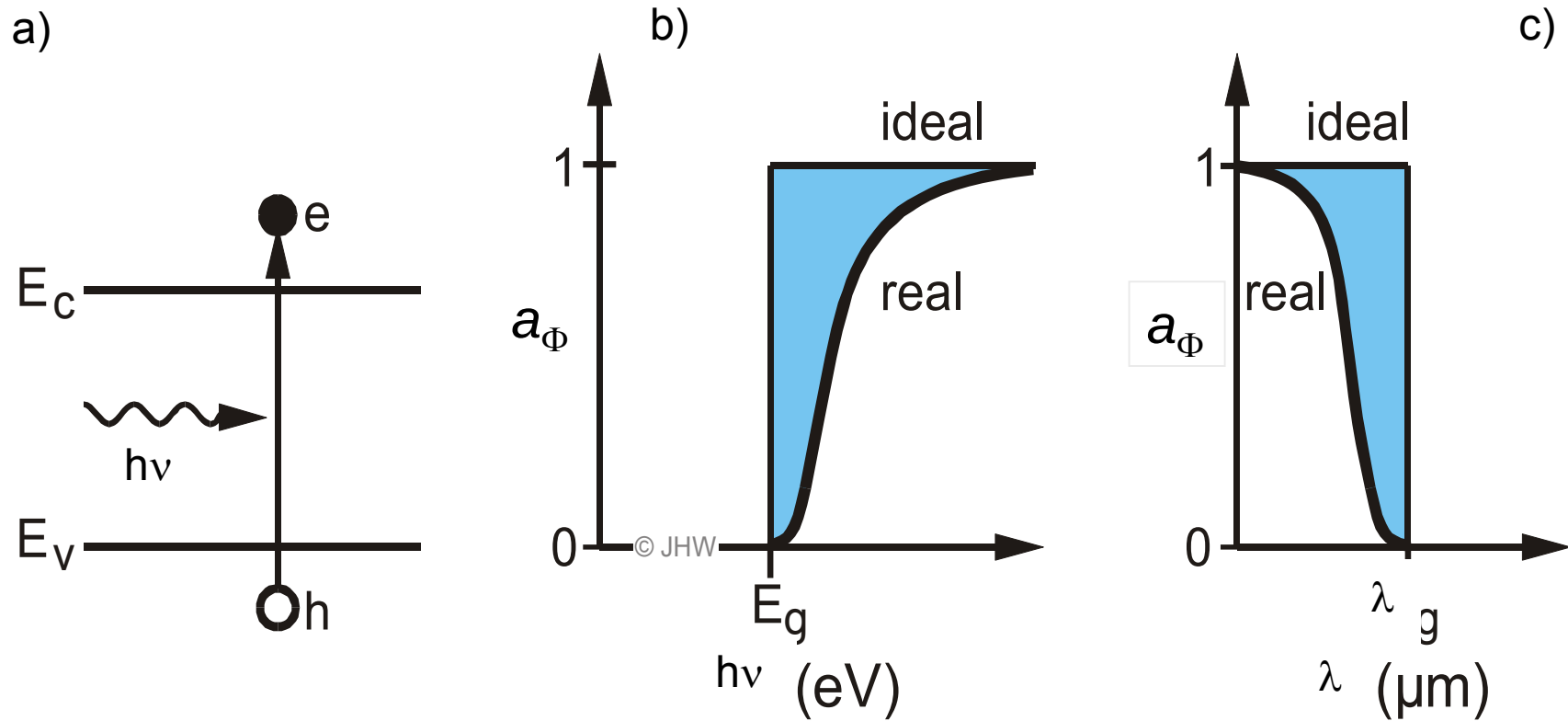


Fig. 4.7: a) absorption in a semiconductor,
b) step function of absorbance vs. photon energy,
c) versus wavelength.

b) the emitted spectrum of a semiconductor

Within the simplest model (step-like absorption, no reflection), the spectrum is similar to the one of a black body. However, the spectrum is cut off for $\lambda > \lambda_g$. For $T = 300$ K, the black body radiation takes a maximum at about $11 \mu\text{m}$. Temperature variations only weakly change the semiconductor's spectrum.

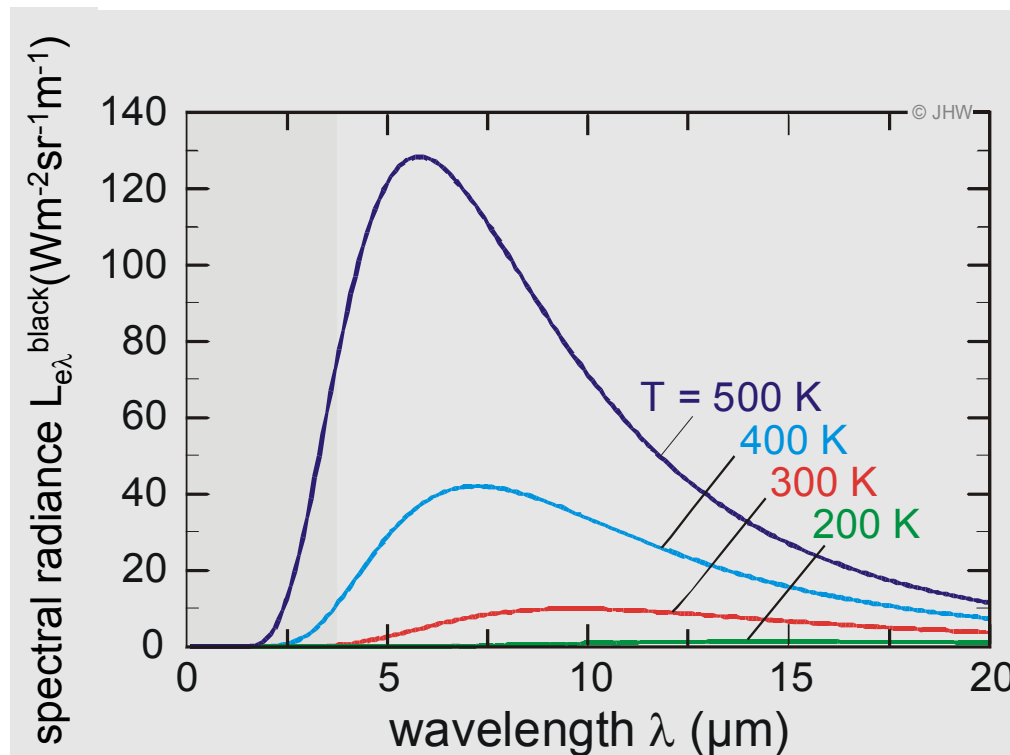


Fig. 4.8: Black body spectrum near room temperature.

A semiconductor with $E_g = 0.31$ eV has the same spectrum as the black body for $\lambda < 4 \mu\text{m}$.

4.4 Emissivity of non-black bodies

$$r_{\phi} + a_{\phi} + t_{\phi} = 1$$

in thermal equilibrium:

$$\text{Emissivity } \varepsilon(\lambda, T) = \text{Absorbance } a_{\phi}(\lambda, T)$$

A black body does not transmit and reflect radiation



$$a_{\phi}(\lambda, T) = \varepsilon(\lambda, T) = 1$$

If a surface is not transparent, $t_{\phi} = 0$,

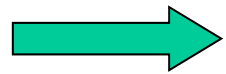


$$a_{\phi}(\lambda, T) = \varepsilon(\lambda, T) = 1 - r_{\phi}(\lambda, T)$$

For partly reflecting surfaces it holds that



$$a_{\phi}(\lambda, T) = \varepsilon(\lambda, T) \leq 1$$



The radiance of real surfaces is always smaller than the radiance of a black body for all temperatures and all wavelengths.



Surfaces with high reflectivity (bright surfaces) emit less radiation at a given temperature than surfaces with high absorption (dark surfaces).

- If the emission coefficient ε of a radiating surface does not depend on wavelength λ , the surface is called a *gray radiator*.
Example: Carbon
- If the emission coefficient ε of a radiating surface depends on wavelength λ , the surface is called a *selective radiator*.
Examples: Tungsten, gas mantle

Wolfram, Glühstrumpf

4.5 Incandescent lamps

Principle of (electric) incandescent lamps:

- Electric current heats a thin filament or thin metal wire to a certain temperature T .
- The higher the temperature, the more radiation is emitted by any body. For a black body, the radiation at the maximum temperature increases with T^5 .
- Efficient light emitters require that the part of *visible* radiation is as high as possible. Due to the highest sensitivity of the eye around 500 nm, the maximum of the emitted radiation has therefore also to be around 500 nm. As a consequence of Wien's displacement law, a black body in the visible needs a temperature around 6000-7000 K. Figure 4.9 gives details.

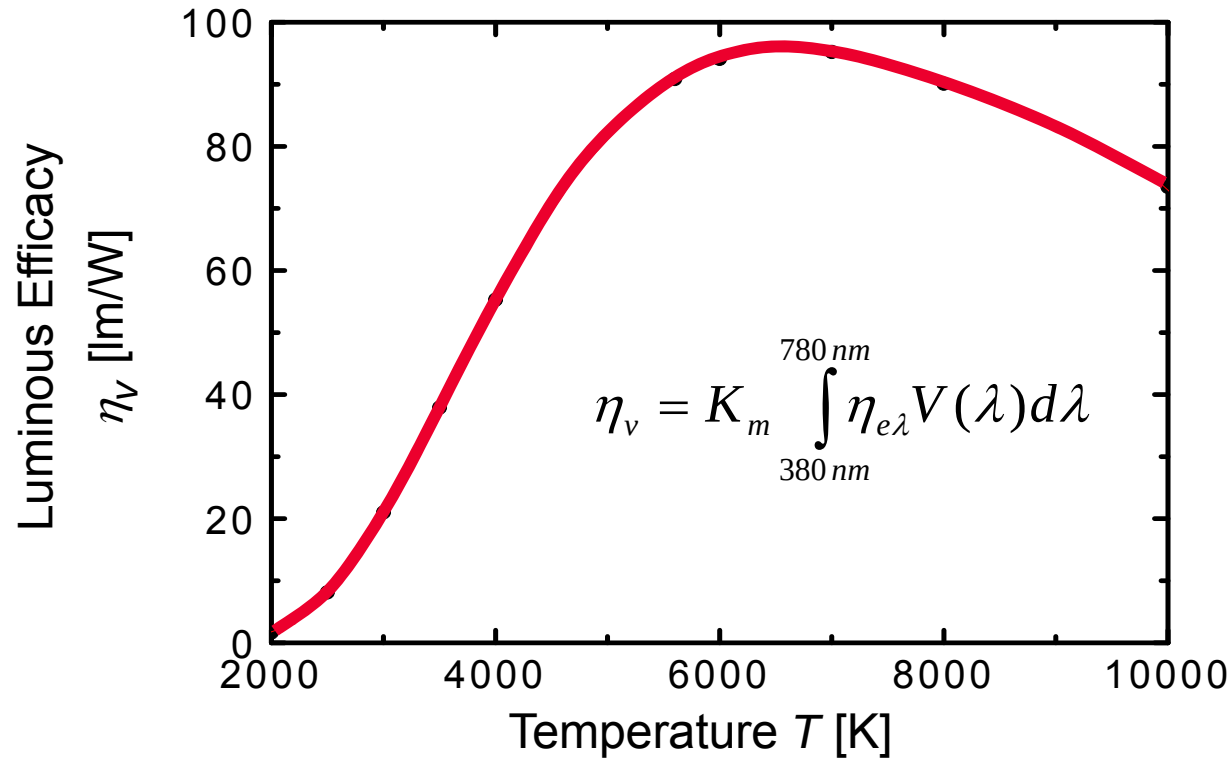


Figure 4.9: The luminous efficacy of a black body is highest between 6000 and 7000 K. For higher temperatures, the black body spectrum shifts too much into the (ultra)violet; for lower temperatures the spectrum shifts too much to the (infra)red. (Consequence of Wien's displacement law).

According to Fig. 4.9, the temperature of a black body should be between 6000 and 7000 K for a high luminous efficacy. Unfortunately, the melting point of all known materials is far below 6000K. Therefore, solid light sources operate always below 6000K.

For temperatures T below $T = 5000$ K the luminous efficacy of a black body increases rapidly with temperature T .

Requirement for efficient (black body) light source:

- high melting temperature
- low vapor pressure

Table 4.1 compares different metals.

Table 4.1: Properties of metals with high melting temperature

Material	Melting point	Vapor pressure at 3000 K
Platinum (Pt) [4.4]	2045 K	
Osmium (Os) [4.5]	3318 K	2.7 Pa
Tantalum (Ta) [4.6]	3269 K	0.05 Pa
Carbon (C) [4.7]	3773 K	192 Pa
Tungsten (W) [4.8]	3683 K	0.009 Pa



Tungsten is best suited material!

Kohlefaden

The first electric incandescent lamps used carbon filaments, because the production of carbon filaments is much less complicated than that of tungsten.

Table 4.2: History of incandescent lamps

Year	Type of lamp	Efficacy	Lifetime
1854	Goebel – Carbon filament lamp	0.8 lm/W	100 h
1879	Edison – Carbon filament lamp	2.1 lm/W	100 h
1881	Edison – Carbon filament lamp	1.6 lm/W	300 h
1882	First carbon filament lamp in Germany	2.8 lm/W	600 h
1902	Auer – Osmium filament lamp	4.4 lm/W	800 h
1905	Bolton – Tantalum filament lamp	3.7 lm/W	800 h
1906	Just und Hanamann – Tungsten filament lamp	5.5 lm/W	800 h
1910	Coiled tungsten wire lamp	6.3 lm/W	1000 h
1913	Gas filled coiled tungsten wire lamp (Argon + Nitrogen)	10.3 lm/W	600 h
1932	Gas filled lamp with double coiled tungsten wire	13.2 lm/W	1000 h
1936	Lamp filled with Krypton instead of Argon	13.9 lm/W	1000 h

Today's incandescent lamps use tungsten wires.

Following Fig. 4.7, the wire temperature should be as high as possible, if tungsten behaved as a black body. The requirement for a temperature as high as possible also is correct for tungsten, however, tungsten *does not behave as a black, but as a selective body*. The absorbance a_Φ = emissivity ε are both smaller than one and depend on wavelength λ (see Fig.4.10).

- In the visible the emissivity $\varepsilon \approx 0.5$ is higher than in the infrared with $\varepsilon \approx 0.2-0.3$. (advantage!)
- The higher the temperature T of tungsten, the more this advantage degrades: For higher T , the emissivity ε becomes smaller in the visible and larger in the infrared regime.



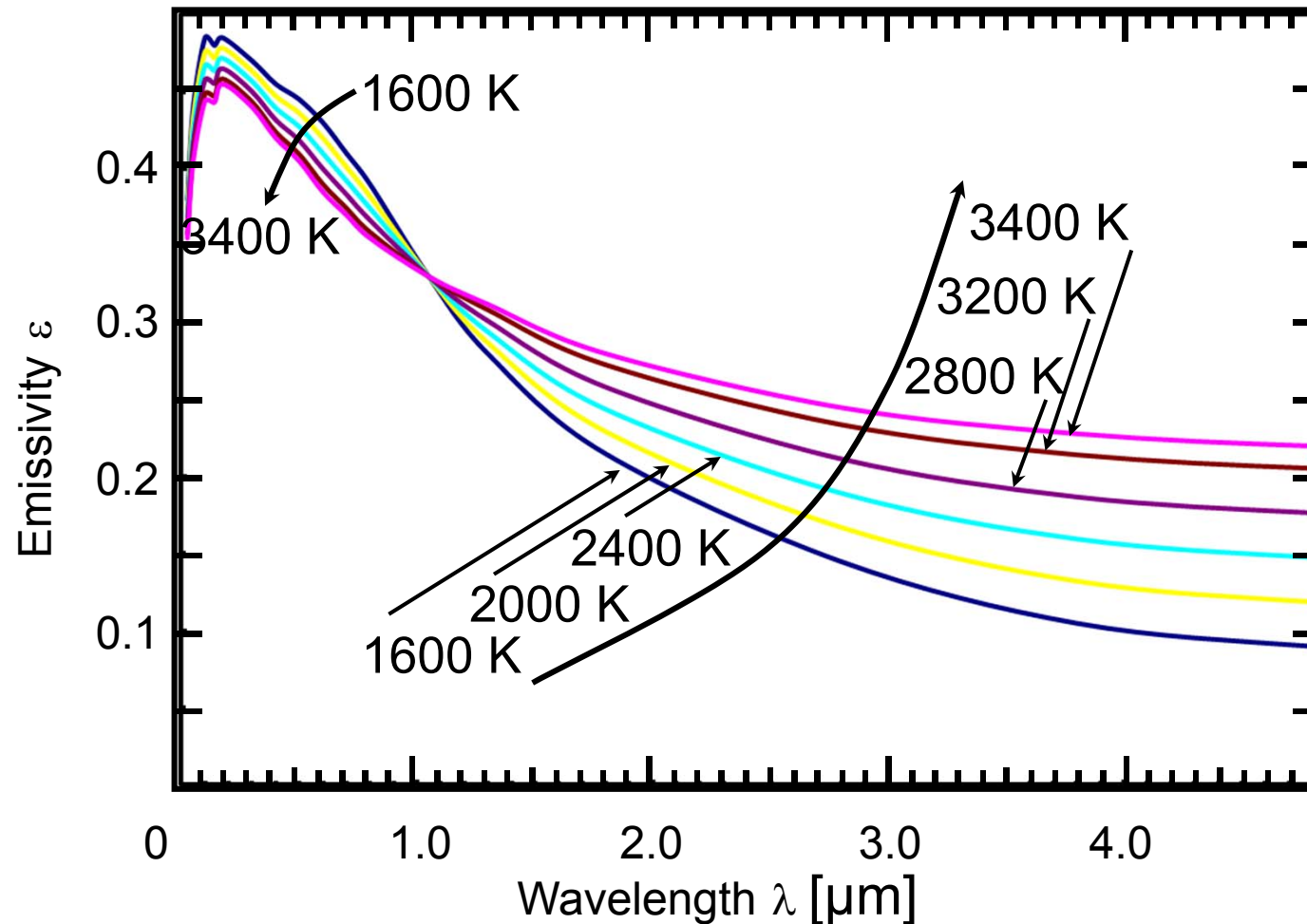


Figure 4.10: Tungsten is of advantage for light generation: The emissivity in the visible ($\lambda = 380 \dots 780 \text{ nm}$) is larger than in the infrared [4.9]. For higher temperatures, part of this advantage is lost.

Due to the wavelength dependence $\varepsilon(\lambda)$ in Fig. 4.10, the luminous efficacy of tungsten is higher than the one of a black body.

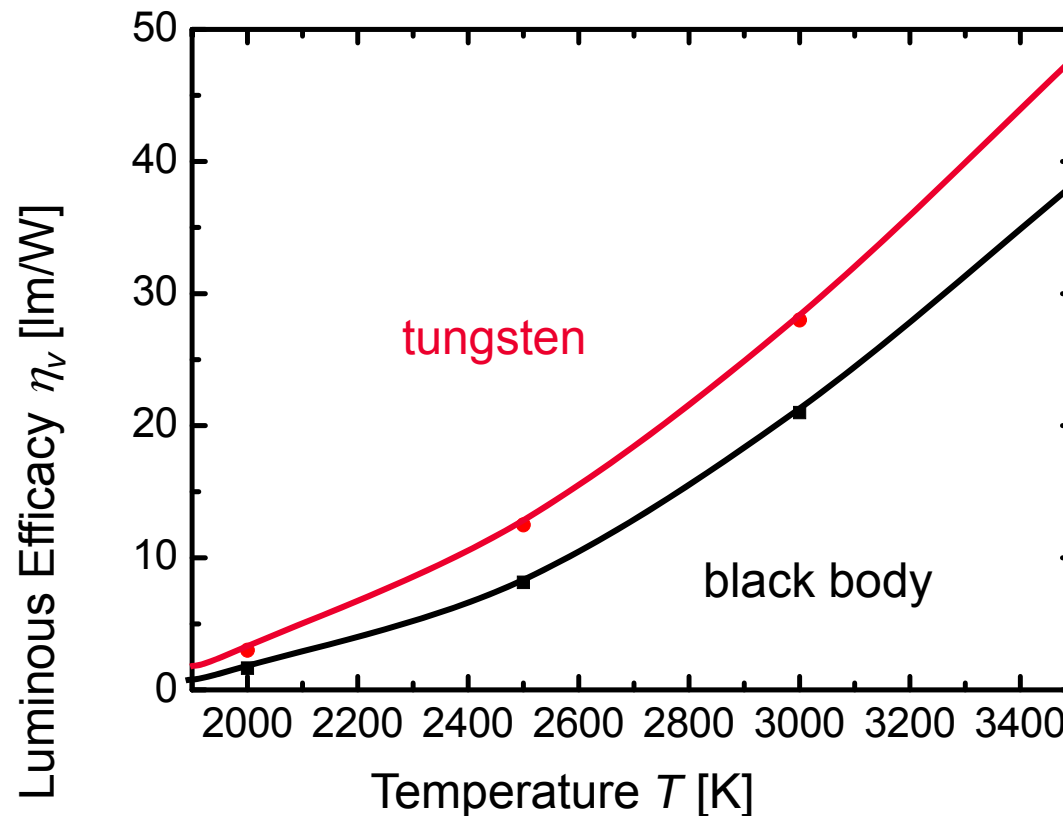


Figure 4.11: For the same temperature, a tungsten wire has a higher luminous efficacy than a black body.

The wavelength dependence $\varepsilon(\lambda)$ in Fig. 4.10 for tungsten leads to an optical efficiency as well as to a visual efficiency which are both higher than for a black body (Figs. 4.12, 4.13).

a) Optical efficiency O :

$$O = \frac{\int_{380nm}^{780nm} \phi_{e\lambda} d\lambda}{\int_0^{\infty} \phi_{e\lambda} d\lambda}$$

= the percentage of light (flux $\phi_{e\lambda}$ emitted between 380 nm and 780 nm) in comparison to the totally emitted radiation (flux $\phi_{e\lambda}$ over all wavelengths)

b) Visual efficiency V :

$$V = \frac{\int_{380nm}^{780nm} \phi_{e\lambda} V(\lambda) d\lambda}{\int_0^{\infty} \phi_{e\lambda} d\lambda}$$

= the percentage of light *weighted with the eye's sensitivity* $V(\lambda)$, in comparison to the totally emitted radiation

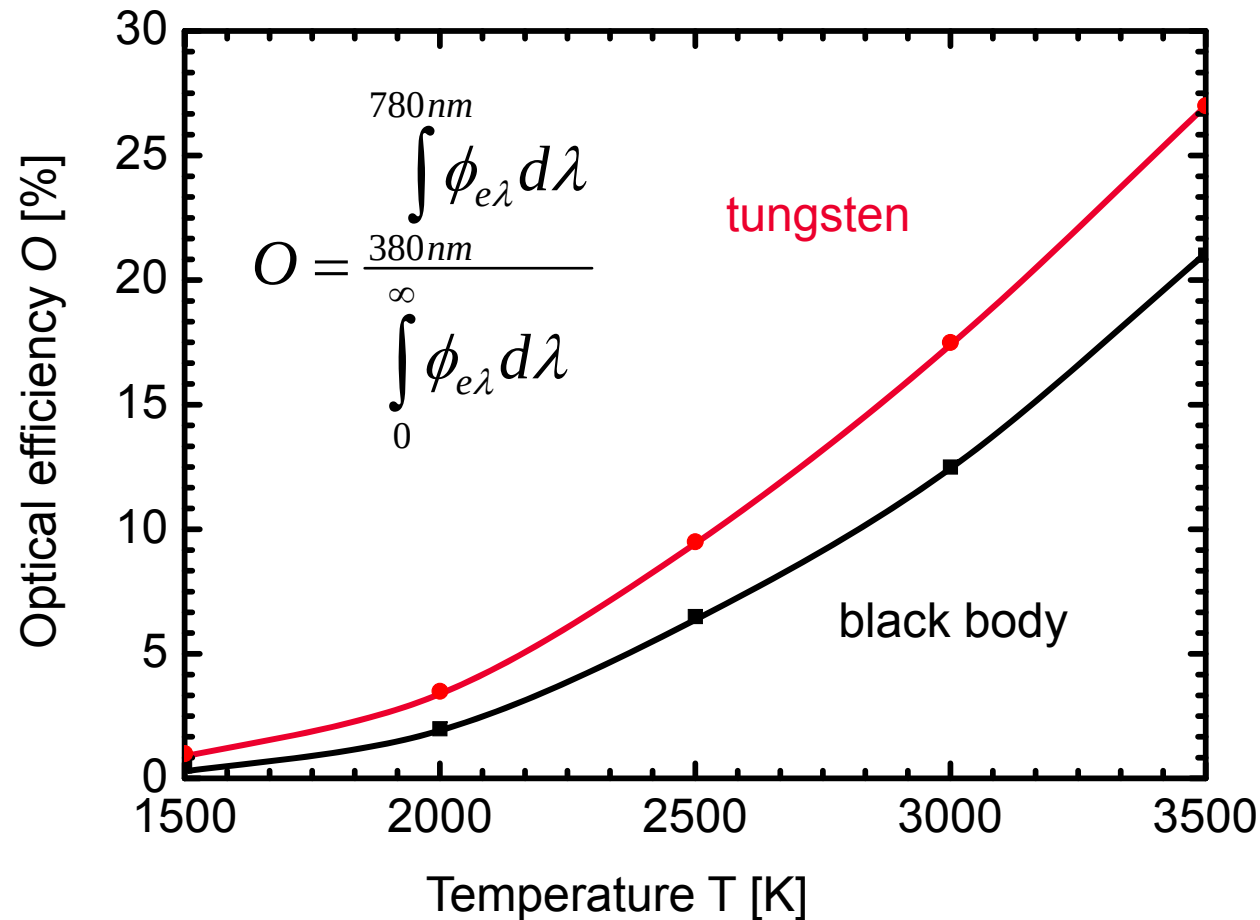


Figure 4.12: For $T = 3000$ K, tungsten emits 17 % of its radiation in the visible; a black body emits only 12 % in the same regime.

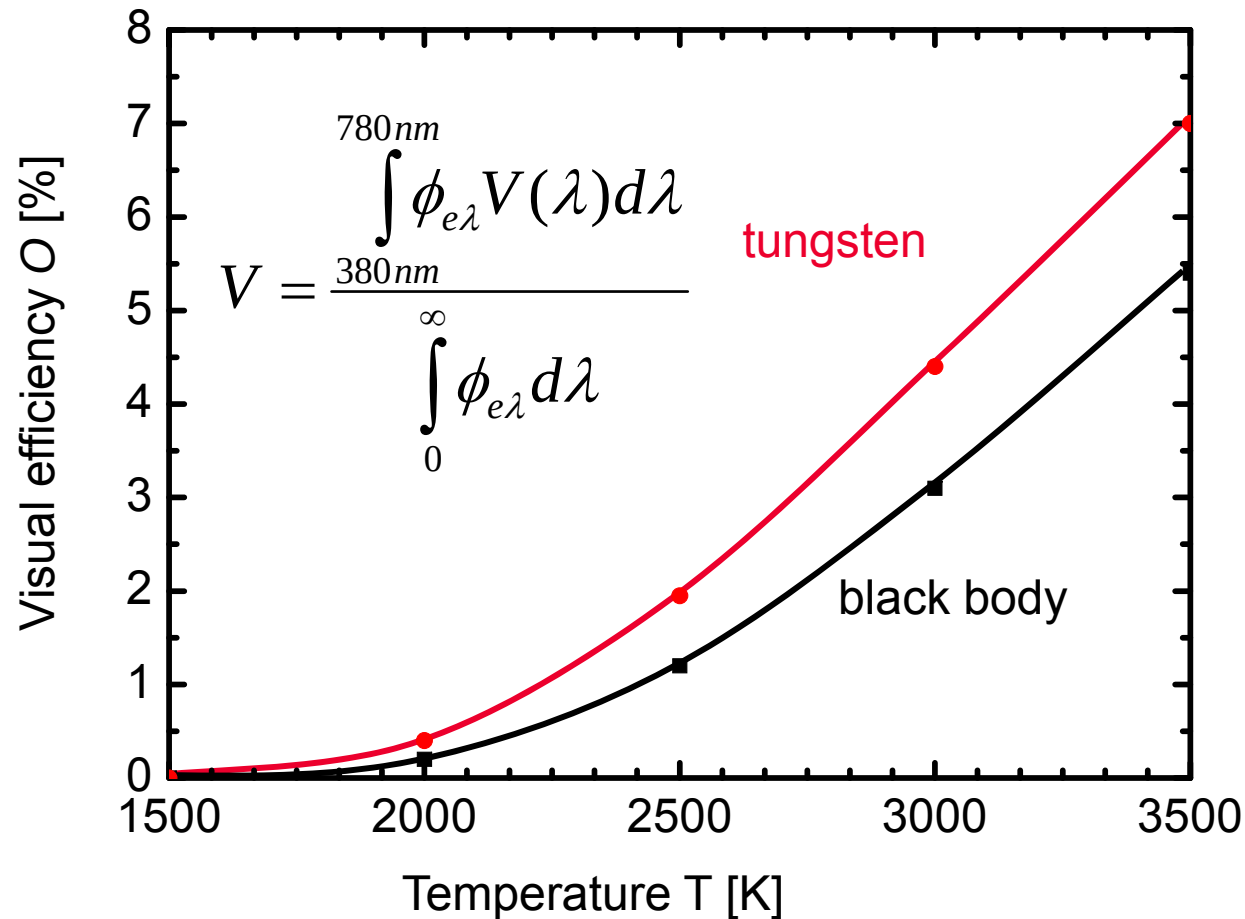


Figure 4.13: For $T = 3000$ K, the human eye detects only 4.5 % of the total radiation from a tungsten wire, and only 3 % from a black body.

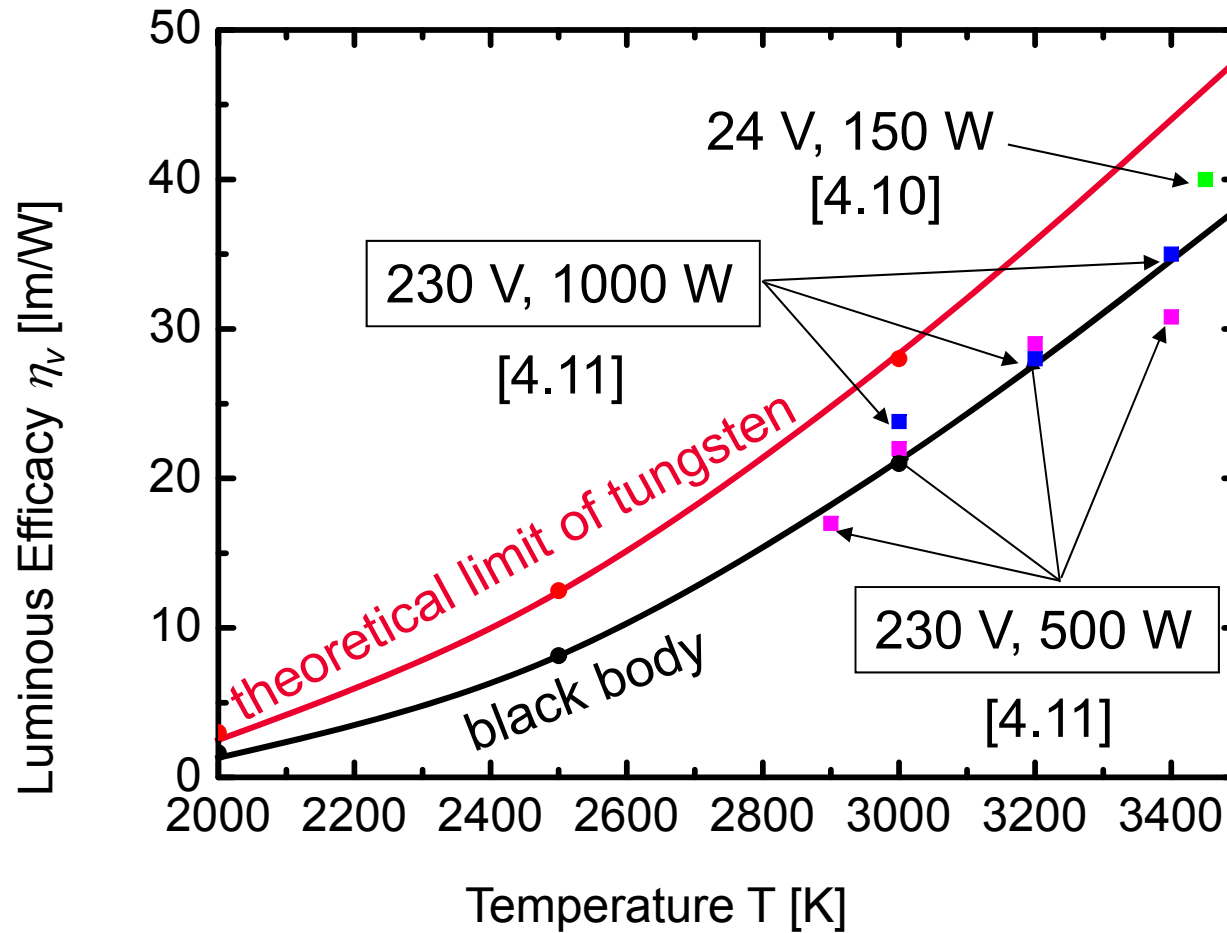
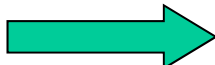


Fig 4.14: Measured efficacies of tungsten incandescent lamps for different filament temperatures.

A) Tungsten filament

- $T_{tungsten} < T_{melt.}; \quad T_{melt} = 3683 \text{ K}$
- High tungsten wire temperatures
  high luminous efficacy η_v
- High tungsten wire temperatures
  low lifetime



Lifetime τ decreases drastically with increasing luminous efficacy!

Empirical law

$$\tau \eta_v^{10} = \text{const.}$$

Table 4.3 gives information about wire temperatures of different types of tungsten incandescent lamps.

Table 4.3: Wire temperatures of tungsten incandescent lamps

Type of lamp	Wire temperature [K]
Vacuum lamp	2200 – 2700
Gas filled lamp	2600 – 3000
Halogen lamp	2900 – 3450

Table 4.4: Efficacy and lifetime of tungsten incandescent lamps

Type of lamp	Luminous efficacy	Lifetime
Standard lamp 15 W	6 lm/W	1000 h
60 W	12.2 lm/W	1000 h
100 W	13.8 lm/W	1000 h
200 W	15.8 lm/W	1000 h
Krypton filled 60 W	13.0 lm/W	1000 h
Krypton filled 100 W	14.7 lm/W	1000 h
Halogen lamp 230 V, 60 W	14.0 lm/W	2000 h
Halogen lamp 230 V, 100 W	16.0 lm/W	2000 h
Halogen lamp 230 V, 250 W	21.0 lm/W	2000 h
Halogen lamp 230 V, 2000 W	22.0 lm/W	2000 h
Halogen lamp with IR reflecting coating, 230 V, 400 W	23.8 lm/W	3000 h
Halogen lamp, 36 V, 400 W	30.5 lm/W	300 h
Halogen lamp, 36 V, 400 W	36.3 lm/W	150 h
Halogen lamp, 36 V, 400 W	40.0 lm/W	50 h

B) Evaporation of tungsten filament

Why does the filament burn out?

General considerations

- Electrical power dissipated in the filament is

$$P_{el} = U^2/R,$$

U = applied voltage, R = resistance of the wire

$$R \sim l / d^2$$

d = diameter, l = length of tungsten wire

$U, l = \text{const.}$  $P_{el} \sim d^2,$

- Power of emitted radiation P_{rad} is proportional to radiating surface A of the wire

$$P_{rad} \sim A, \quad A \sim d.$$

 $P_{rad} \sim d$

©JHW



P_{el} decreases more rapid than emitted radiation power P_{rad} if diameter of the wire decreases via evaporation:

- ➡ Temperature of the wire must decrease!
- ➡ Evaporation rate decreases!
- ➡ Lifetime of the wire should be infinite!

What is the reason for lamp failure?

Assume *imperfect* tungsten wire, illustrated in Fig. 4.14:

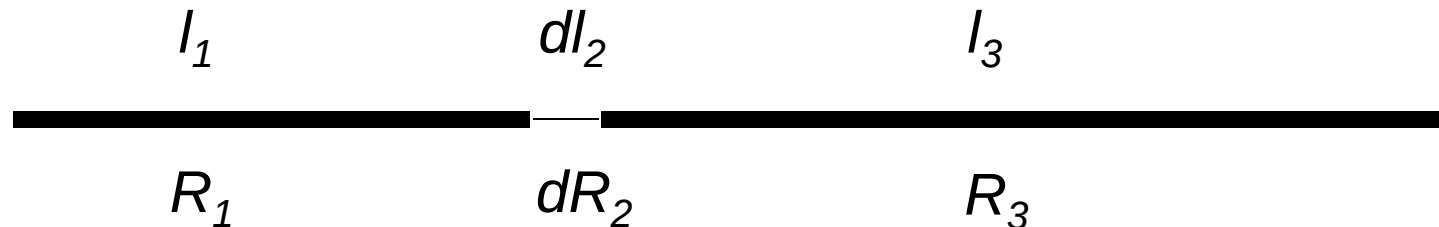


Figure 4.14: Tungsten wire with constant resistance per unit length: $R_1 / l_1 = R_3 / l_3$, but with a small part of length dl_2 with higher resistance per unit length: $dR_2 / dl_2 > R_1 / l_1$.

Total resistance R_{tot} of wire does not depend on resistance per unit length dR_2 / dl_2 as long as $dl_2 \ll l_1 + l_3$

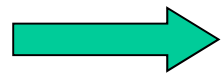
$$R_{tot} = R_1 + dR_2 + R_3 = R_1 + R_3$$

Electric current I is constant, even if resistance dR_2 increases by evaporation

$$I = U / R_{tot}$$

Dissipated electrical power per unit length dl of wire dP_{el} / dl is then given by electric current I :

$$dP_{el} / dl = I^2 dR / dl$$



Small part of wire with higher resistance per unit length dissipates more electrical power per unit length than rest of wire.

- ➡ Small part of the wire evaporates more rapid than rest of wire and thus becomes thinner.
- ➡ Total resistance does not increase significantly, electric current stays constant!
- ➡ Dissipated electric power in small part increases more and more until the wire melts and burns out at the so-called “*hot-spot*”.

Consequence

“Perfectly” manufactured wires have longer lifetimes: Variation of wire thickness has to be avoided!

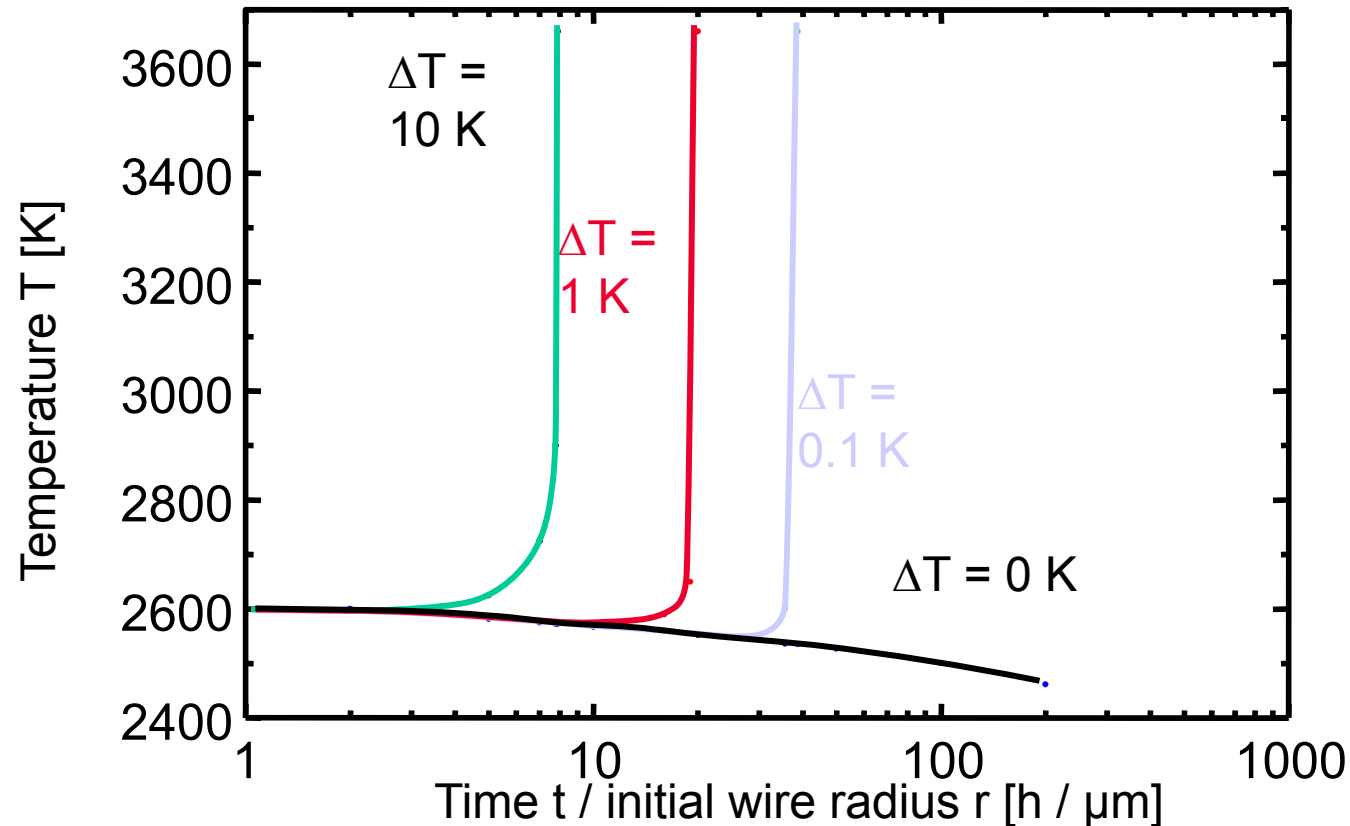


Figure 4.15: Calculated time dependent temperature evolution in a hot-spot. ΔT is *initial* temperature difference between hot-spot and wire temperature: The cooler the hot-spot, the longer the lifetime. Lifetime t is proportional to wire radius r ($t / r = \text{const.}$): The thicker the wire, the longer the lifetime.

Evaporation rate decreases if wire is operated in inert gas.

Reason:

Viscosity η_{gas} of any gas increases with temperature according to:

$$\eta_{\text{gas}} \sim T^{1/2}$$

If tungsten wire is surrounded by a medium of *high viscosity*, then diffusion of evaporated tungsten atoms through the medium is hindered.



Probability, that a tungsten atom returns back to tungsten wire instead of drifting apart increases.

High gas pressure and inert gases with high atomic or molecular mass m_a result in higher viscosity.



Use: Nitrogen (N_2 , $m_a = 28$) , Argon (Ar, $m_a = 40$), Krypton (Kr, $m_a = 84$) or Xenon (Xe, $m_a = 131$)

expensive

more expensive

and high gas pressure.



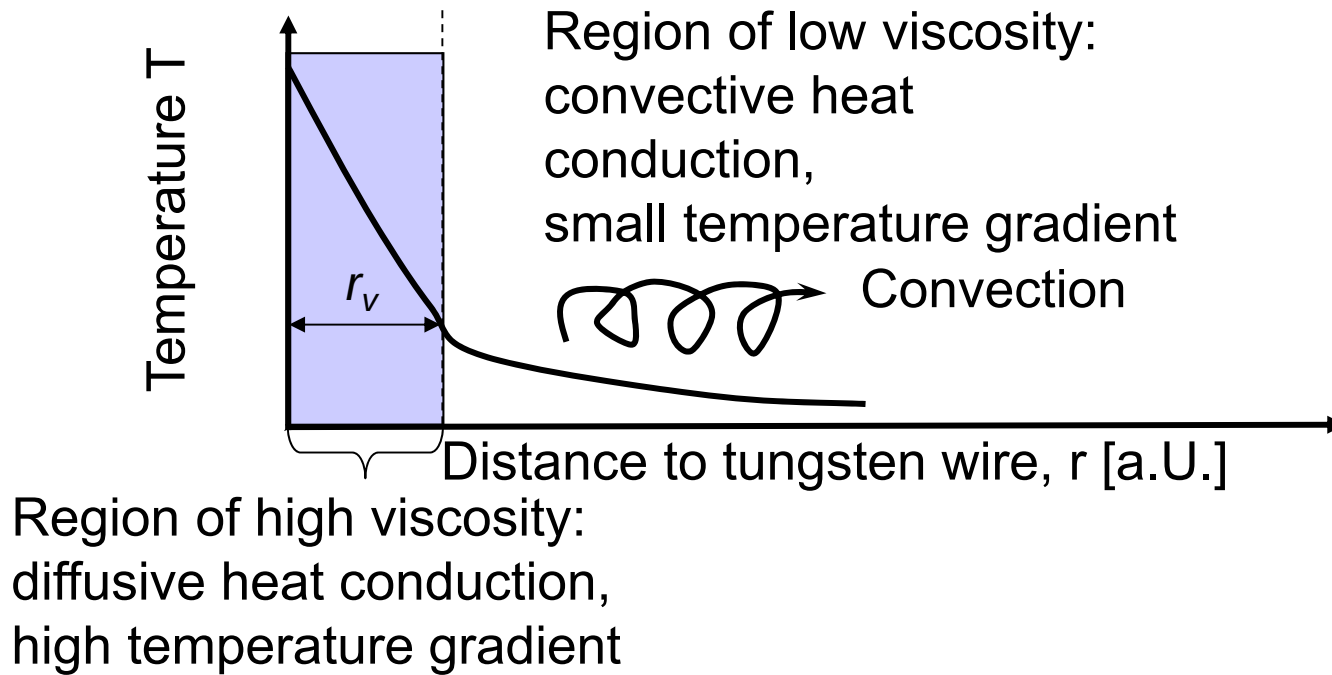


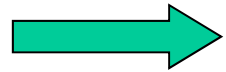
Figure 4.16: Energy or heat loss (cooling of wire) governed by diffusive heat conduction in high viscous layer around wire: radius of high viscous layer $r_v \gg r$.

- ➡ Heat loss of wire does not depend on wire diameter!
- ➡ Increase wire diameter to reduce heat loss.
- ➡ Use a coiled wire, which has a larger *effective* diameter.

How to increase efficiency of tungsten incandescent lamps?

Summary:

- Use tungsten wire of constant thickness
- Use a coiled tungsten wire
- Use inert gas of high molecular or atomic mass
- Fill glass bulb with high pressure



Problem: mechanical stability of glass bulb if filled with high pressure: cracking, explosion.

Solution: use very tiny glass bulbs with thick walls.



Problem: evaporated tungsten causes blackening of small bulb surface.

Solution:



C) Halogen incandescent lamps

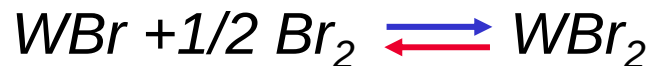
Bromine filling avoids blackening of the glass bulb by chemical reactions of tungsten atoms and bromine.

Simplified reaction kinetics:



—→ Direction at low temperatures

—→ Direction at high temperatures



Reactions takes place at higher temperatures ($T > 2000$ K) close to tungsten wire. At high temperatures (≈ 2900 K, red arrow) WBr molecules dissociate to elements bromine and tungsten.

Reaction takes place at medium temperatures $1000 \text{ K} < T < 2000 \text{ K}$ between tungsten wire and wall.

$WBr_2 + Br_2 \rightleftharpoons WBr_4$ Reaction takes place at low temperatures, $T \approx 900$ K, close to bulb wall.

During operation temperature of bulb wall must exceed 800 K.

➡ Vapor pressure of WBr_4 at temperatures $T > 800$ K exceeds partial pressure.

➡ WBr_4 stays in gaseous phase and blackening of the inner bulb surface is avoided.

Dissociation of WBr molecules to bromine and tungsten does not take place at hot-spots, but at regions with much lower temperature (below 2900 K).



➡ Lifetime of a lamp is practically the same than that of a lamp without halogen.



Halogen lamps are filled with a few millibars (mbar) of the bromine and up to 6 bars of either *Kr* (Krypton) or *Xe* (Xenon).

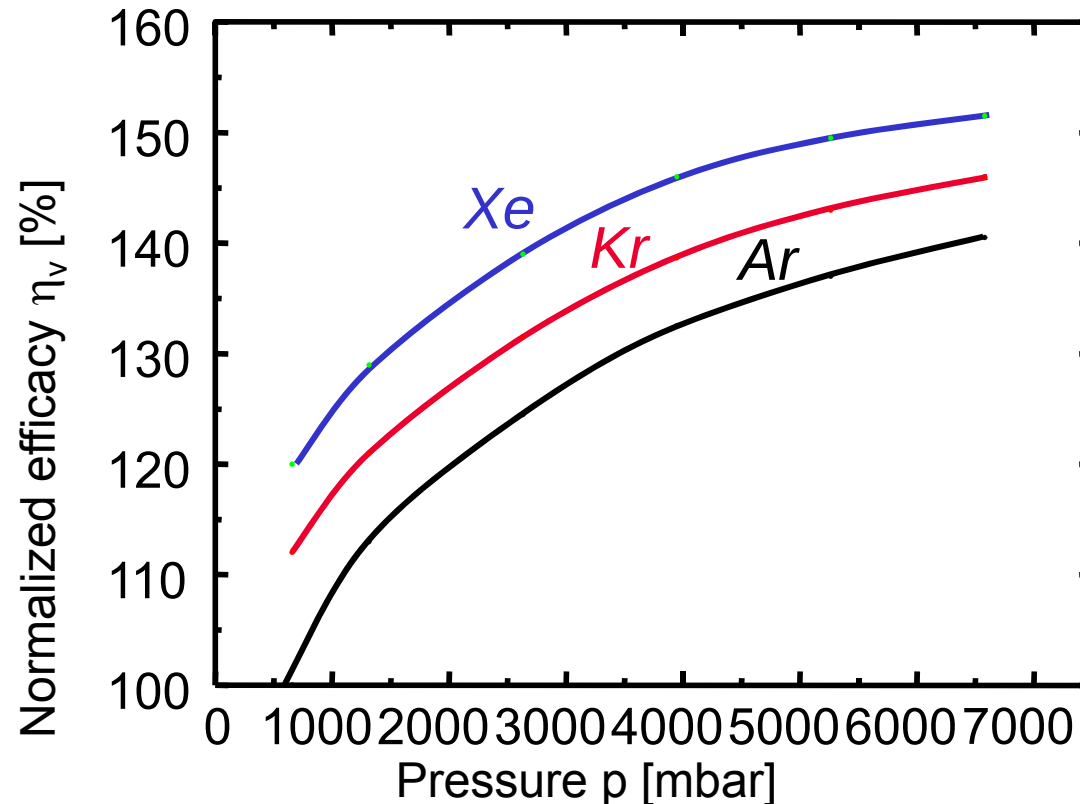


Figure 4.17: Efficacy of halogen lamps for *constant lifetimes* increases with with filling pressure. The higher molecular mass of *Xe* compared to *Kr* or *Ar* increases the efficacy significantly.

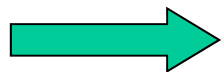
Summary:

- Efficacy increases with wattage of lamp: diameter of tungsten wire increases, with the result of higher temperatures at constant lifetimes.
- Gas fillings with *Kr* instead of *Ar* increase the efficacy.
- Halogen lamps are fabricated using very small and pressure resistant housings with thick glass walls.
- They are filled with up to 6 bars of *Kr* or *Xe*
- Bromine avoids blackening of the small glass bulb, which would become rapidly opaque without the halogen.
- Halogen lamps are operated at wire temperatures up to 400°C hotter than those of standard lamps. Efficacy increases by about 30 %.
- Highest efficacy of commercial halogen lamps is 40 lm/W at a lifetime of 50 h.

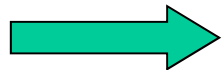


Future developments:

Infrared (IR) reflecting coating of the glass bulb reduces radiation losses, by reflecting the “waste” IR-radiation back onto the tungsten wire surface. By this means part of the electrical heating of the tungsten wire can be substituted by IR-absorption heating.



Increase of luminous efficacy by 30%



Prototypes: $\eta_V > 40 \text{ lm/W}$, with lifetime $\tau > 1000 \text{ h}$

Problems: Coating technology not ready for mass production!
Price too high!



4.6 Gas discharge light sources

Applications:

- Industry lighting
- Street lighting
- House lighting
- Car lighting

Advantages:

- Lifetime several 10^4 h
- Luminous efficacy up to 200 lm/W

Disadvantages:

- Expensive
- Low color rendering index at high luminous efficacies

Farbwiedergabeindex



4.6.1 Basics of gas discharges

Gases do not conduct electric current at

- room temperature,
- ambient pressure,
- low electric field $E < 1 \text{ kV/mm}$

If certain voltage U_B is applied between two electrodes gases “break down” and become conductive.

Breakdown voltage U_B decreases for

- low gas pressure (down to a certain value, then it increases again. Example: vacuum)
- increasing temperature
- decreasing electrode spacing: increasing electric field

Breakdown occurs within a time interval $\Delta\tau$, where

$$10 \text{ ms} > \Delta\tau > 0.1 \text{ ns}.$$



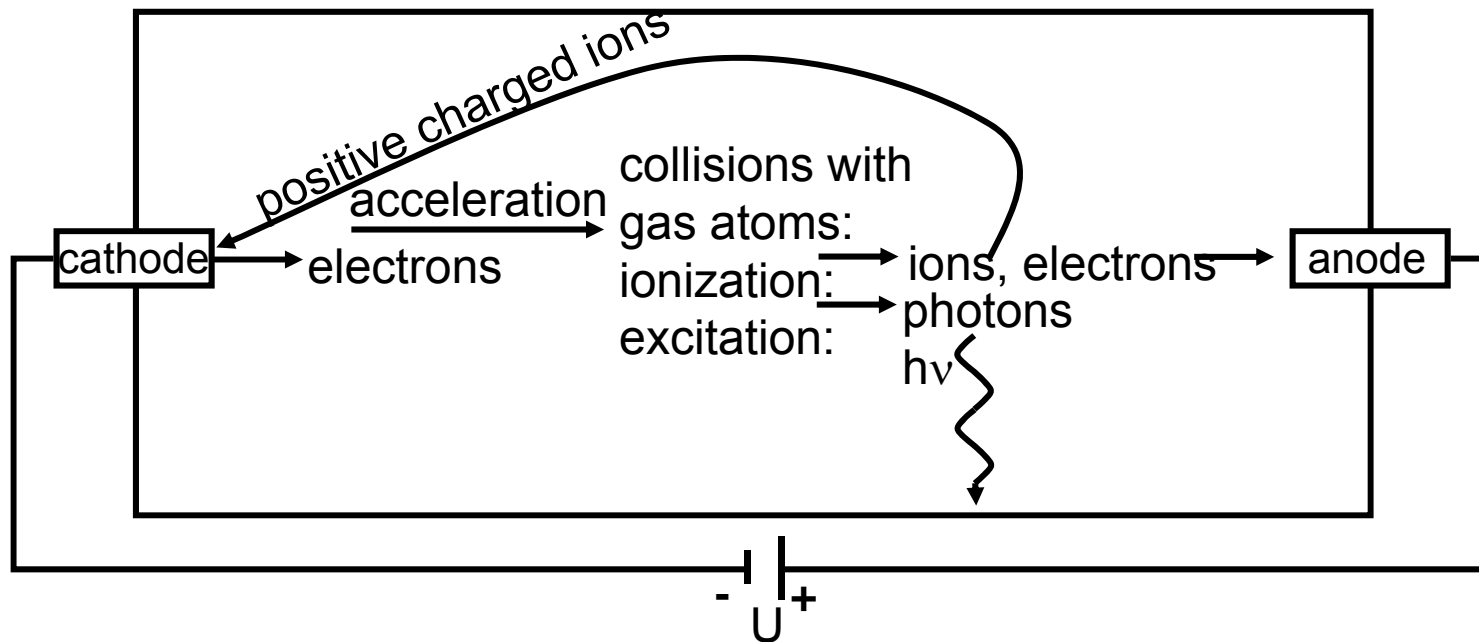


Figure 4.18: Elementary processes in an electric discharge *after breakdown*: cathode emits electrons, electric field accelerates electrons, electrons with sufficient energy collide inelastically with atoms, and ionize or transfer them to an excited state. Ionization leads to electron multiplication and (positive) ions, which drift back to the cathode. Excited atoms return to the ground state by emitting a photon.

A) Emission of electrons at the cathode

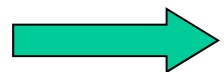
Thermionic emission of electrons from the cathode is given, according to Richardson, by

$$j_s = A T^2 \exp(-q\phi_a/kT).$$

With ϕ_a = work function of the cathode, k = Boltzmann's constant, T = cathode temperature, and Richardson's constant

$$A = 4 \pi e m_e k^2/h^3 = 120 \text{ A/(cm}^2 \text{ K}^2),$$

where, e = charge of electron, m_e = mass of electron, h = Planck's constant.



High temperatures T and low work function ϕ_a increase electron current density j_s .

Discharges with thermionic electron emission are called *arc discharges*. Bogenentladung

The γ -effect

describes the probability γ , that one positive ion impinging the cathode surface liberates one electron.

Coefficient γ depends on the gas filling and especially on the cathode material, but is in general very low,

$$\gamma < 1\%.$$

in front of cathode: ion current $\gg$ electron current

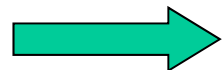
Consequences:



Ion density $\gg$ electron density



Positive space charge in front of cathode



Increased electric field in front of cathode





Main part of voltage drop ΔU between cathode and anode occurs close to the cathode

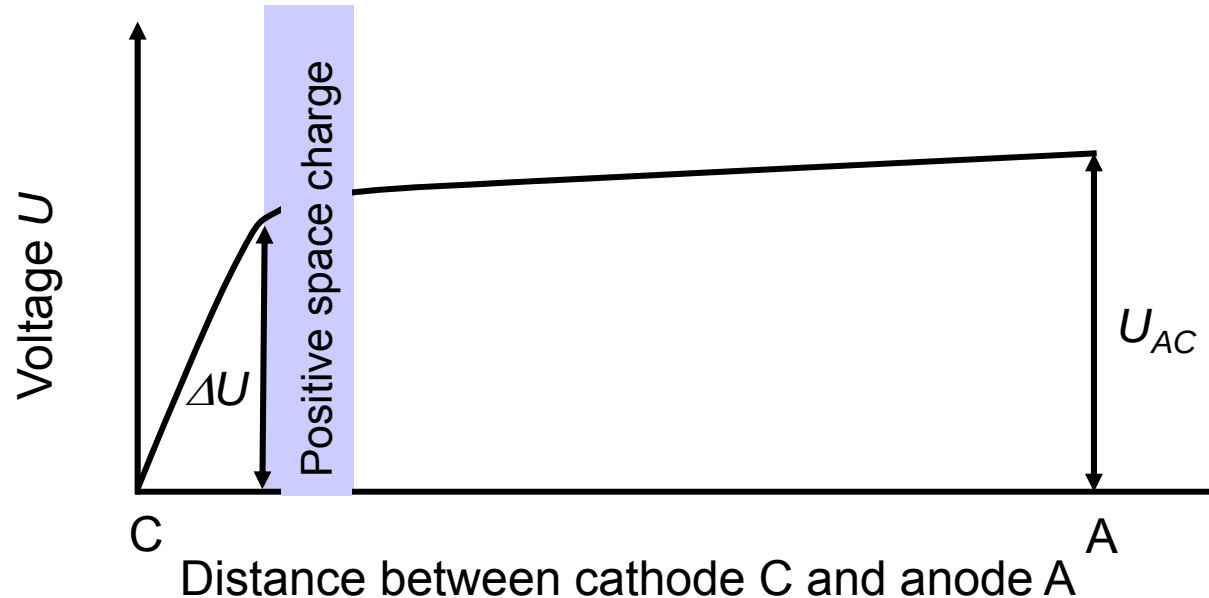
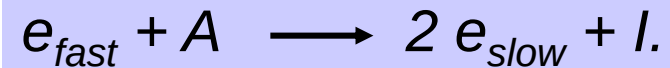


Figure 4.19: Electrons are accelerated in high electric field between cathode C and positive space charge region, where they obtain sufficient energy to ionize atoms, which built up the space charge. The ions gain energy while drifting in the electric field towards the cathode, where they can liberate electrons from the cathode.

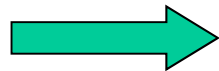
Discharges where electrons are emitted via γ -effect are called *glow discharges*. **Glimmentladung**

B) Processes between cathode and anode

Accelerated electrons e_{fast} ionize atoms A , ions I are generated,

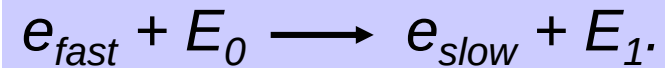


Electron density increases.



In case of glow discharge: required positive ions are produced.

Accelerated electrons e_{fast} transfer atoms from the ground state of energy E_0 to an excited state of energy E_1



Excited atoms return to the ground state by emitting a photon of energy $h\nu = E_1 - E_0$, see Fig. 4.20.

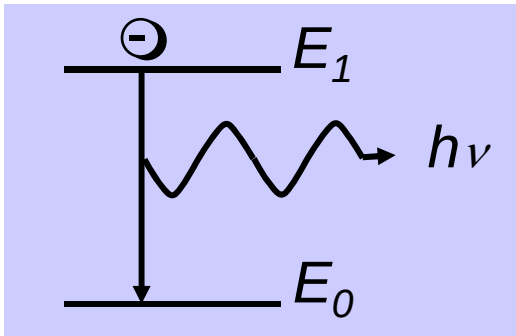


Figure 4.20: Excited atom in state E_1 returns to the ground state E_0 and emits photon of energy $h\nu = E_1 - E_0$.

C) What happens at the anode?

Electrons enter the crystalline lattice of the metallic anode. Here they release their potential energy, defined by the work function Φ_a of the metal. “Eintrittsarbeit”

Figure 4.21 illustrates the effect.



Electrons heat anode.

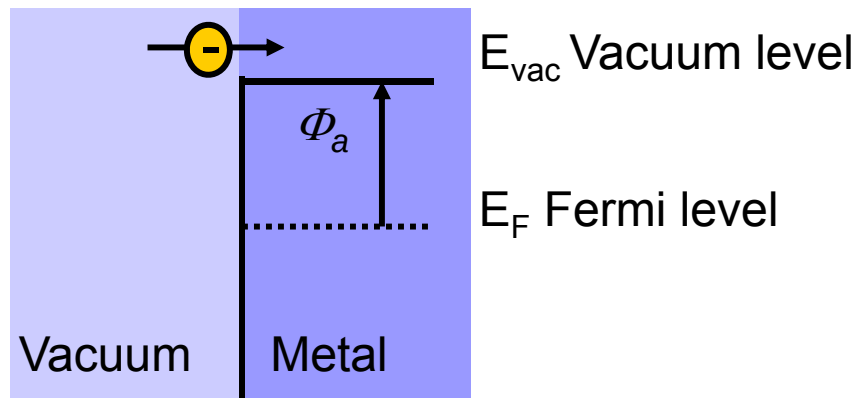


Figure 4.21: Electron enters the anode and releases energy $E_{anode} = e \Phi_a$.

D) Low pressure discharges

Characterized by:

- Gas pressure $p \lesssim 10$ mbar
- Discharge current $I < 100$ mA
- Discharge diameter limited by discharge tube diameter
- Temperature of electrons $T_e \gg$ Gas temperature T_{gas} ,

$$T_e \approx 10 \text{ eV } (\cong 10^5 \text{ K}); T_{gas} \approx 300 - 500 \text{ K}$$

E) High pressure gas discharges

Characterized by:

- Gas or vapor pressure $1 \text{ bar} < p < 100 \text{ bar}$
- Discharge current $1 \text{ A} < I < 100 \text{ kA}$
- Temperature of electrons $T_e \approx$ Gas temperature T_{gas}

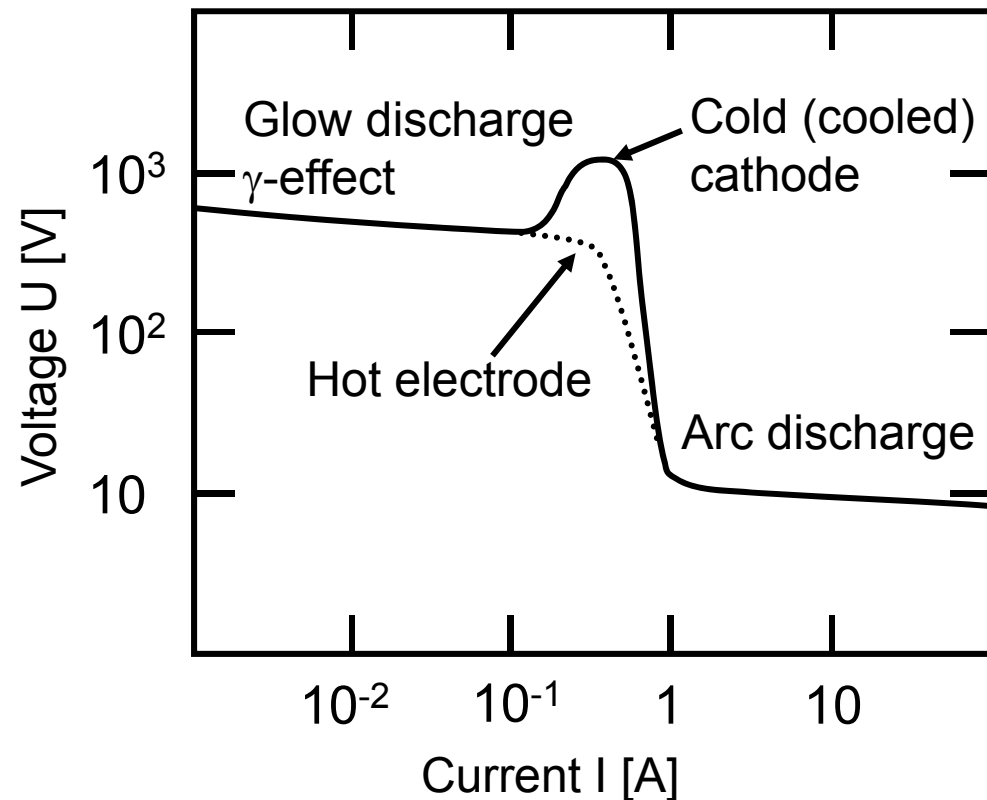
$$T_e \approx T_{gas} \approx 5000 \text{ K}$$



F) I/U characteristic of discharges

Figure 4.22 shows typical I/U characteristic of electric discharge.

Figure 4.22: At low discharge currents I the applied voltage is mainly determined by the cathode voltage drop (~ 400 V). Increasing current leads to cathode heating (dotted curve): Glow discharge transfers to arc discharge. Intensive cathode cooling maintains glow discharge.



for higher discharge currents at the expense of increasing voltage (solid curve).

$dU/dI < 0$  Resistor (dc-voltage), inductor, choking coil (ac-voltage) needed to limit discharge current!

4.6.2 Low pressure gas discharge lamps

A) Fluorescent lamp Leuchtstofflampe

Principle:

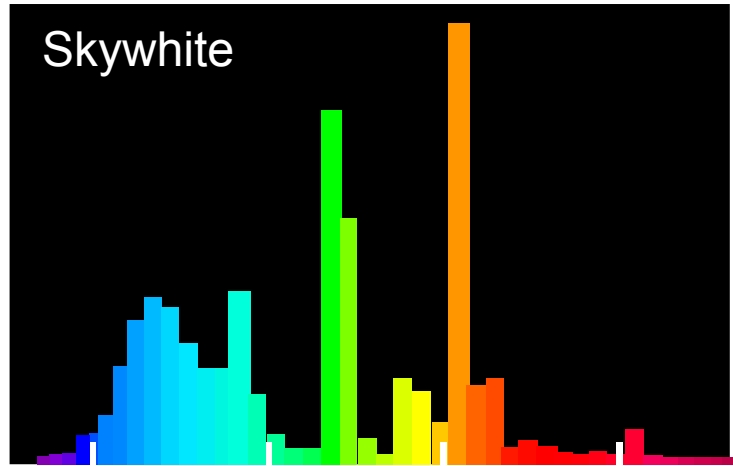
- Filled with 4 mbar argon (*Ar*) and mercury (*Hg*) or (since a few years) with mercury amalgam (not toxic, fully recyclable).

- Operation temperature 40 °C results in mercury vapor pressure $p_{Hg} = 6 \times 10^{-3}$ Torr.
- Low pressure arc discharge excites *Hg*-atoms
- Argon only needed to match impedance of discharge and to enable ignition (break down) of cold lamp.
- Excited *Hg*-atoms emit *invisible* UV-radiation mainly at 253.7 nm (80 %) and 185 nm (15 %).
- Phosphor coating of glass tube transfers UV radiation into visible radiation.

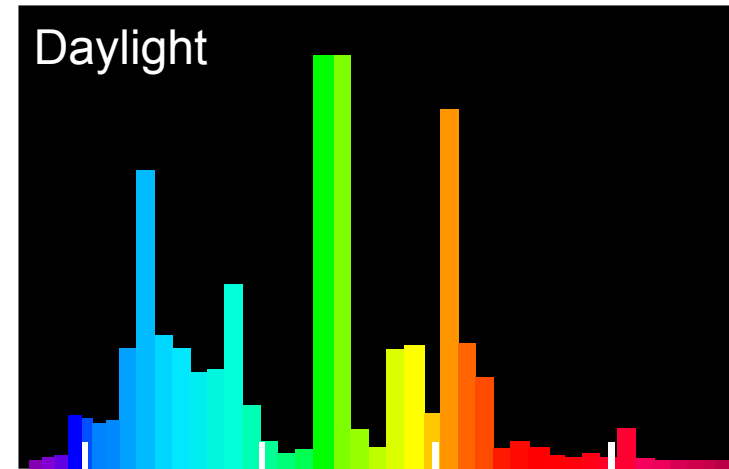
Table 4.5 gives examples of high efficient phosphors with quantum efficiencies up to 80%.

Host material	Doping	Emission color
Y_2O_3	Eu^{3+}	red
$\text{MgAl}_{11}\text{O}_{19}$	Tb^{3+} , Ce^{3+}	green
$\text{Mg}_2\text{Al}_{16}\text{O}_{27}$	Ba^{2+} , Eu^{3+}	blue

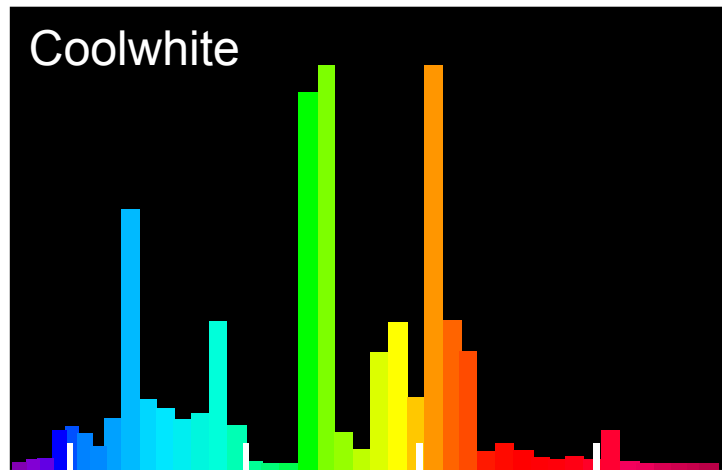
Comparison of Fig. 4.23 and 4.24 illustrates the dependence of luminous efficacy on the color rendering index R_a . High efficient rare earth doped phosphors have good but not excellent color rendering properties. The so-called DE LUXE phosphors show excellent color rendering properties but a much lower luminous efficacy.



400 nm 700 nm
 $T_C = 8000 \text{ K}$, $R_a = 86$, $\eta_V = 98 \text{ lm/W}$

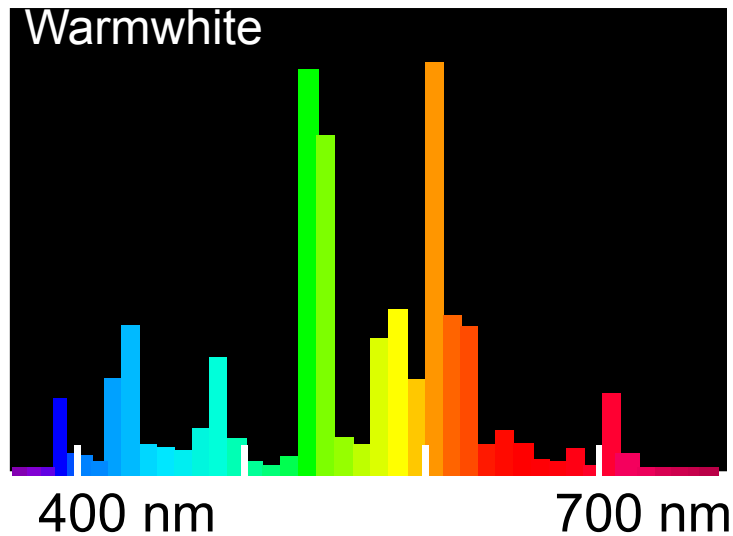


400 nm 700 nm
 $T_C = 6000 \text{ K}$, $R_a = 86$, $\eta_V = 98 \text{ lm/W}$

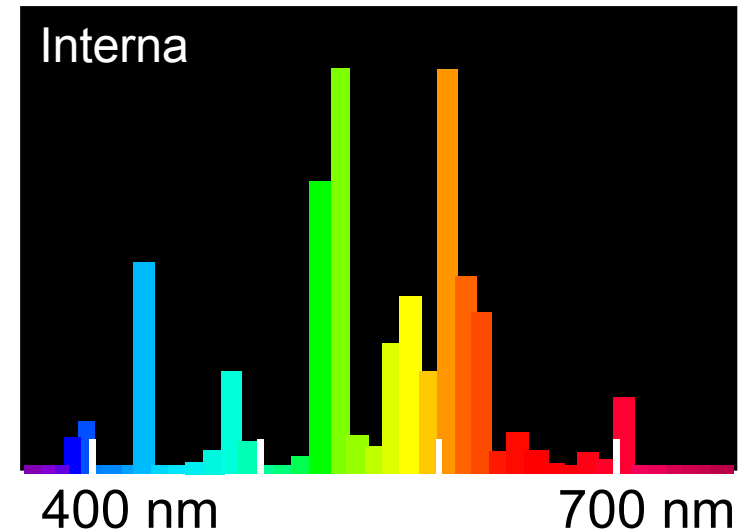


400 nm 700 nm
 $T_C = 4000 \text{ K}$, $R_a = 86$, $\eta_V = 103 \text{ lm/W}$

Figure 4.23: Spectral distribution of high efficient rare earth doped phosphors with *good* color rendering properties, color rendering index $R_a = 86$, with color temperature T_C and luminous efficacy η_V [4.12].

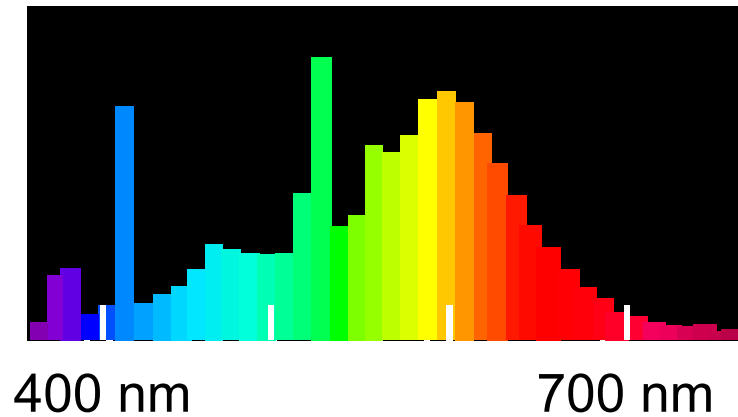


$T_C = 3000 \text{ K}$, $R_a > 86$,
 $\eta_V = 103 \text{ lm/W}$

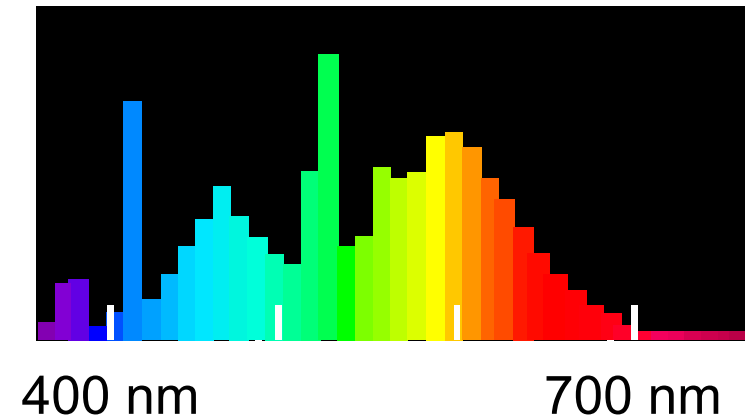


$T_C = 2700 \text{ K}$, $R_a > 86$,
 $\eta_V = 103 \text{ lm/W}$

Figure 4.24: Spectral distribution of high efficient rare earth doped phosphors with *good* color rendering properties, color rendering index $R_a = 86$, with color temperature T_C and luminous efficacy η_V [4.12].



$T_C = 3000 \text{ K}$, $R_a = 98$,
 $\eta_V = 65 \text{ lm/W}$



$T_C = 4000 \text{ K}$, $R_a = 98$,
 $\eta_V = 65 \text{ lm/W}$

Figure 4.25: Spectral distribution of DE LUXE phosphors with *excellent* color rendering properties, color rendering index $R_a = 98$, but *lower* luminous efficacy η_V , with color temperature T_C [4.13].

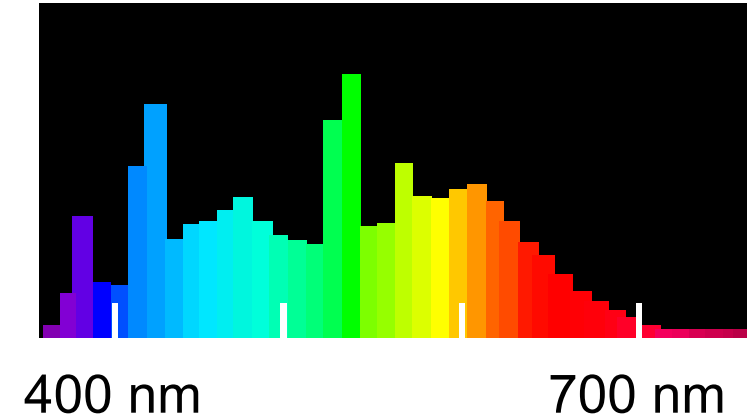
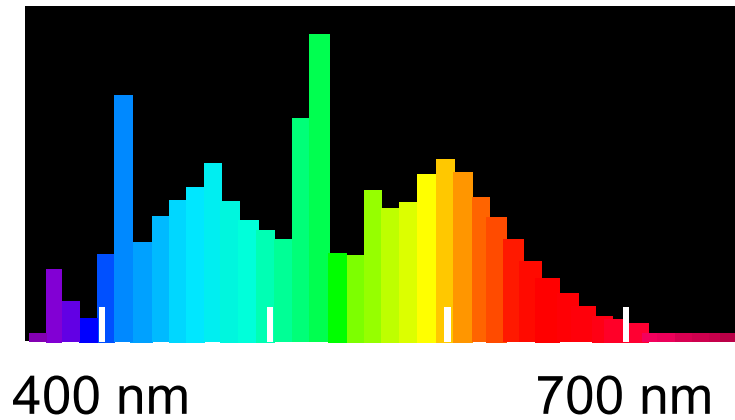


Figure 4.26: Spectral distribution of DE LUXE phosphors with *excellent* color rendering properties, color rendering index $R_a = 98$, but *lower* luminous efficacy η_v , with color temperature T_c [4.13].

Different mixture compositions of the phosphors allow to vary the color temperature T_c of the emitted light in a wide range between the color temperature $T_c = 8000$ K of sky light, and the color temperature $T_c = 2700$ K of an incandescent lamp.

Properties of fluorescent lamps:

- ac-operation of lamp: both electrodes act periodically as cathode and anode.
- Ignition is provided by ohmic heating of electrodes together with a high voltage pulse generated by a starter.
- Electrodes consist of tungsten wire coated with a thin layer of *BaO*, *SrO*, and *CaO* mixture.
- Coating reduces work function and improves thermionic emission of electrons.
- Tungsten wire of electrodes is contacted on both sides.



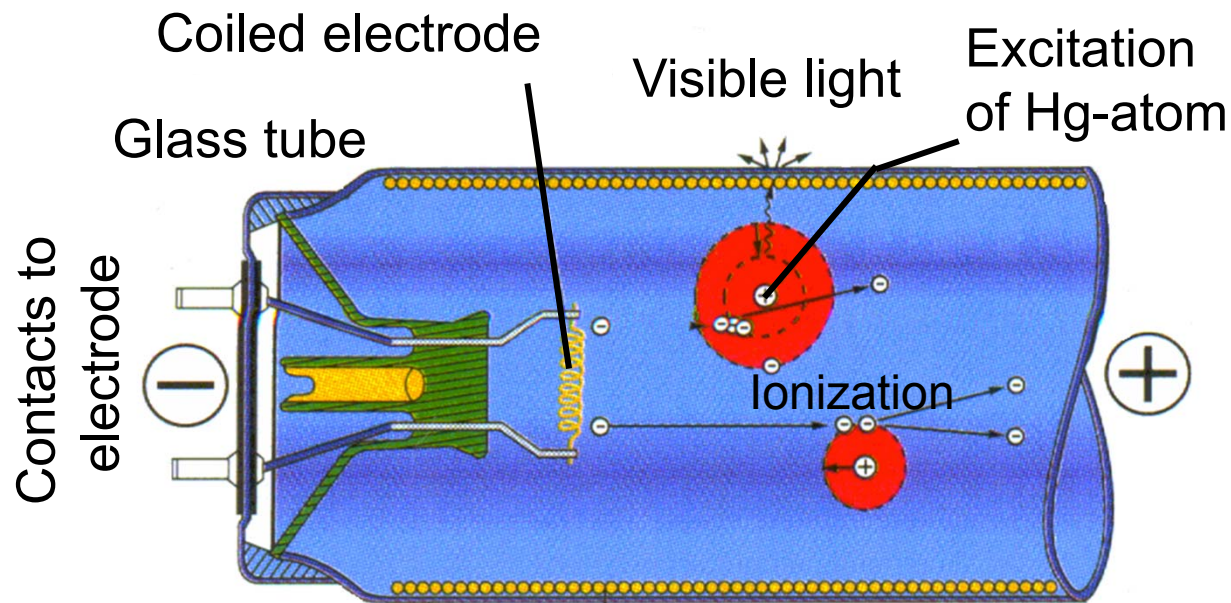


Figure 4.27: Coiled electrode is heated by electric current before ignition to achieve immediate arc discharge conditions. Electrons are accelerated by the electric field and excite mercury (*Hg*) atoms, which emit UV-radiation. UV-radiation is transformed to visible light by a phosphor coated layer at the inside of the glass tube.

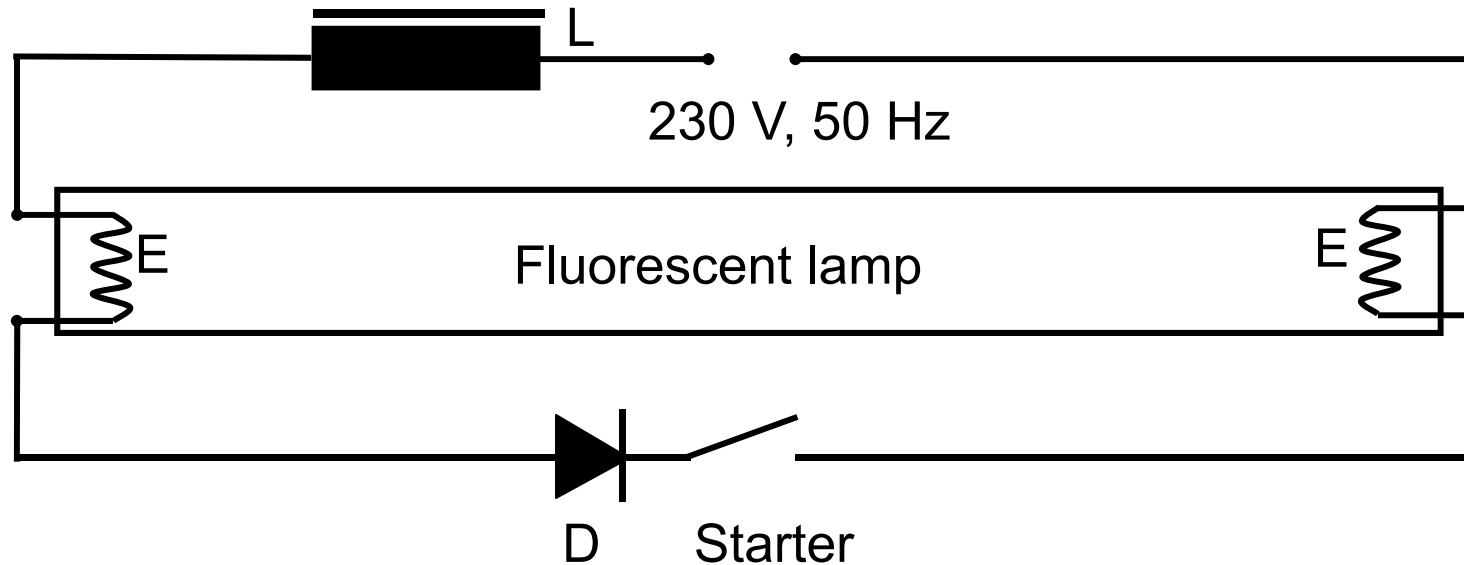


Figure 4.28: Improved starting arrangement for a fluorescent lamp. After applying the main voltage the starter closes the circuit and the current through the choking coil with inductivity L increases. The diode D rectifies the ac circuit current, with the consequence of increasing current, which efficiently heats the electrodes and rapidly enables arc discharge conditions. After certain heating time the starter opens the circuit and the rapid current drop dl/dt causes a high voltage pulse $U_{ind} = -L \, dl/dt$, which finally ignites the lamp.

Typical discharge characteristic of a 36 W fluorescent lamp

- Maximum ac-discharge current: 0.43 A
- Operation voltage: 103 V
- Current I and voltage U are not in phase: inductive ballast

New generation of fluorescent lamps:

- Fluorescent lamps with high efficacies $\eta_v > 100 \text{ lm/W}$ are fed by electronic control gears (elektronische Vorschaltgeräte) (ECG).
- ECGs generate ac voltage with a frequency of several 10 kHz, which reduces voltage losses at the electrodes.
- ECGs increase the lifetime of the lamps from 10,000 up to 20,000 hours, by reducing electrode erosion (high frequency reduces γ -effect induced sputtering of electrode material).

B) Low pressure sodium vapor lamp

Natriumdampf-Niederdrucklampe

Advantage of sodium (*Na*) against mercury (*Hg*):

- Energy difference between the lowest excited state E_1 and ground state E_0 of *Na* corresponds to visible photons of 589 nm (see Fig. 4.29), close to the maximum of the eye's sensitivity.
- *Hg* emits invisible UV-photons!

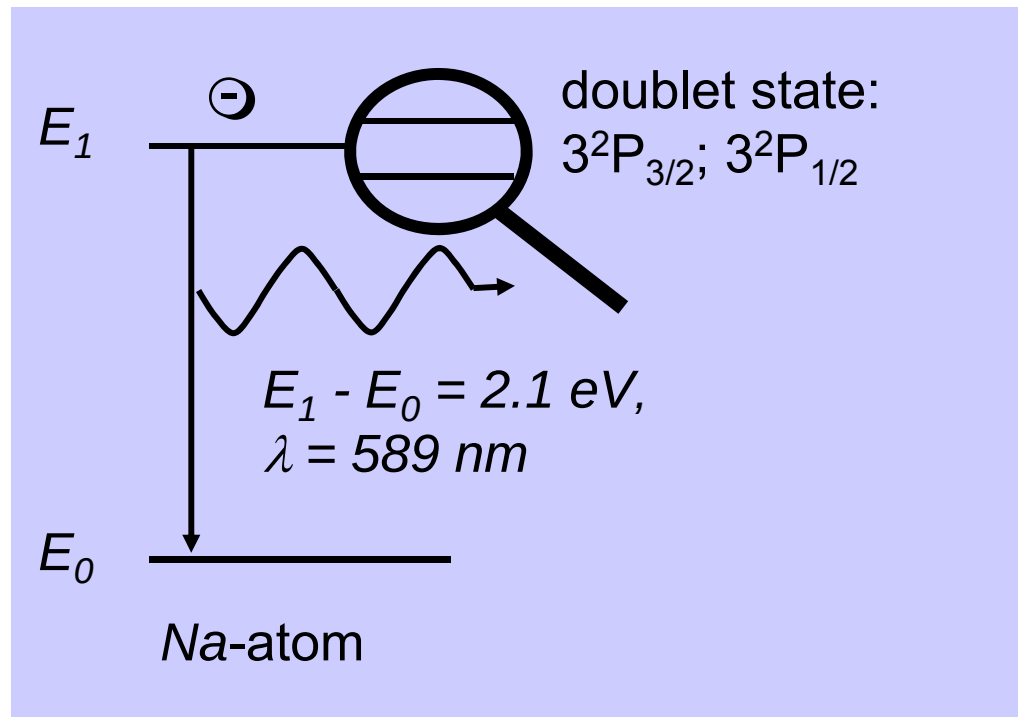


Figure 4.29: Simplified energy-level diagram of sodium (Na). Transition of electron from the *first* excited state to ground state results in so-called *resonance line* of sodium, which appears under high spectral resolution as a doublet line at 589.0 and 589.6 nm.

- ➡ Radiation can be directly utilized without a fluorescent phosphor!
- ➡ No transformation losses due to phosphor.
- ➡ High luminous efficacy up to $\eta_v = 200 \text{ lm/W}$ [4.14]

Problems:

- Vapor pressure p_{vapor} of Na is very low at room temperature:

$$p_{\text{vapor}}(300 \text{ K}) = 10^{-4} \text{ Torr}$$

- ➡ Coldest part of discharge tube must reach temperature of 260 °C to achieve necessary vapor pressure:

$$p_{\text{vapor}}(530 \text{ K}) = 3 \times 10^{-3} \text{ Torr}$$

- High efficacy of lamp result in low heat losses: discharge tube keeps “cold”.



- 
1. Dewar housing isolates discharge tube
 2. IR reflecting coating shields infrared radiation.

Wavelength of IR radiation:

Wien's law: $\lambda_{max} T = \text{const.} = 500 \text{ nm} \times 6000 \text{ K}$


$$\lambda_{max} (530 \text{ K}) \approx 5.5 \text{ } \mu\text{m}$$

A thin layer of SnO_2 is used as IR reflecting coating.

Figure 4.30 illustrates that tin-oxide coating reflects IR radiation above $5 \text{ } \mu\text{m}$ to more than 80% and is almost transparent for the emission wavelength at 589 nm.

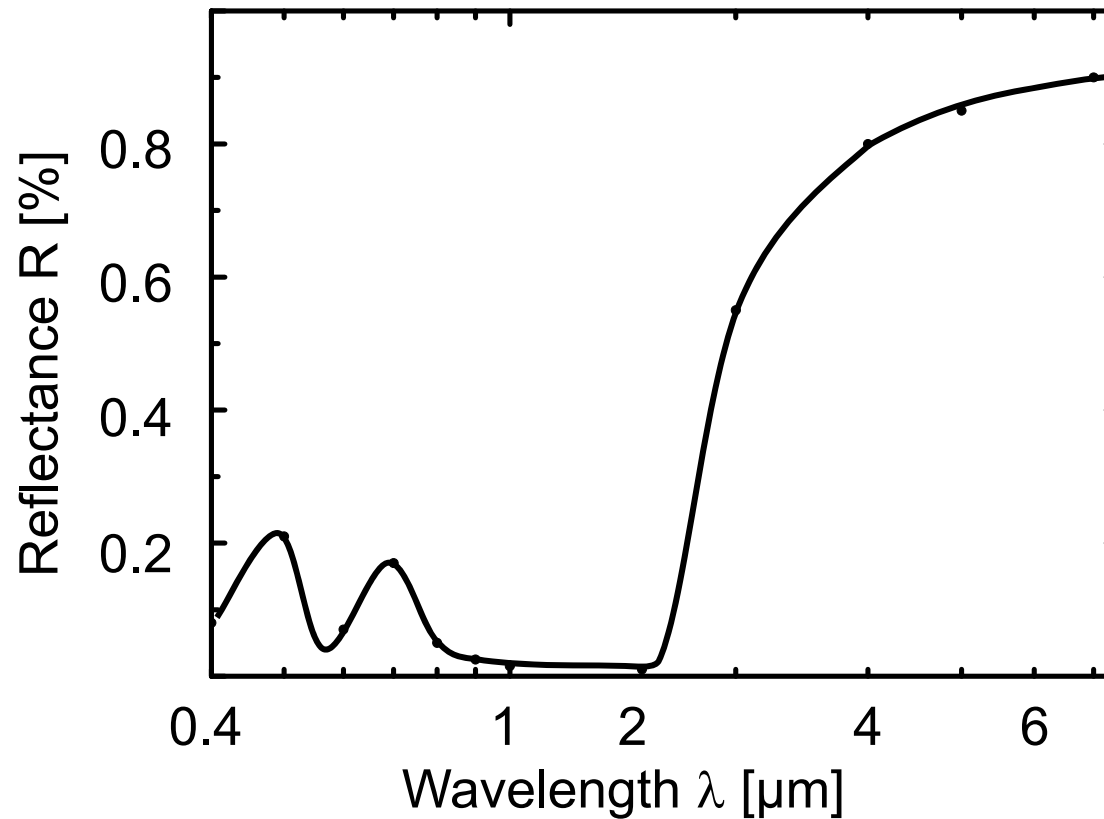


Figure 4.30: A thin stacked multi-layer of $\text{SnO}_2 / \text{SiO}_2$ deposited on the inside of glass housing of a low pressure Na -vapor lamp reflects IR radiation above 5 μm to more than 80 %. The coating is almost transparent for the emitted light at a wavelength of 589 nm.

Features and characteristics of low pressure *Na*-vapor lamp:

- The inside of lime glass discharge tube is lined with 50 μm thick borate glass layer

➡ Borate glass is *Na* resistant. Prevents in-diffusion of *Na* atoms and ions.

- Lamp filled with 5 mbar of neon.

➡ Enables ignition: *Na* vapor pressure is too low in cold lamp.

➡ Elastic collisions of electrons with *Ne* atoms randomize electron velocity and increase probability of inelastic electron - *Na* collisions.

- Electrical input power range: 27 W - 127 W
- Luminous flux: 3,500 lm - 25,000 lm Lichtfluß
- Monochrome light: color vision impossible, $R_a = 0$!

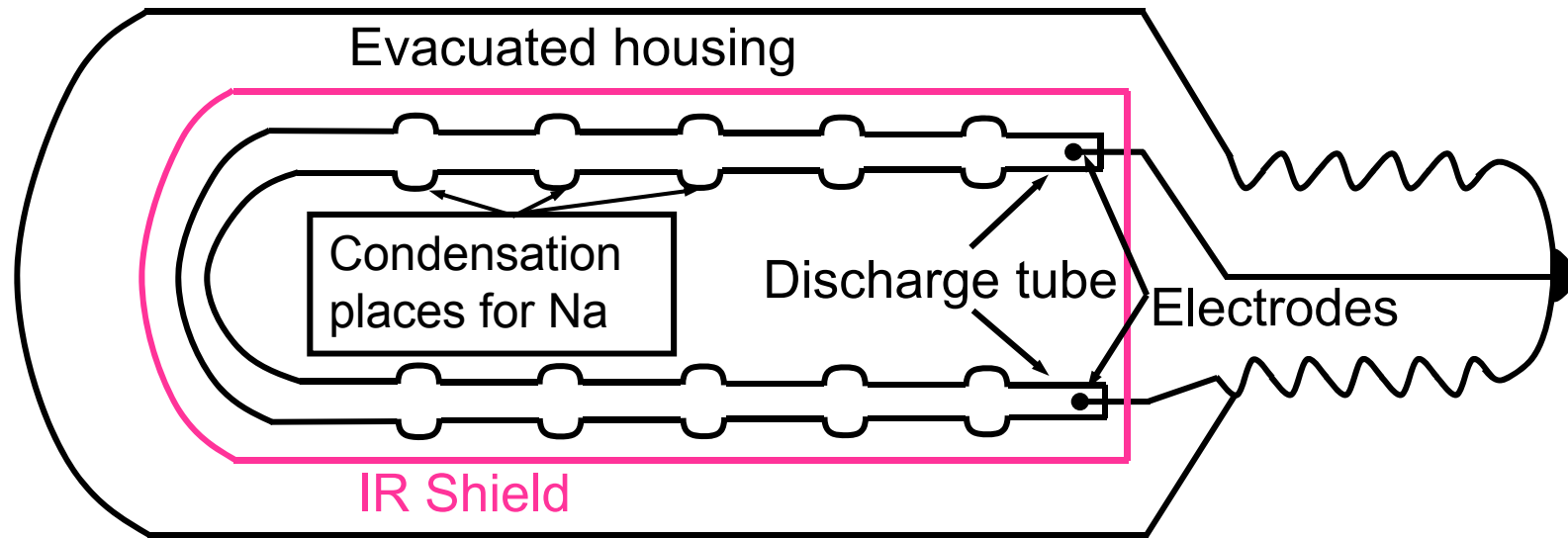


Figure 4.31: Cross section of low pressure sodium vapor lamp. Discharge tube is filled with 5 mbar neon (Ne) as buffer gas. Ne enables ignition of the lamp when the Na vapor pressure is low and reduces mean free path of electrons during operation. Discharge tube is equipped with bulges to enable condensation of excess liquid Na . IR reflecting coating reflects black-body radiation of hot (530 K) inner discharge tube. Evacuated outer housing prevents convective heat loss.

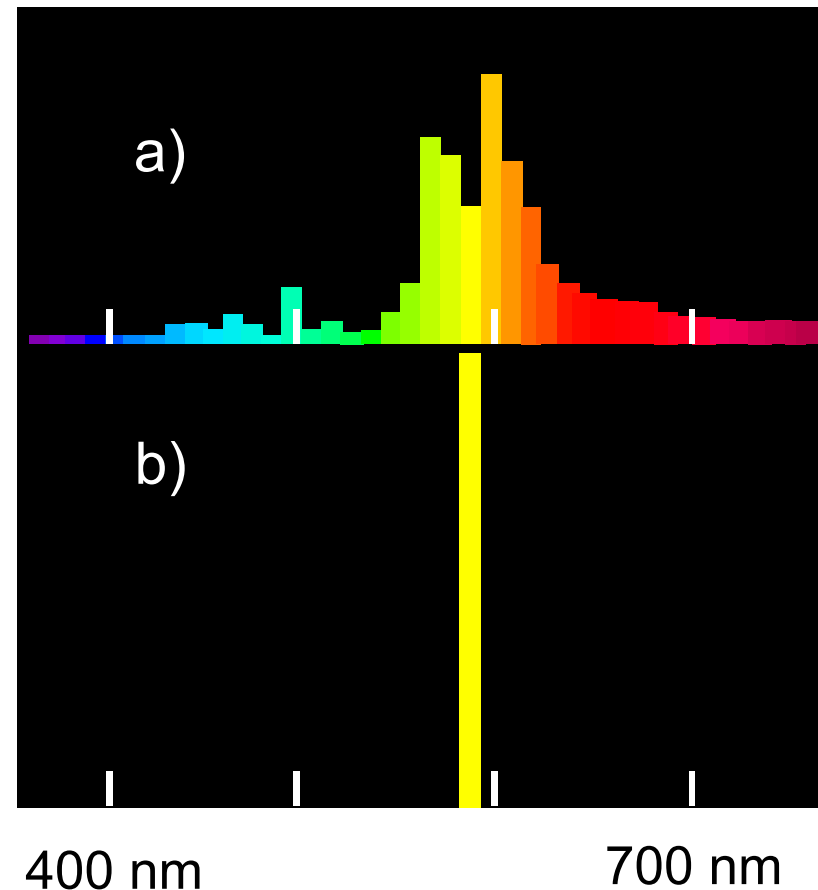
4.6.3 High pressure gas discharge lamps

A) High pressure sodium vapor lamp

Advantages against low pressure *Na*-vapor lamps:

- High pressure arc discharge: Increased power density
 - ➡ IR shielding and thermal isolation not necessary
 - ➡ Increased luminance Leuchtdichte
 - ➡ Increased luminous flux: 6,500 lm - 130,000 lm
Lichtfluß
- High pressure broadens emission line of *Na* (pressure broadening). Figure 4.27 compares the spectrum of low pressure with high pressure sodium vapor lamp.
 - ➡ Improved color rendering index: $R_a = 20$

Figure 4.32: Emitted spectra of sodium vapor a) high pressure b) low pressure lamp. High sodium vapor pressure and high gas temperature in a) broaden Na resonance line, improving color rendering of light. Minimum at center of the transition (589 nm) corresponds to “self-absorption” of colder Na vapor regions surrounding the center of discharge. Here lower temperatures lead to less pressure broadening and not excited Na-atoms absorb this part of the spectrum.



Problem:

Temperature of discharge tube exceeds 1000 K

Quartz glass cannot be used: Na atoms and ions diffuse into quartz and the quartz becomes opaque (brownish).

➡ Sintered aluminum oxide (Al_2O_3) ceramics is used

spröde

- Difficult to handle: obdurate material
- Complicated to manufacture: high temperature sintering process reduces material volume by 30 %, vacuum tight feed-throughs etc..

Details of high pressure Na-vapor lamp:

- Xenon buffer gas filling $30 \text{ mbar} < p_{\text{gas}} < 300 \text{ mbar}$
- Arc current $I \leq 10 \text{ A}$
- Luminous efficacy $\eta_v \leq 150 \text{ lm/W}$
- Luminance $L_v \leq 700 \text{ cd/cm}^2$

Low pressure Na vapor lamp:
 $L_v \leq 10 \text{ cd/cm}^2$

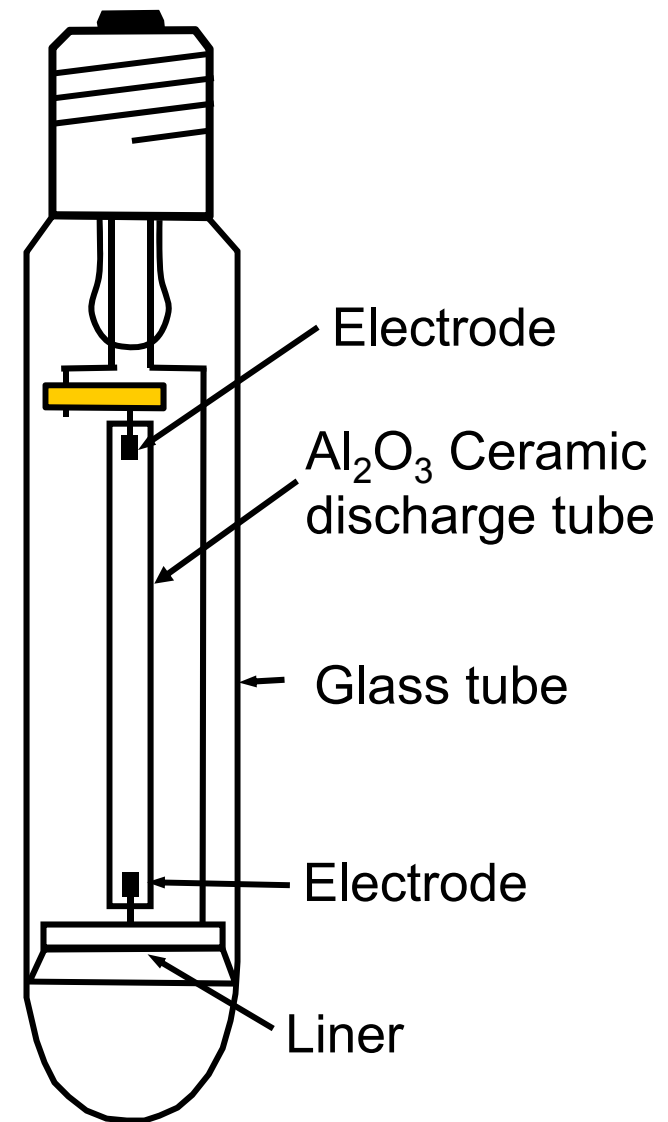


Figure 4.33: Image and drawing of high pressure sodium vapor lamp.

B) High pressure metal halogen gas discharge lamps

Transition from lowest excited state to ground state (resonance line) of all gases, especially noble gases, is in the far UV (VUV).

(V corresponds to vacuum, because air absorbs light beyond a wavelength of $\lambda < 200$ nm.)

 Gases unsuitable for discharge light sources

All metals, especially the rare earths, show strong transitions in the visible and near UV.

 In principle many metals are excellently suitable, but:
Vapor pressure too low, even at temperatures above 1000K.

Exceptions: Mercury, sodium



Solution:

Molecules (salts) of metals and halogens show low vapor pressure. They dissociate in the discharge and metal atoms are excited by electron collisions

Examples:

InJ_3 , TiJ , DyJ_3 , NaJ

Advantage [4.16]:

- Mixing of metal-halogen salts enables (almost) any light color
- with excellent color rendering index, $R_a > 90$.
- Luminous efficacy $\eta_v \approx 100 \text{ lm/W}$.
- Luminance $L_v : L_v > 6000 \text{ cd/cm}^2$.
- Quartz glass can be used instead of ceramics

Example of high pressure metal halogen gas discharge lamps:
So-called *Xenon lamp* for car headlights is displayed
in Fig. 4.28, the spectrum of the emitted light in Fig. 4.29.



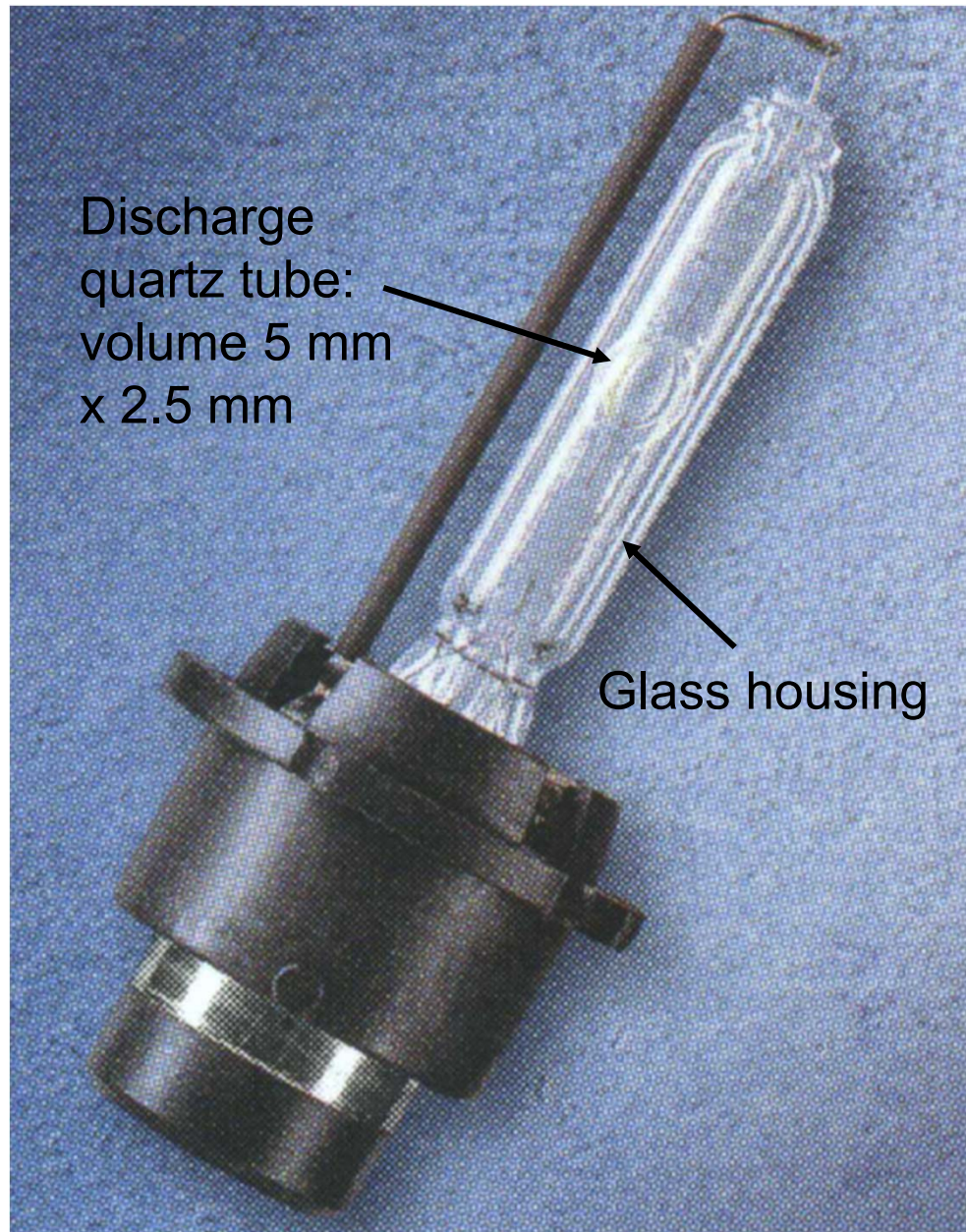


Figure 4.34: High pressure metal halogen vapor lamp is filled with Xenon buffer gas of pressure $p_{Gas} = 10$ bar. After ca. 5 seconds of operation with overpower of factor 4 the metal halogen salt mixture evaporates and the lamp emits light of “white” color. Color temperature $T_C = 4200$ K, luminous efficacy $\eta_V = 91$ lm/W, lifetime $\tau = 2000$ h.

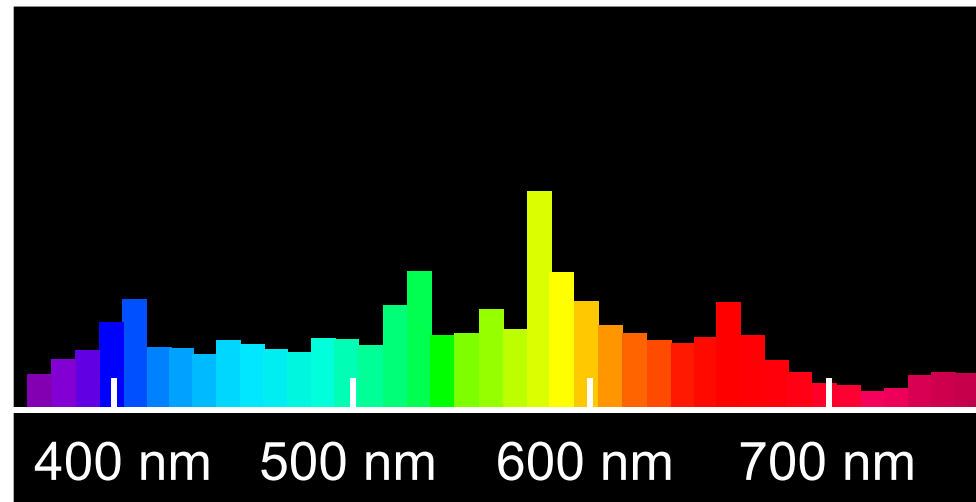


Figure 4.35: Spectrum of high pressure metal halogen vapor car headlight lamp. A purpose developed mixture of metal halogen salts (mainly DyJ_3 and NaJ) reduces the color temperature T_C of the older types from $T_C = 5500$ K to $T_C = 4200$ K and increases the acceptance of “Xenon” light even for those who cannot allow themselves to by such expensive cars.

4.7 Literature, Internet Links

[4.1]: Niedrig, H. (Hrsg.), *Bergmann Schäfer Lehrbuch der Experimentalphysik, Band 3*, 9. Aufl., (Walter de Gruyter, Berlin New York 1993), S. 629 - 634.

[4.2]: Niedrig, H. (Hrsg.), *Bergmann Schäfer Lehrbuch der Experimentalphysik, Band 3*, 9. Aufl., (Walter de Gruyter, Berlin New York 1993), S. 627.

[4.3]: Niedrig, H. (Hrsg.), *Bergmann Schäfer Lehrbuch der Experimentalphysik, Band 3*, 9. Aufl., (Walter de Gruyter, Berlin New York 1993), S. 626.

[4.4]: <http://www.chemicalelements.com/elements/pt.html>

[4.5]: <http://www.chemicalelements.com/elements/os.html>

[4.6]: <http://www.chemicalelements.com/elements/ta.html>



[4.7]: <http://www.chemicalelements.com/elements/c.html>

[4.8]: <http://www.chemicalelements.com/elements/w.html>

[4.9]: Niedrig, H. (Hrsg.), *Bergmann Schäfer Lehrbuch der Experimentalphysik, Band 3*, 9. Aufl., (Walter de Gruyter, Berlin New York 1993), S. 635.

[4.10]: <http://catalog.myosram.com>

- Licht für Foto, Film, TV, Bühne und Optik
- Halogen-Niedervoltlampen

[4.11]: <http://catalog.myosram.com>

- Licht für Foto, Film, TV, Bühne und Optik
- Halogen-Netzspannungslampen



[4.12]: http://www.eur.lighting.philips.com/int_en/prof/prod/lamps/mastertld/product/specifications/standard/spectral.html

[4.13]: <http://pc.lamping.info/pc01/DEindex.html>

- Leuchtstofflampen
- Für spezielle Anwendungen
- TL-D 90 de Luxe

[4.14]: <http://pc.lamping.info/pc01/DEindex.html>

- Entladungslampen
- SOX
- MASTER SOX-E

[4.15]: <http://pc.lamping.info/pc01/DEindex.html>

- Entladungslampen
- SON
- MASTER SON-T PIA PLUS



[4.16]: <http://catalog.myosram.com/>

- Die Fahrzeuglampen
- Scheinwerfer-Autolampen
- XENARC: Xenon-Gasentladungs –Lampen und –Systeme
- D1-System Lampe & EVG

5. Light Emitting Diodes

5.1 Working principle

Light emitting diodes use the mechanism of *spontaneous* emission due to radiative band/band recombination of electrons and holes. Spontaneous emission is the inverse process of absorption. Figure 5.1 compares the two processes for a general two-level system.

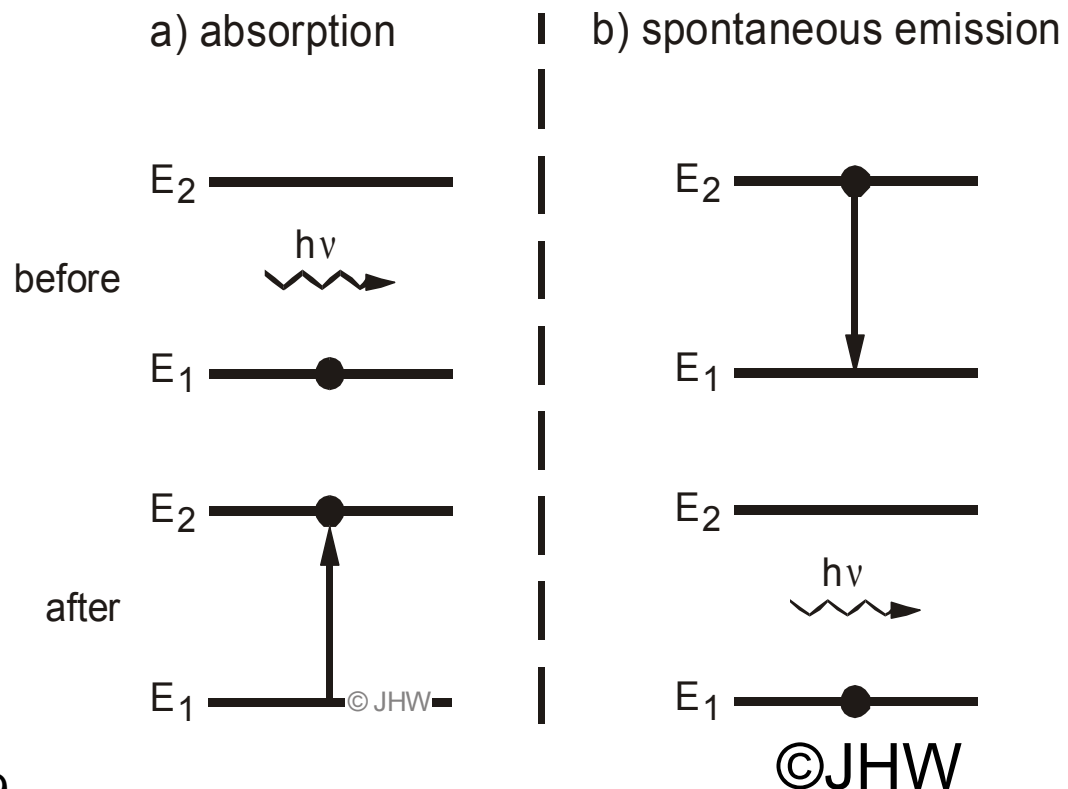


Figure 5.1: Absorption and spontaneous emission for a system with two discrete electron levels E_1 , E_2 .

In Fig. 5.1, the energies E_1 , E_2 are mono-energetic and radiation with the photon energy $h\nu = E_1 - E_2$ is emitted. In a semiconductor, these two energies correspond approximately (but not exactly) to the conduction band edge E_c and the valence band edge E_v . The emitted photon energy $h\nu$ of luminescence diodes with band gap E_g is

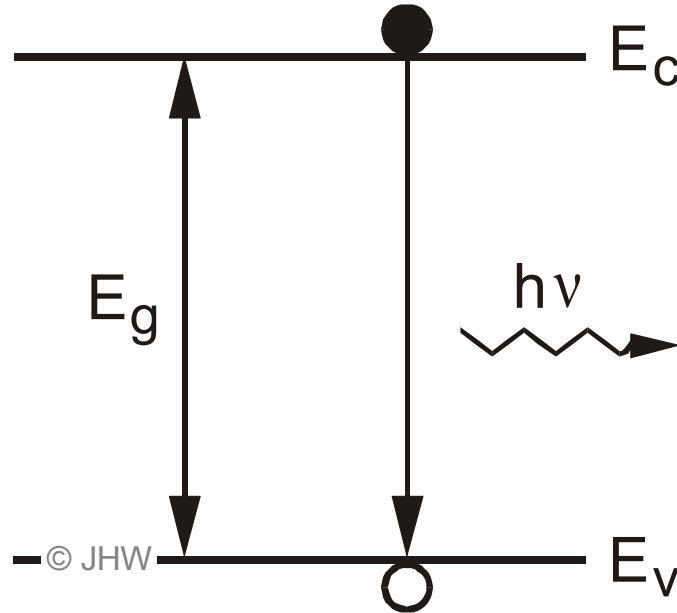
$$h\nu \approx E_c - E_v = E_g.$$


Figure 5.2: Spontaneous emission in a semiconductor.

Generation of *visible* radiation requires

$$\begin{array}{ll} \lambda < 780 \text{ nm} & \Rightarrow E_g > 1.60 \text{ eV} \\ \lambda > 380 \text{ nm} & \Rightarrow E_g < 3.26 \text{ eV} \end{array}$$

Efficient generation of radiation requires many electrons *and* holes at the same site and at the same time! As a consequence, the semiconductor must be in *non-equilibrium*!

In equilibrium it holds that $np = n_i^2$.

As a consequence, in n-type material, n is large, p is small,
in p-type material, p is large, n is small,
in intrinsic material, n and p are small.

In all these cases, the emitted radiation is low (it is just the equilibrium selective body radiation of a semiconductor described in chapter 4.3.2 and 4.4).

Emission of strong radiation requires $np > n_i^2$ by injection of carriers, for example across a pn-junction. The recombination zone extends one diffusion length into the bulk of the n-type and p-type region.

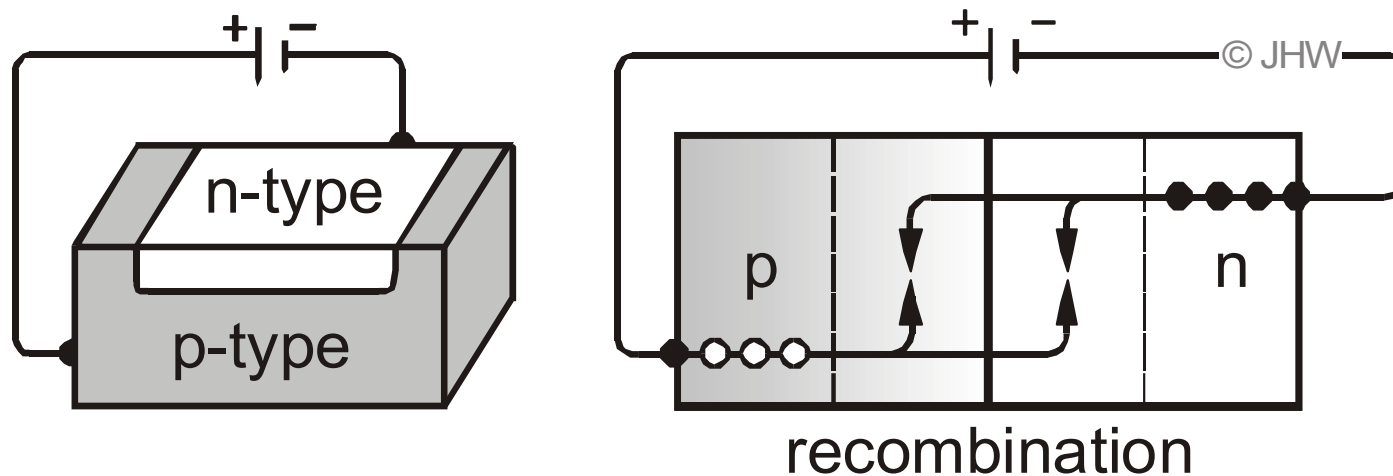


Figure 5.3: Carrier injection into junction causes recombination.

The requirement $np > n_i^2$ corresponds to $E_F^n > E_F^p$ for the quasi Fermi levels E_F^n (for electrons) and E_F^p (for holes).

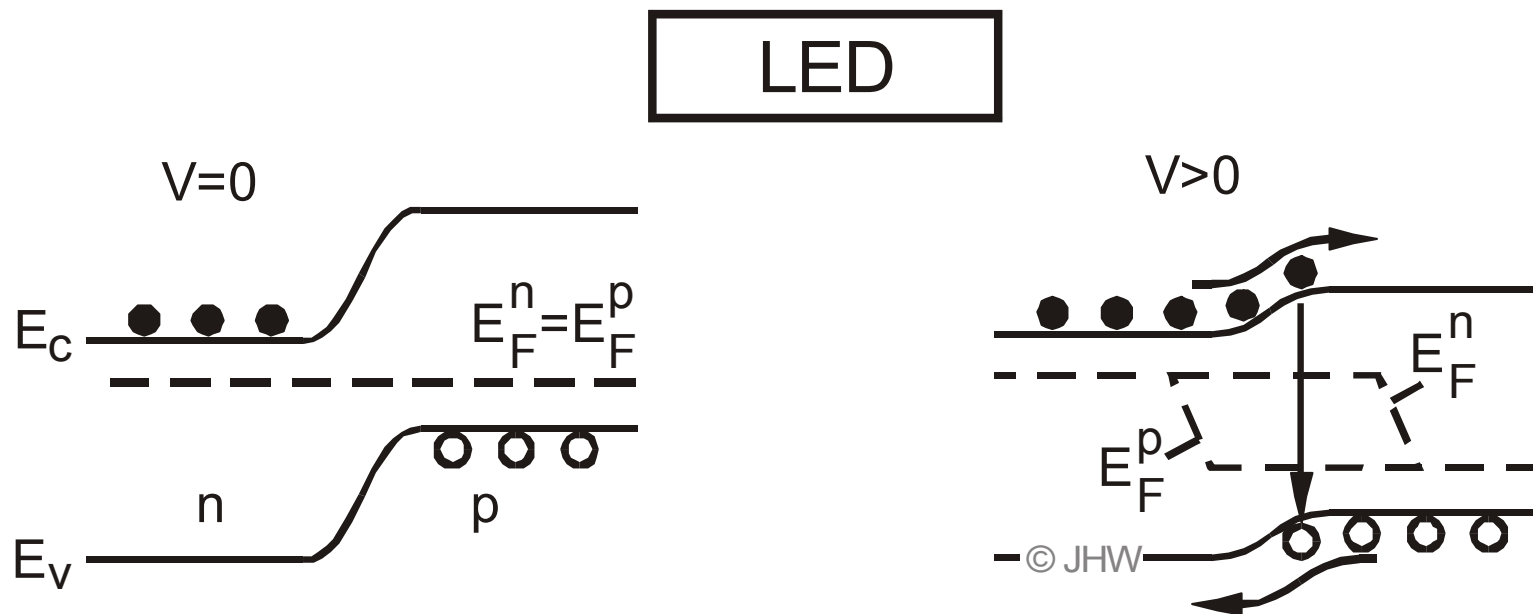


Figure 5.4: Band diagrams for LED without and with bias voltage V .

5.2 The emitted spectrum of an LED

cannot be described by Planck's black body equation. The exact shape of the emitted radiation depends on the energy distribution of electrons in the conduction band and holes within the valence band. For Fig. 5.5b, it holds

$$n_E = D_c(E)f_n(E)$$

and

$$p_E = D_v(E)f_p(E)$$

with the density of states $D_c(E)$, $D_v(E)$, and the Fermi functions $f_n(E)$, $f_p(E)$ for electrons and holes.

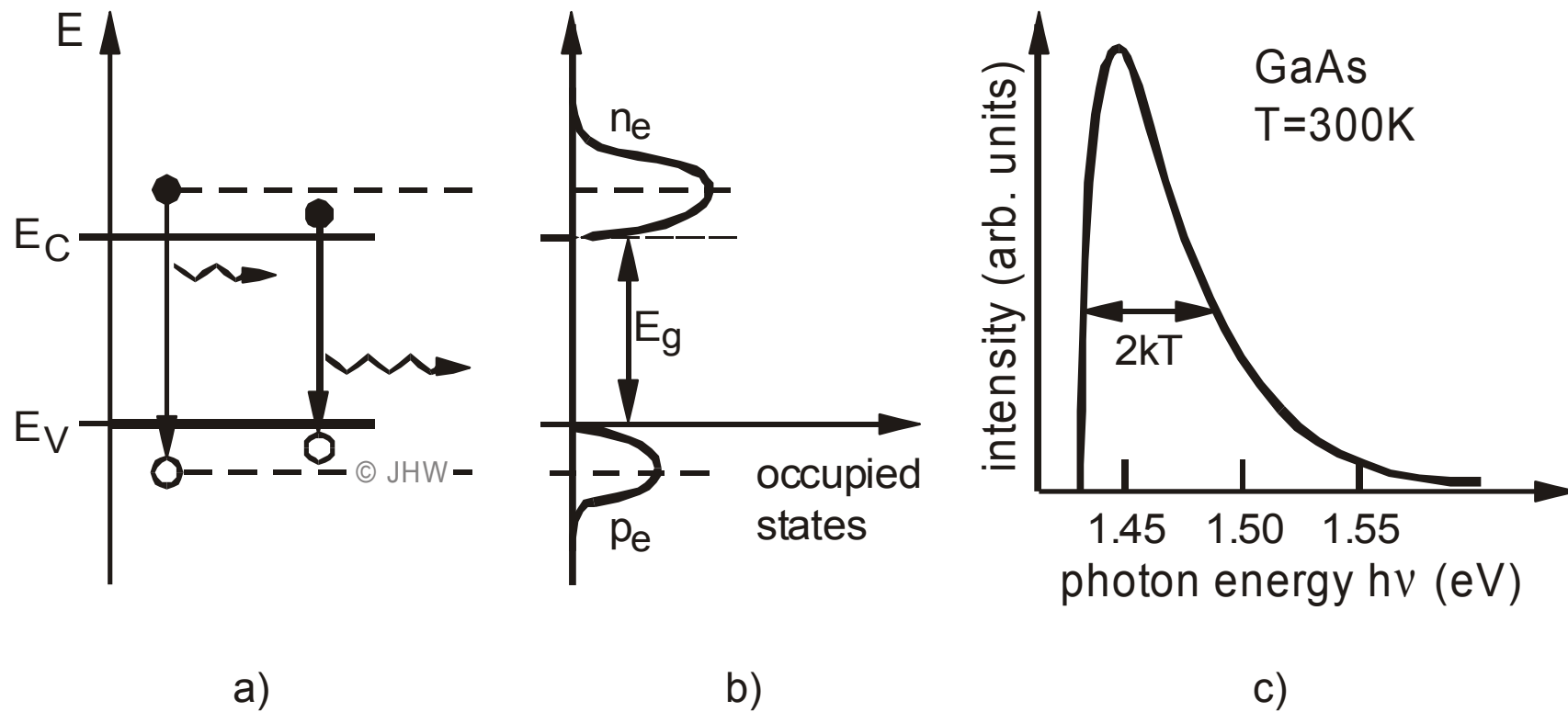


Figure 5.5: a) electron energies b) occupied states and c) emitted spectrum of an LED.

According to Fig. 5.5a, electrons and holes recombine not directly from the band edges but between slightly higher energies. The emitted spectrum was calculated by van Roosbroek and Shockley and is given by

$$\Phi(h\nu) \propto (h\nu)^2 \sqrt{h\nu - E_g} e^{-\frac{h\nu - E_g}{kT}} \text{ for } h\nu > E_g$$

The energetic width $\Delta(h\nu)$ amounts to $\Delta(h\nu) \approx 2 kT = 52 \text{ meV}$ at room temperature. The maximum of the radiation is about one $kT \approx 26 \text{ meV}$ above the band gap energy E_g . This effect stems from the energetic width of the Fermi distribution function of electrons and holes which spans about $1kT$.

On the wavelength axis, the energetic width $\Delta(h\nu)$ corresponds to a width $\Delta\lambda$. From $\lambda = c/\nu$ it follows

$$\Delta\lambda = \frac{\partial\lambda}{\partial\nu} \Delta\nu = -\frac{c}{\nu^2} \Delta\nu = -\frac{1}{c} \frac{c^2}{\nu^2} \Delta\nu = -\frac{\lambda^2}{c} \Delta\nu = -\frac{\lambda^2}{hc} \Delta(h\nu).$$

with $\Delta(h\nu) \approx 2kT$ it follows for the width $\Delta\lambda$ that

$$|\Delta\lambda| = \frac{2kT}{hc} \lambda^2 = \frac{52\text{meV}}{1.24\text{eV } \mu\text{m}} \lambda^2 = \frac{1}{24\mu\text{m}} \lambda^2.$$

The width of the radiation increases with the square of the center wavelength! For example, a GaAs LED with $\lambda = 870$ nm has a width $\Delta\lambda = 32$ nm. Diodes emitting at $1.3 \mu\text{m}$ or $1.5 \mu\text{m}$ (which are optimum wavelengths for communication via glass fibers) have a width of $\Delta\lambda = 70$ nm and $\Delta\lambda = 94$ nm, respectively. These widths are too large for optical data communication. Therefore, lasers with typical widths $\Delta\lambda < 0.1$ nm are used for optical data communication.

5.3 Materials for LEDs (and lasers)

5.3.1 III/V-Compounds (GaAs, GaP, InAs etc.)

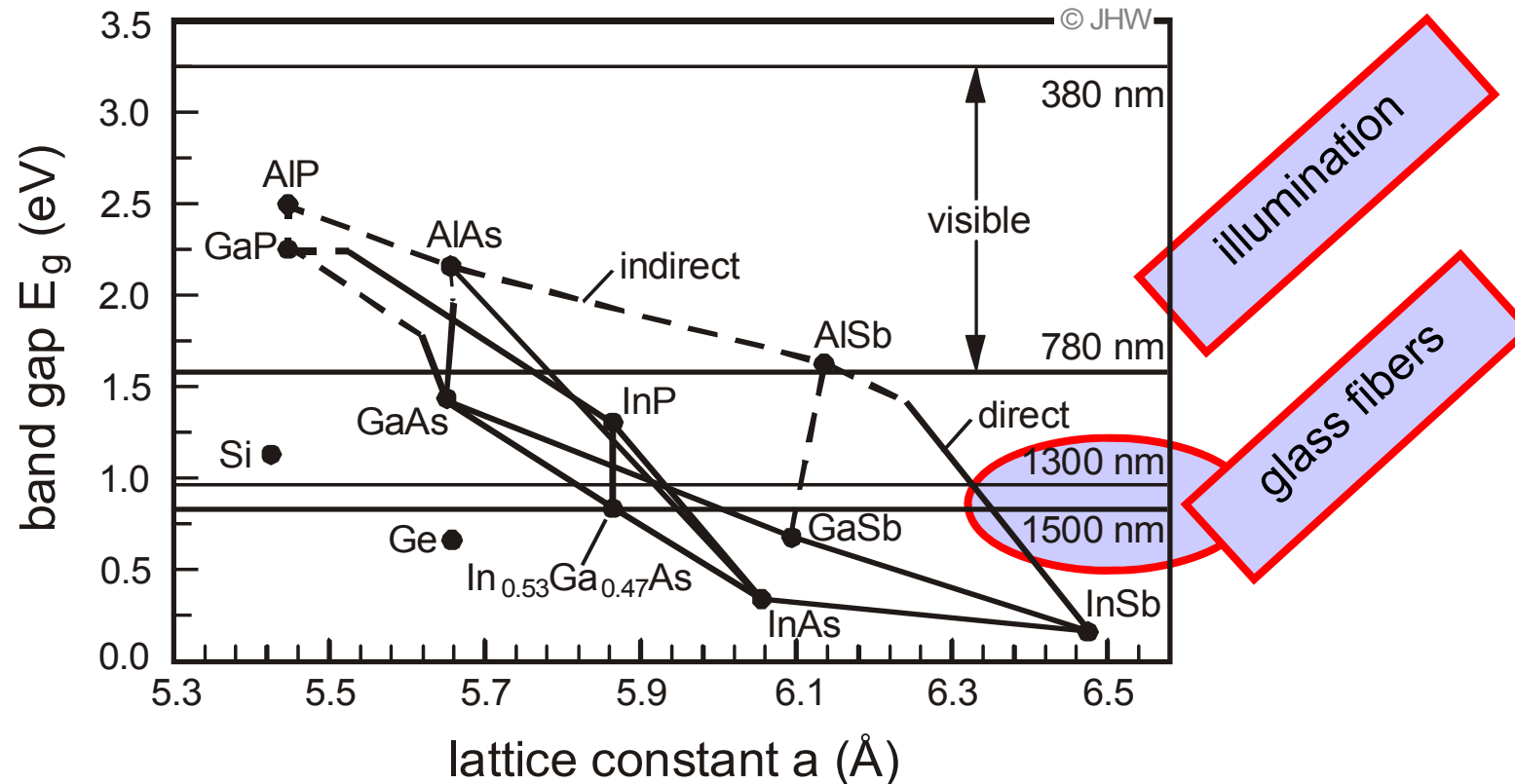


Figure 5.6: Binary and ternary compounds for optoelectronic devices. Dashed lines represent indirect band gaps.

A wide range of materials can be fabricated by alloying III/V-compounds. For example the system *GaAs* and *AlAs* is completely miscible. For all compositions $Al_xGa_{1-x}As$, the lattice constant is almost unchanged, whereas the band gap can be varied over a wide range. As a consequence, all compositions of $Al_xGa_{1-x}As$ grow without lattice defects on *GaAs* substrates. The materials *GaP* and *InP* are also miscible, however, for these compositions, the lattice constant changes over a wide range. More freedom than with a binary alloy is obtained by mixing four materials as shown in Fig. 5.7.

The following parameters change upon alloying:

- the band gap E_g
- the lattice constant a
- the band structure (direct, indirect)
- the thermal expansion coefficient

With the band gap E_g , we choose the wavelength of the emitted light. However, the value E_g is coupled to a certain lattice constant a for a particular alloy composition. Therefore, we are faced with the challenge to find an appropriate substrate because the light emitting material (for example any composition in the *InGaAs* system) is usually grown by epitaxy. Unfortunately, there are not many materials which could be considered as substrate.

Requirements for the substrate:

- availability in large areas (> 3 inches)
- defect free (no dislocations etc.)
- lattice matched to epitaxial layer
- similar thermal expansion coefficient as epitaxial layer.

Unfortunately, not many materials of the III/V-compounds are appropriate. Only *GaAs*, *InP* and *GaP* are available in the required size. Silicon as the most used material in microelectronics does not fit to the lattice constant of the III/V-materials.

Figure 5.6 demonstrates the following interesting features:

- There is no *direct* band gap III/V-semiconductor with a band gap $E_g > 2.3$ eV. As a consequence, no blue light can be generated with these materials. Here, new materials such as nitrides or organic materials come into play.
- *GaP* has the highest band gap. Unfortunately, *GaP* has only an indirect band gap and is therefore not very efficient in emission.
- For a certain mixture, ternary alloys of *GaP* and *InP* fit on a *GaAs* substrate.

- For a certain mixture, ternary alloys of *GaAs* and *GaSb* (or *InAs*) fit onto an *InP* substrate.
- The alloy $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}$ is direct, fits on *InP* and emits at 1.5 μm which is an ideal wavelength for glass fibers.
- A semiconductor for emission at 1.3 μm cannot be grown by a ternary alloy, but requires four, instead of three elements, i.e. a quaternary alloy.

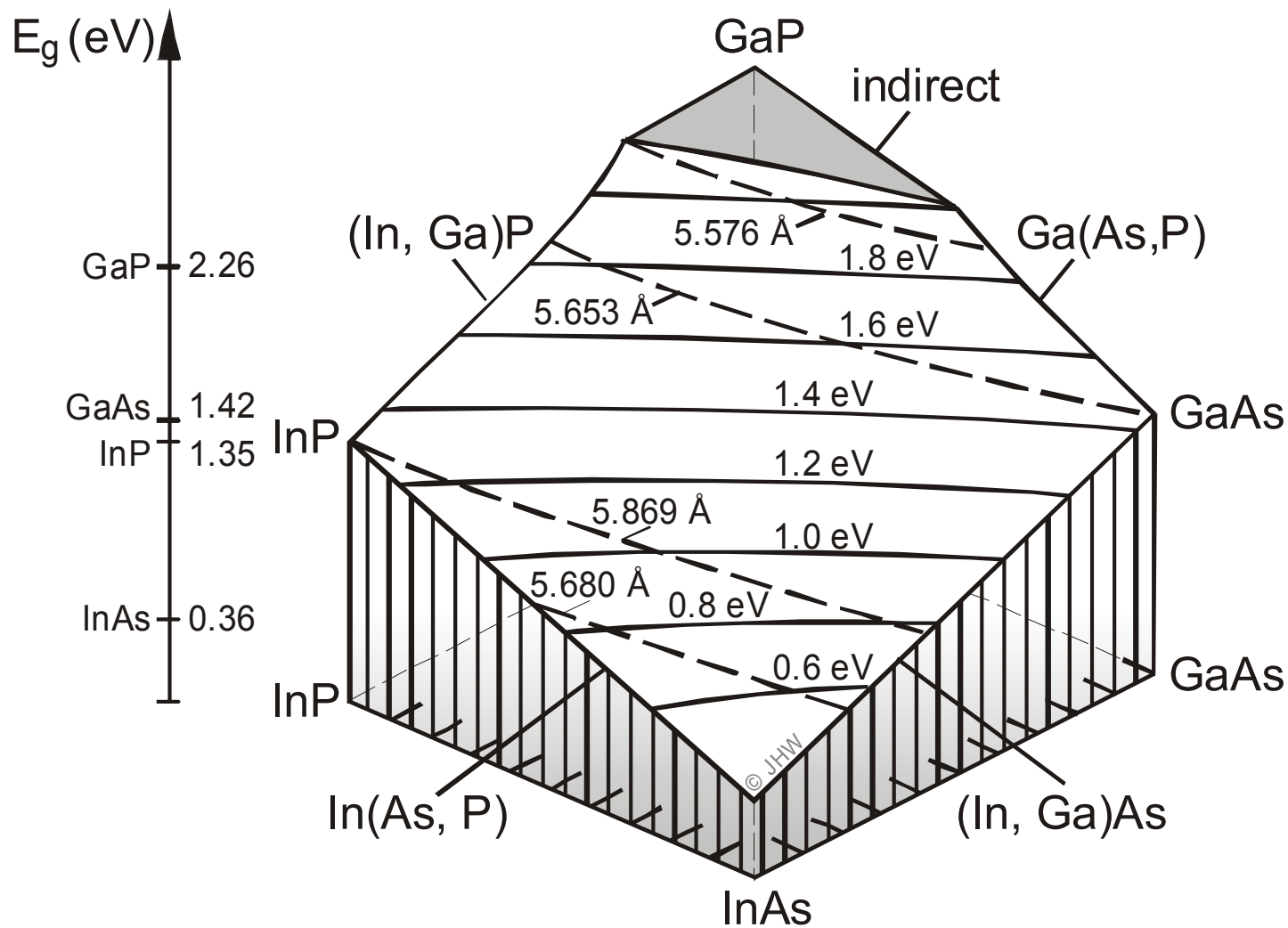


Figure 5.7: Quaternary alloys in the system $In_{1-x}Ga_xAs_yP_{1-y}$.

Within the quaternary system $In_{1-x}Ga_xAs_yP_{1-y}$ the band gap is adjustable between $E_g = 0.36$ eV ($InAs$) and $E_g = 2.26$ eV (GaP), which corresponds to $\lambda = 3.4$ μm to $\lambda = 0.55$ μm . For glass fibers, the wavelengths at $\lambda = 1.3$ μm (0.95eV) and $\lambda = 1.5$ μm (0.83 eV) are most important. Light emitting diodes and lasers with these two wavelengths can be grown on InP substrates.

On InP substrates, the minimum possible E_g is 0.75eV (1.6 μm).

On InP substrates, the maximum possible E_g is 1.35 eV (0.92 μm).

On $GaAs$ substrates, the minimum possible E_g is 1.42 eV (0.87 μm)

On $GaAs$ substrates, the maximum possible E_g is 1.85 eV (0.67 μm).

No green and blue light is possible with this material system!

5.3.2 Materials for blue light (LEDs and lasers)

Why blue light?

- for color displays (RGB)
- generation of white light (via LUCOLEDs)
- for optical storage systems with higher density (CDs etc.).

The smaller the wavelength λ of a scanning laser in a CD, the smaller can be the distance d between two pits on the CD. The resolution is limited by diffraction of the laser beam at the edges of the lens of the scanning system according to

$$d_{\min} = 1.22 \frac{\lambda}{n \sin \Theta}.$$

materials of the
past

SiC , $Zn(S,Se)$
research essentially given up, due
to low luminescence efficiency
(hampered by defects)

materials of today

GaN , InN , $In_xGa_{1-x}N$
first good material in 1995
- low sensitivity to defects
- on SiC -substrate (Siemens, Cree)
- on Al_2O_3 -substrate (Nichia)

a) basics of GaN technology

Table 5.1 Basic parameters of nitrides; data scatter significantly.

Material	Bandgap E_g at 300K [eV]
<i>GaN</i> (wurzite)	3.17-3.45
<i>GaN</i> (zinc blend)	3.44-3.54
<i>InN</i>	0.6*

*[http://physikwww.upb.de/ag/ag-as/papers/Schley et al. pss \(c\) -ICNS-7.pdf](http://physikwww.upb.de/ag/ag-as/papers/Schley%20et%20al.%20pss%20(c)%20-ICNS-7.pdf)

Table 5.2 Substrate materials for nitrides have a large lattice mismatch.

Material	Lattice constant a [Å]
<i>GaN</i> (wurzite)	3.17-3.19
<i>InN</i>	3.533
Sapphire	2.747
<i>6H-SiC</i>	3.082

b) substrate materials

$MgAl_2O_4$: 10% lattice mismatch to GaN results in lower crystalline quality of epitaxial films compared to sapphire.

→ not used any more!

ZnO : Lattice matches perfectly to $In_{0,22}Ga_{0,78}N$
Chemical reactions with NH_3

→ Promising, but commonly not in use!

6H-SiC:

3.5% lattice mismatch to GaN
Good electrical and thermal conductance
Expensive
Costly surface preparation

→ Occasionally used!

Siemens and Cree use this substrate material (royalty reasons).

Sapphire:
(Al_2O_3)

13% lattice mismatch to GaN

Cheap

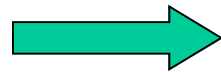
Transparent

Available as 6" wafer with good quality

High temperature resistance

Simple surface preparation

Isolator



Substrate of Nichia

GaN:

Fabrication of wafers from melt not yet possible

(melting point $T_m = 2800$ K,

N_2 vapor pressure $p_{N_2} > 100$ kbar)



Promising, unsolved technological problems!

c) buffer layer

No substrate with appropriate lattice constant available. Buffer layer reduces defect density and thus improves crystalline quality of epitaxial *GaN* film.

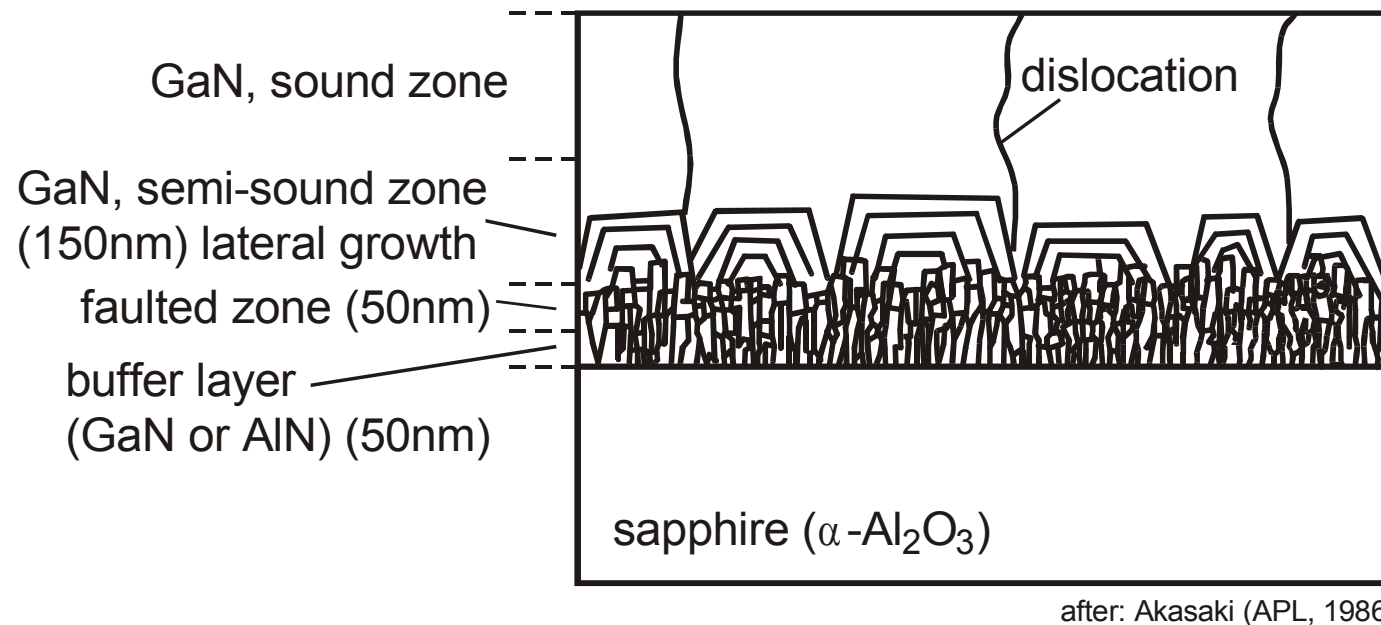


Figure 5.8: A 30 nm to 100 nm thick *AlN*- or *GaN*-buffer layer deposited at a temperature $T_D = 500^\circ\text{C}$ reduces defect density and resulting dislocation density in the active semiconductor material.

d) epitaxial lateral over-growth (ELOG)

So-called ELOG-layer on top of the buffer layer further reduces the defect density [5.1].

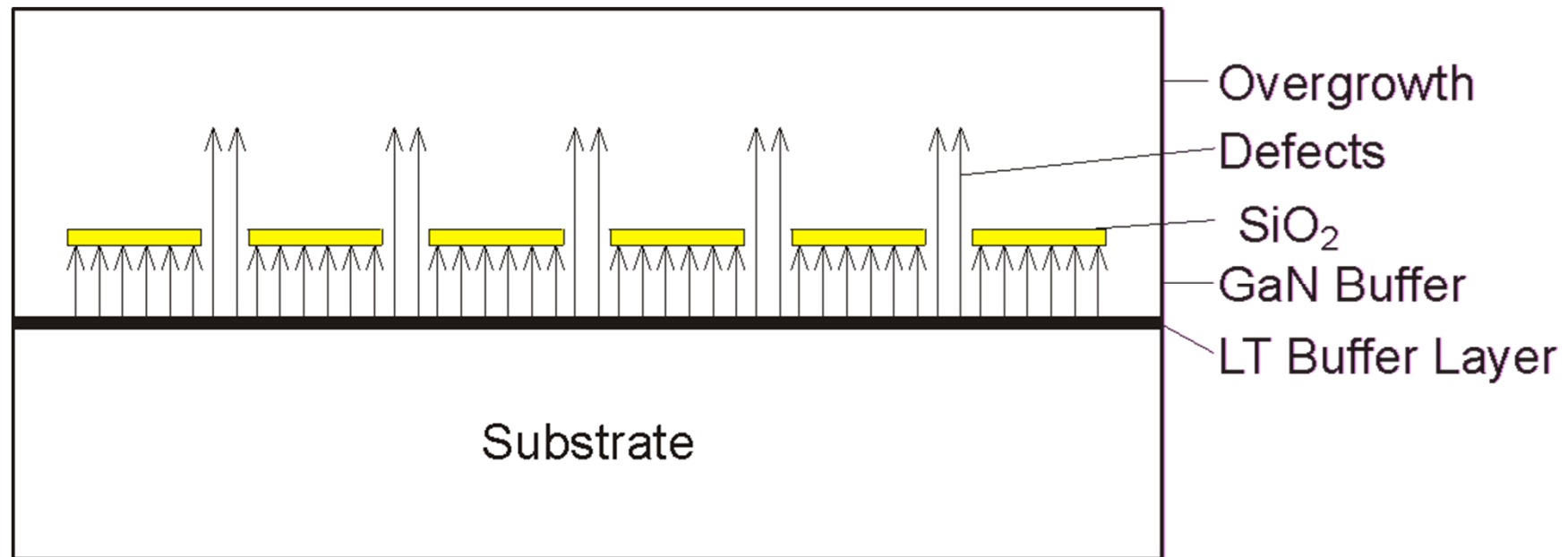


Figure 5.9: ELOG layer consists of thin SiO_2 stripes. The overgrowing GaN contains almost no defects.

e) structure of InGaN light emitting diodes

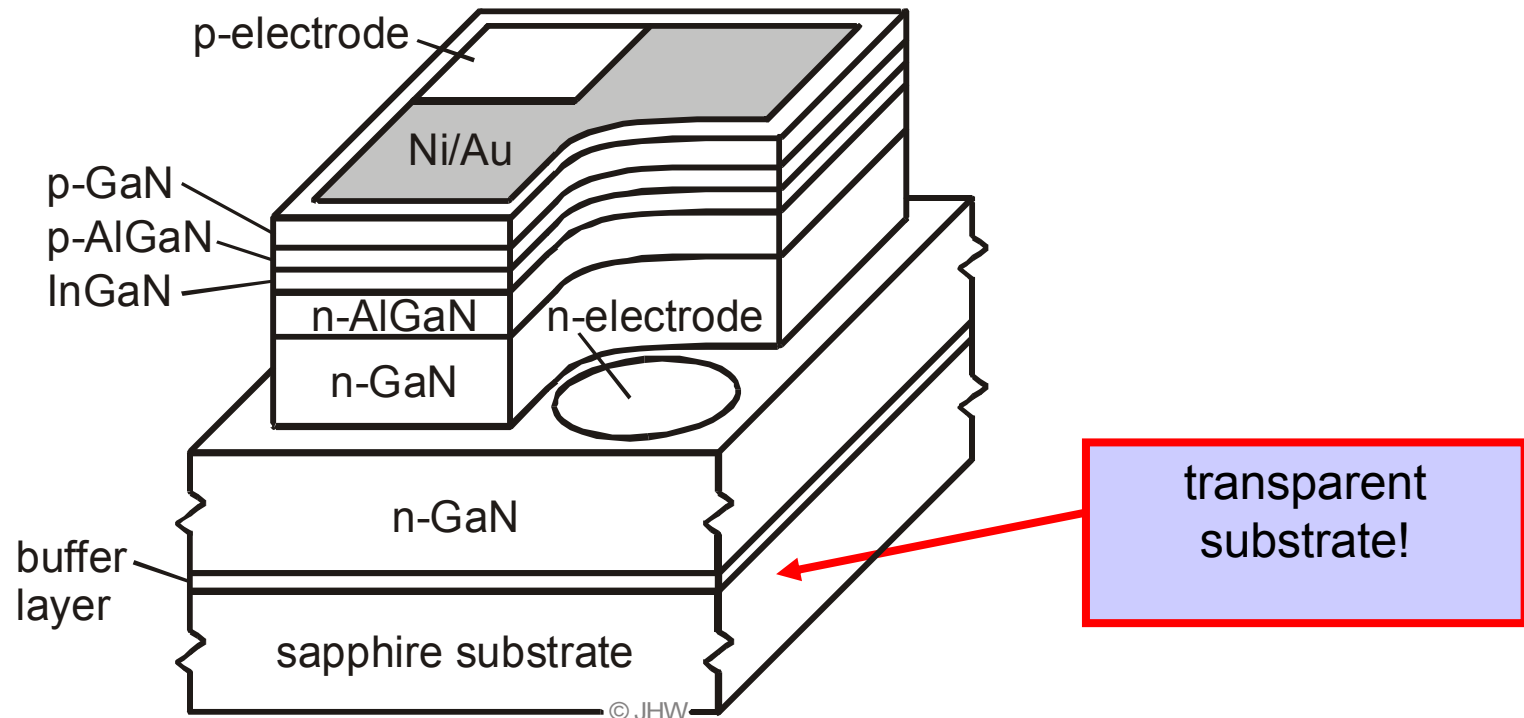


Figure 5.11: A thin layer of *InGaN* sandwiched between two layers of n- and p-type *AlGaN* provides the optically active zone, where recombination takes place. Isolating sapphire substrates requires p- and n- front contacts. *Ni/Au* layer is extremely thin and therefore transparent.

f) properties of blue LEDs

Table 5.3: Comparison of LEDs made from today's and "historical" semiconductor materials

Material	Optical power [μ W]	External Quantum Efficiency (EQE) [%]
<i>SiC</i>	20	0.04
<i>ZnCdSe</i>	327	1.3
<i>InGaN</i> (Dec. 2003)	85,000	32

Table 5.4: Nitride-LEDs in comparison with conventional light sources [5.2]

Material	Color	Wavelength [nm]	Optical efficacy [lm/W]	EQE [%]
<i>InGaN</i>	Blue	470	10	11
<i>InGaN</i>	Green	520	34	10
<i>InGaN</i>	Amber	590	14	3.5
Incandescent lamp (100 W)	White		15	
Fluorescent lamp	White		107	
Green fluor- escent lamp	Green	550	150	

g) white LEDs



Figure 5.13:

LED based
lamps with
incandescent
lamp sockets

Two different approaches:

I. Combination of three LEDs (red, green, blue)

Advantage: good color rendering properties
variable color temperature

Disadvantage: expensive
color is temperature dependent

II. LuCoLEDs (Luminescence-Conversion-LEDs):

Blue or UV-LED combined with fluorescent phosphors [5.3, 5.4]

Advantage: Cheap

Disadvantage: Fabrication with defined color temperature problematic. Color temperature changes when ageing.

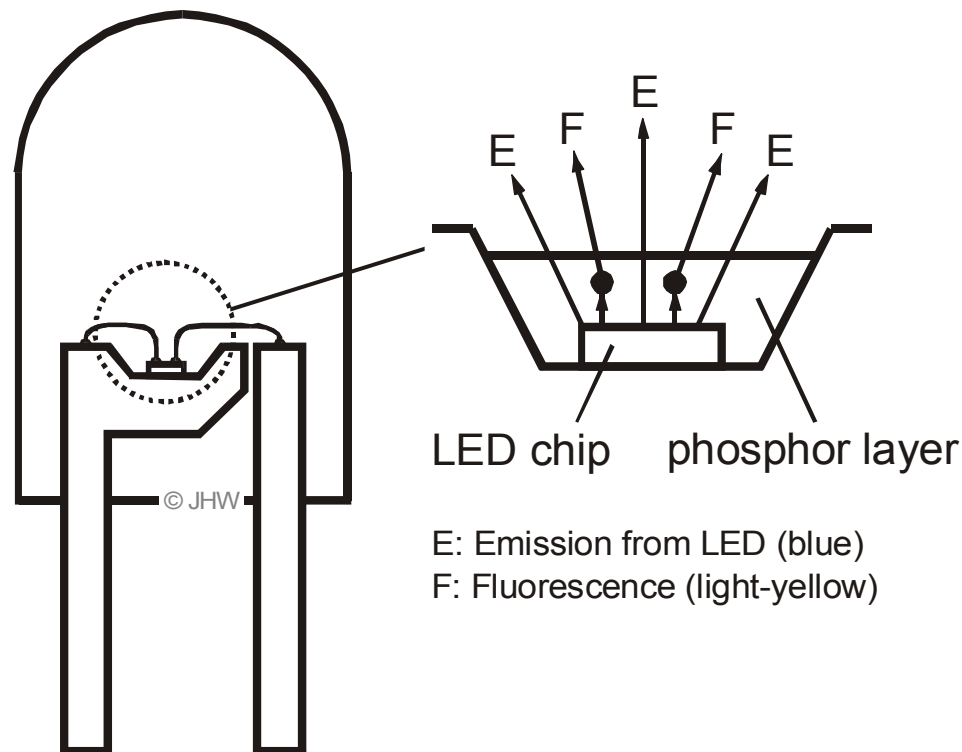


Figure 5.13: Cross section of LUCOLED. A fluorescent phosphor converts part of the blue light to yellow light. Additive color mixing of appropriate parts of blue and yellow light results in white light.

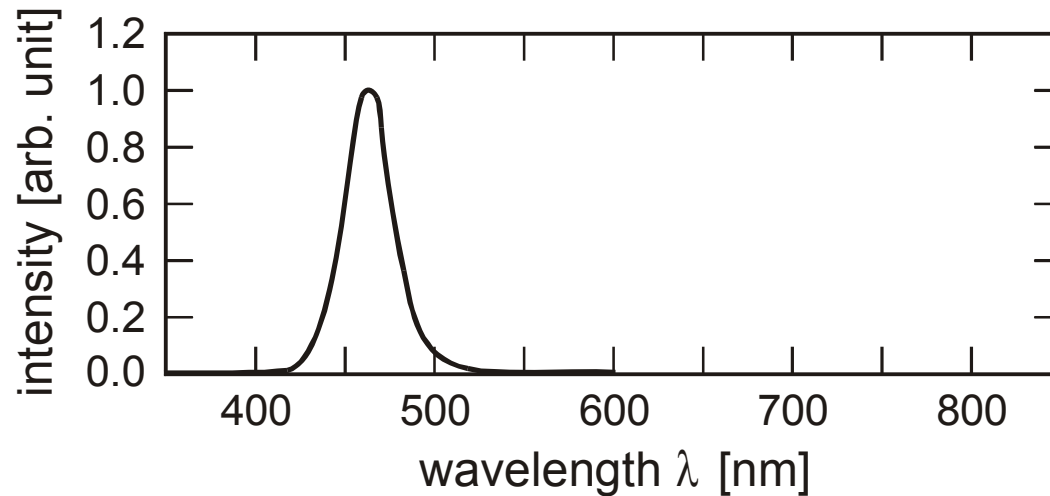


Figure 5.14:
Emission spectrum
of a blue *GaN* LED

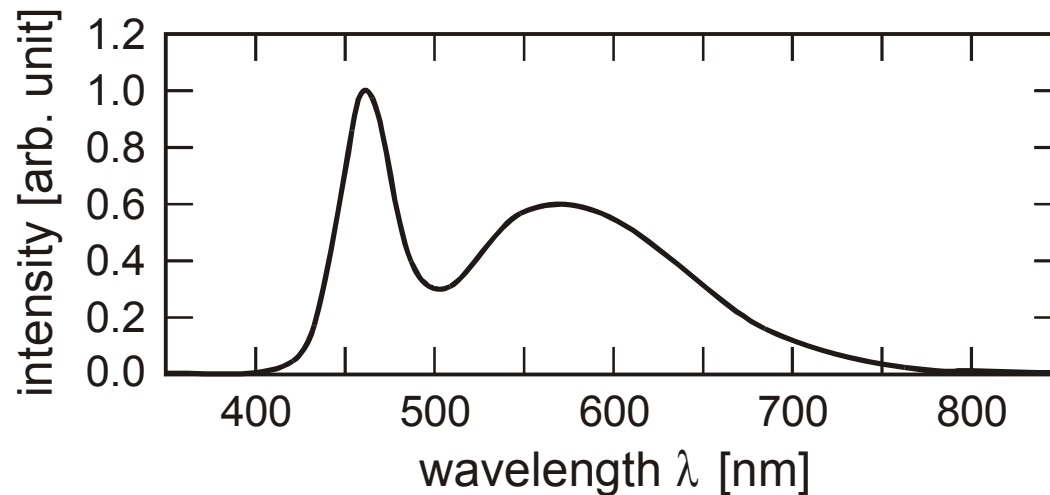


Figure 5.15:
Emission spectrum
of a white
LUCOLED

Luminous efficacy η_v up to 92 lm/W (Oct. 2009) excl. electronic driver[5.2, 5.5]

5.4 Emission efficiency of LEDs

5.4.1 External quantum efficiency (EQE)

The *internal* quantum efficiency (*IQE*), i.e. the number of *created* photons per recombining e/h-pair in LEDs, is high, up to 99 %. However, because of optical reasons, not many of these created photons are able to leave the crystal. Most of them are trapped within the semiconductor. Therefore, the *external* quantum efficiency (*EQE*), i.e. the number of photons that leave the crystal per recombining photon, is low, usually around 3 to 4 % for a LED.

$$EQE = IQE \times \eta_{opt}$$

Three reasons are responsible for the low optical efficiency η_{opt}

- re-absorption
- reflection
- total reflection



a) re-absorption

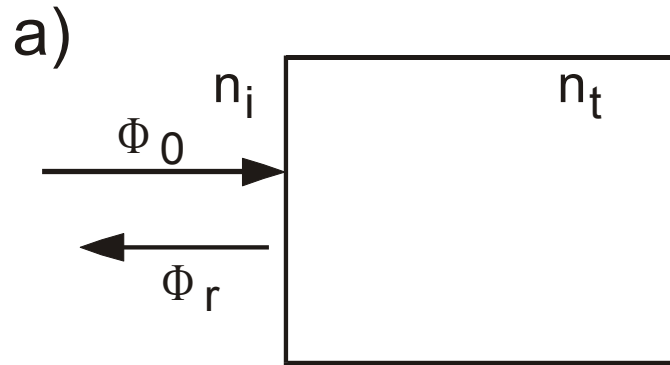
Direct semiconductors have a high absorption constant. As a consequence, only light that is created directly underneath the surface can leave the crystal. For a normal pn-junction LED, re-absorption results in a loss of 10 to 20 %, i.e. in a (non-) absorption efficiency $\eta_{non-abs} = 0.8$ to 0.9 .

way out: use of hetero-structure.

b) reflection

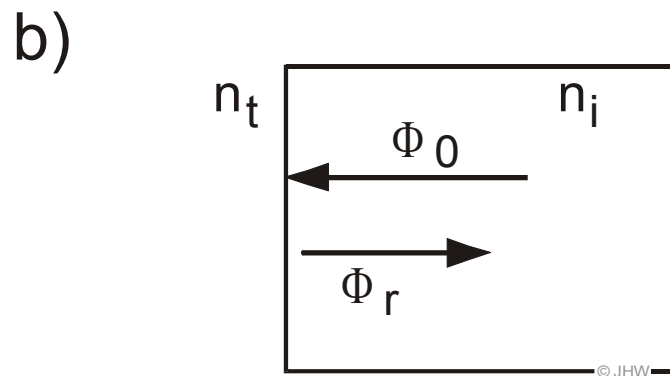
Reflection and transmission of radiation depends on the refraction index of the semiconducting material, n_t , and the refraction index of the surrounding material n_i . Figure 5.15 illustrates two different situations. a) the surface partly reflects incident (from outside) ray of intensity Φ_0 with intensity Φ_r . The remaining part of intensity Φ_t enters the semiconductor. b) A ray of intensity Φ_0 propagating inside the semiconductor is partly reflected back with intensity Φ_r . Only part of intensity Φ_t is transmitted (emitted).





$$r = \frac{\Phi_0}{\Phi_r} = \left(\frac{n_t - n_i}{n_t + n_i} \right)^2$$

$$t = 1 - r$$



Due to the quadratic dependence, r is the same for a) and b)!!!

Figure 5.15: Reflectance r , transmittance t for perpendicular incidence of radiation.

The reflectance (to the inner side!) between a semiconductor (n_{semi}) and air ($n = 1$) is

$$r_{\Phi} = \left[\frac{n_{semi} - 1}{n_{semi} + 1} \right]^2 \cong 0.3 \quad \text{for } n_{semi} \approx 3.5$$

way out: epoxy
with $n = 1.5$.

Table 5.5: Perpendicular reflectance for different interfaces

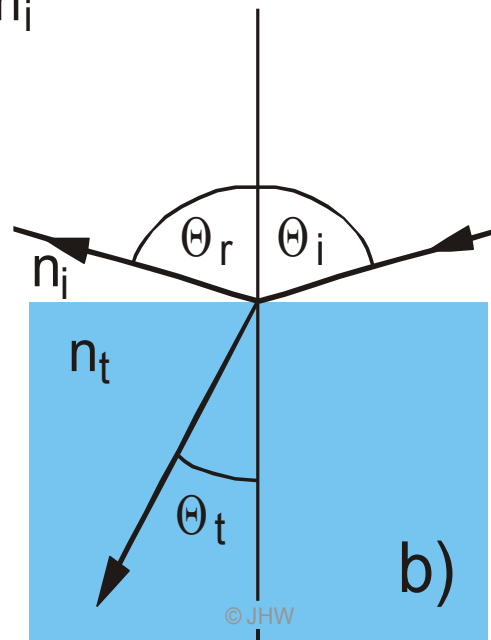
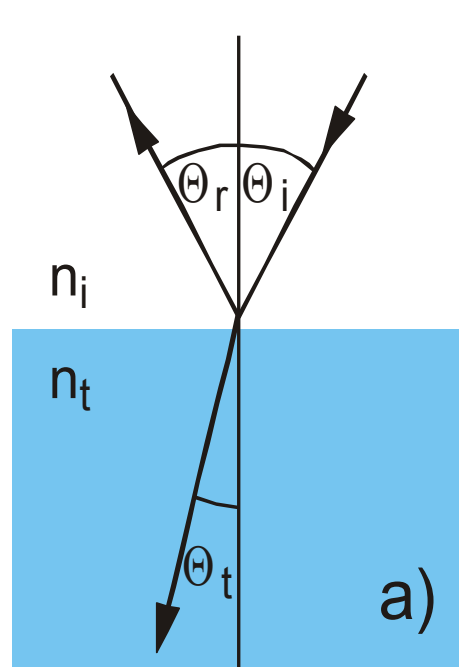
interface	reflectance r	transmission t
glass/ air	4 %	96 %
GaAs/ air	30 %	70 %

c) total internal reflection

refraction = change of light velocity, described by Snell's law for angles of incident (i) and transmitted (t) radiation [5.6].

$$\frac{\sin \Theta_i}{\sin \Theta_t} = \frac{c_i}{c_t} = \frac{n_t}{n_i}$$

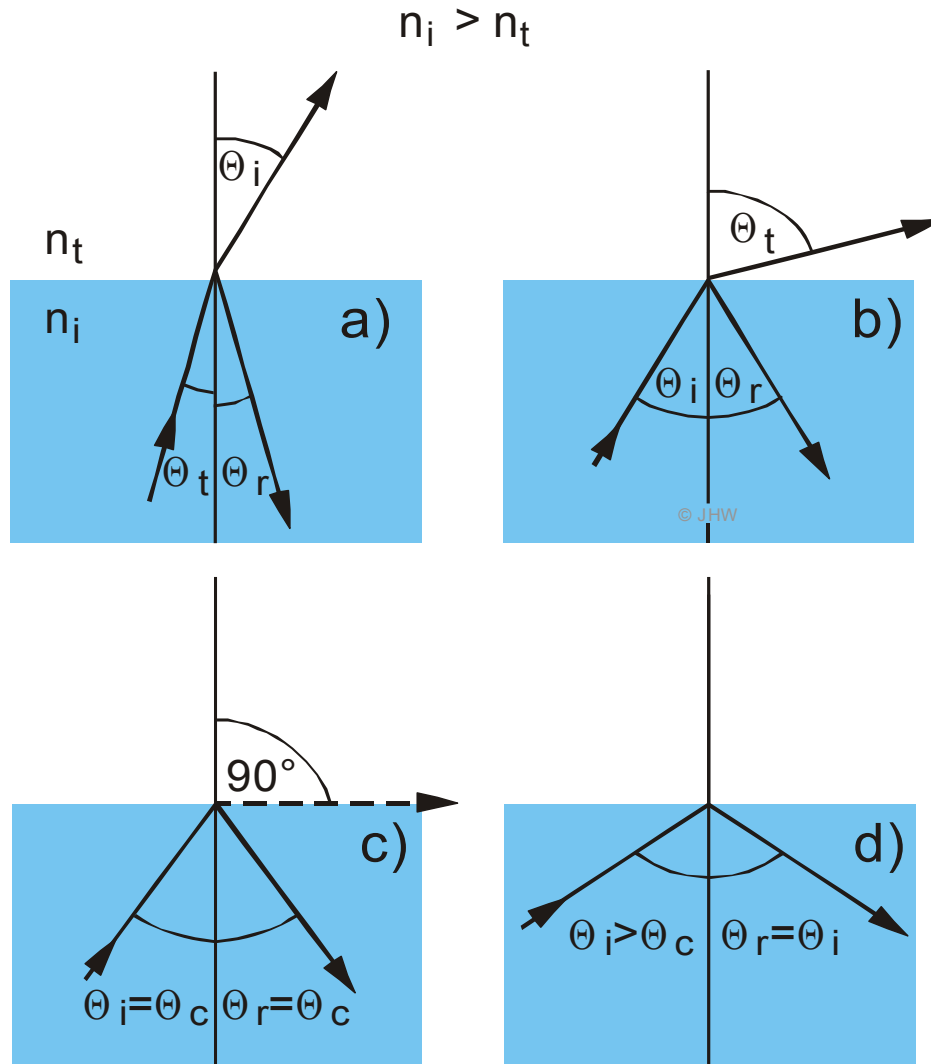
$$n_i \sin \Theta_i = n_t \sin \Theta_t$$



Upon refraction, the ray bends towards the normal.

Figure 5.16: Refraction for two different angles of incident light.

Internal total reflection requires radiation coming from the side with the higher optical density ($n_i > n_t$).



For $\Theta_t = 90^\circ$ and $n_t = 1$, we find

$$\sin \Theta_c = n_i^{-1}$$

for the *critical* angle of the impinging light.

Figure 5.17: Internal total reflection.

Figure 5.18 shows a photon being generated somewhere within the crystal. All directions for the emission of the photon have equal probability. However, only those photons hitting the surface within a cone of half angle Θ_c can leave the crystal. The angle Θ_c is the angle of total reflection. For semiconductors with $n_{semi} = 3.5$ it holds $\Theta_c = 17^\circ$. The small angle is mainly responsible for the low optical efficiency of LEDs.

The ratio of the emitted to the transmitted light is the same as the hemispherical angle Ω_e (spanned by Θ_c) to the total solid angle $\Omega_0 = 4\pi$.

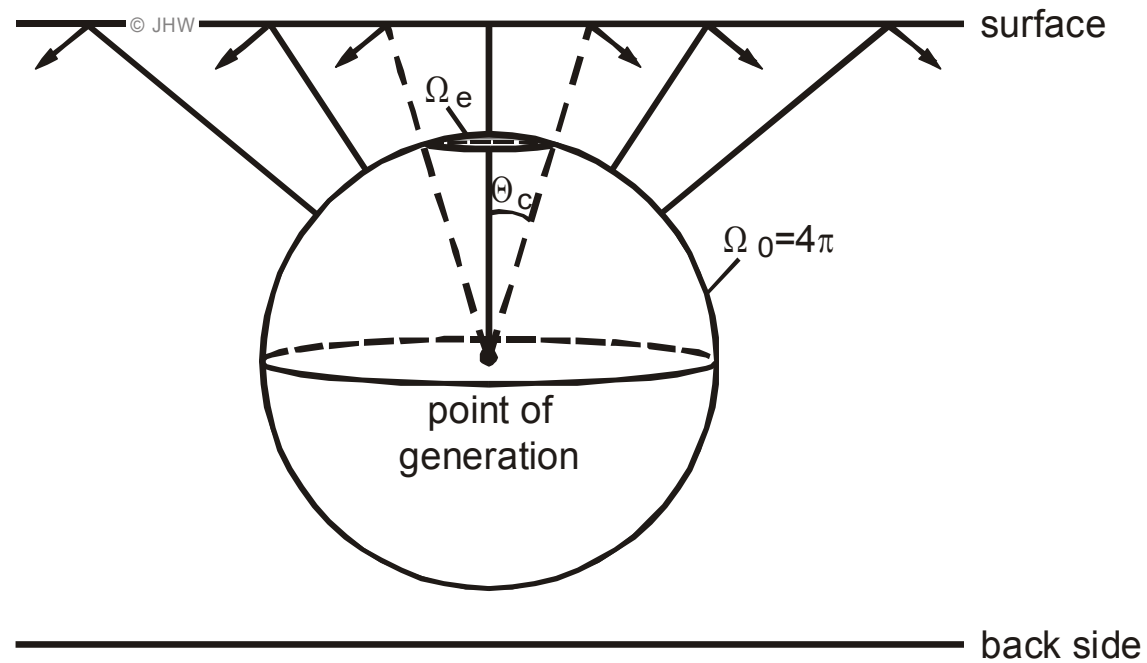


Fig. 5.18 Loss due to total reflection.

$$\eta_{tr} = \frac{\Omega_e}{\Omega_0} = \frac{2\pi(1 - \cos \Theta_c)}{4\pi}$$

$$= (1 - \cos \Theta_c) / 2 \approx 2.2\%$$

way out: encapsulation in an epoxy ($n = 1.5$) increases the angle and (non-)total reflection efficiency to $\Theta_c = 25^\circ$ and $\eta_{tr} = 9.4 \%$.

The total optical efficiency

$$\eta_{opt} = \eta_{non-abs} r_\Phi \eta_{tr}$$

is usually around 5 %. Higher efficiencies require to pre-select preferential emission of photons into the narrow cone of (non-) total reflection.

Table 5.6: Critical angle for total internal reflection in optoelectronic materials

material	index of refraction n_i	critical angle Θ_c
glass	1.5-1.7	35° - 41°
Si	3.45	16.8
GaAs	3.4	17.1
Ge	3.9	14.6

5.5 Organic Light Emitting Diodes (OLEDs)

Advantage [5.7, 5.8]

- simple technology on glass substrates
- cheap, low tech
- lattice matched substrate not necessary
- variable band gap
- flexible

External quantum efficiency: 5 %,
best result: 8 % at 510 nm

Luminous efficacy: > 20 lm/W
best result: 31 lm/W at 510 nm

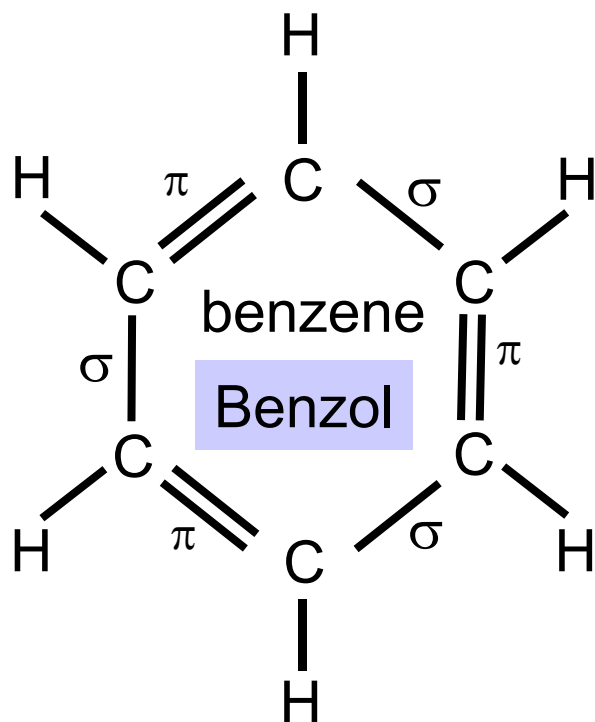
Life time: > 10.000 h

Operating voltage: < 5 V

Problems to be solved:

- stability under humid conditions
- ohmic losses
- luminous efficacy

5.5.1 Chemical bonds of carbon / double bonds



Polymers are derivatives of benzene, where single and double bonds alternate between carbon atoms.

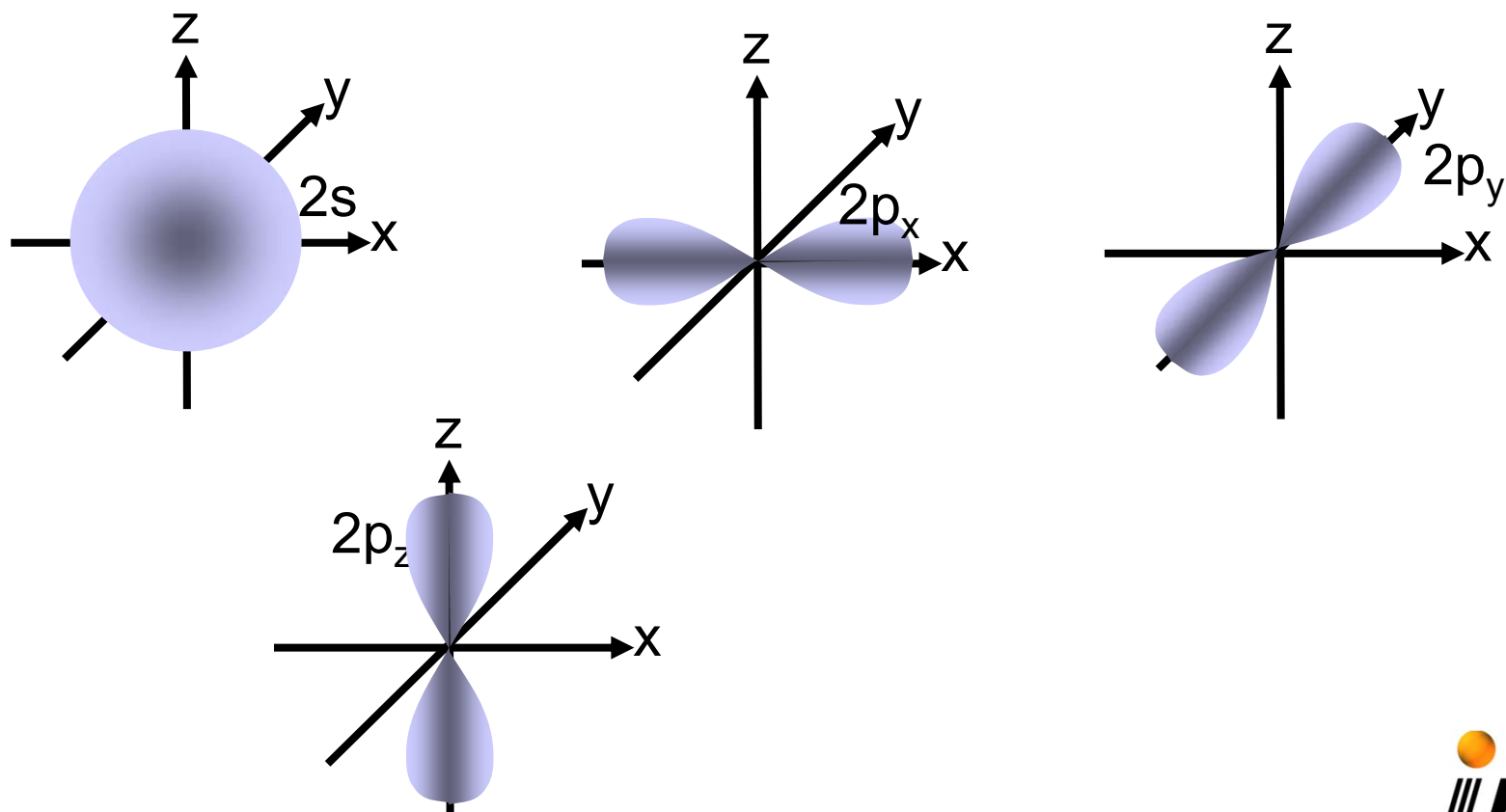
Double bonds consist of one σ and one π -bond. π -bonds are responsible for electric current transport within polymer chain.

Figure 5.19: alternating σ - and π -bonds in a benzene molecule

The four valence electrons of an isolated carbon atom occupy one 2s-orbital and three 2p-orbitals.

σ -bond

Originates from a 2s-orbital and two of three 2p-orbitals via hybridization (sp^2 -hybridization).



sp^2 -hybridization

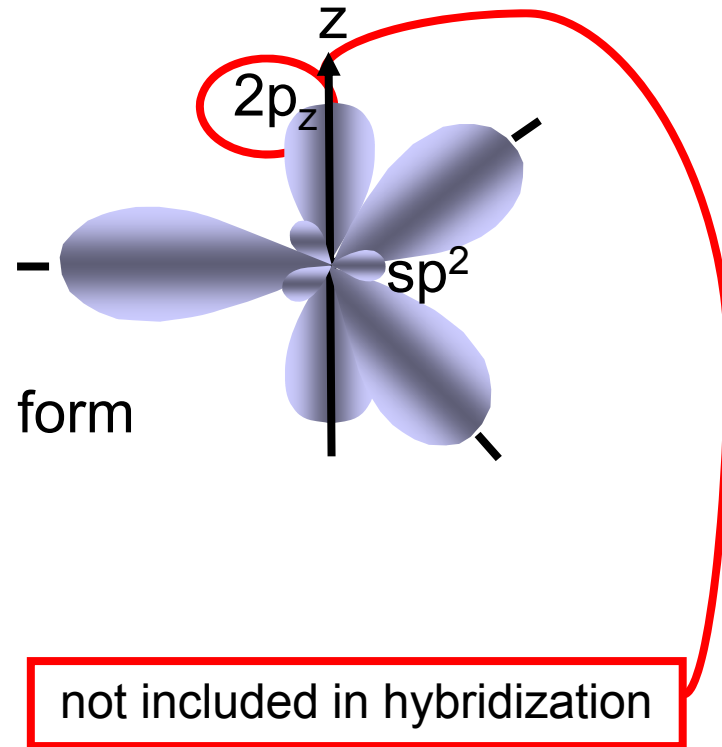
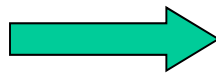


Figure 5.20: $2s$, $2p_x$ and $2p_y$ orbitals form three sp^2 orbitals via hybridization

sp^2 hybridization results in three orbitals located in a plane. The angle between the orbitals is 120 degrees.

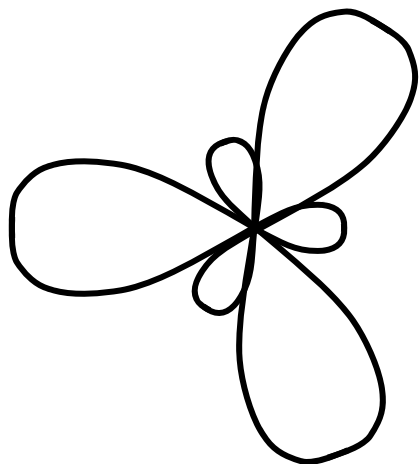


Figure 5.21: sp^2 orbitals, top view

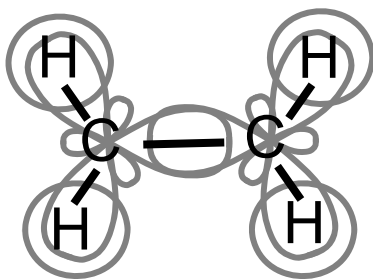


Figure 5.22: Ethene molecule, σ -bonds

Carbon atoms connect (react) with neighboring atoms via sp^2 orbitals. The overlapping orbitals form so-called σ -bonds.

Examples:
benzene, ethene

The single bonds within of benzene ring are σ -bonds.

The p_z -orbitals of Fig. 5.20 are not shown in Figs. 5.21, 5.22. They stand perpendicularly to the paper plane!

π -bond

One of the 2p orbitals ($2p_z$) remains un-hybridized. This orbital is oriented in perpendicular direction to the sp^2 -orbitals.

Example:
ethene

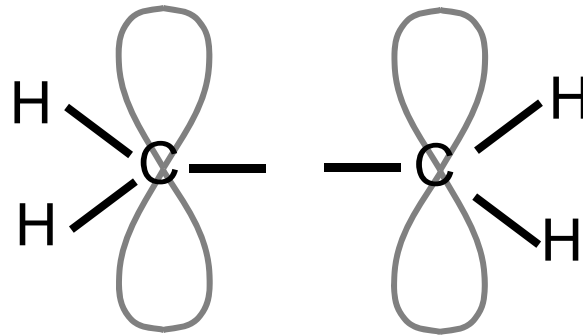


Figure 5.23: Separated CH_2 -molecules (radicals)

If the carbon atoms connect (react), the two $2p_z$ -orbitals overlap and form a so-called π -bond.

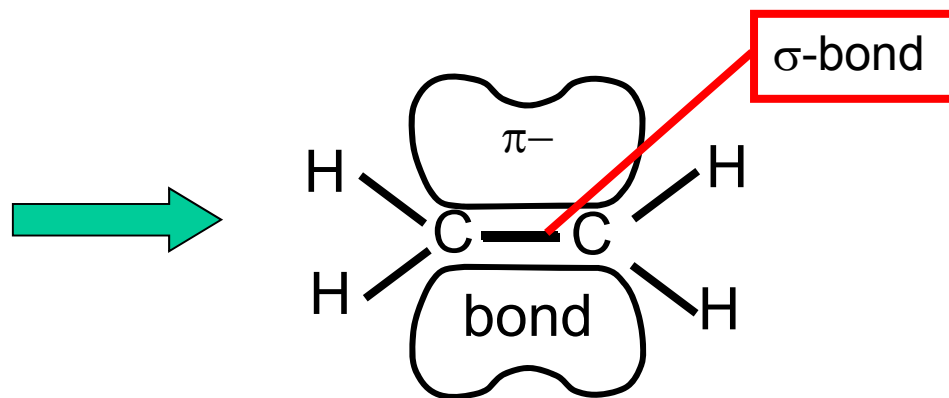


Figure 5.24: Two CH_2 -molecules form an ethene molecule. One of the double bonds is a π -bond.

In case of ethene, the two electrons forming the π -bond are delocalized along the bond.

In case of benzene the delocalization of π -electrons occurs within the whole molecule. Therefore double and single bonds are not distinguishable.

5.5.2 The band gap

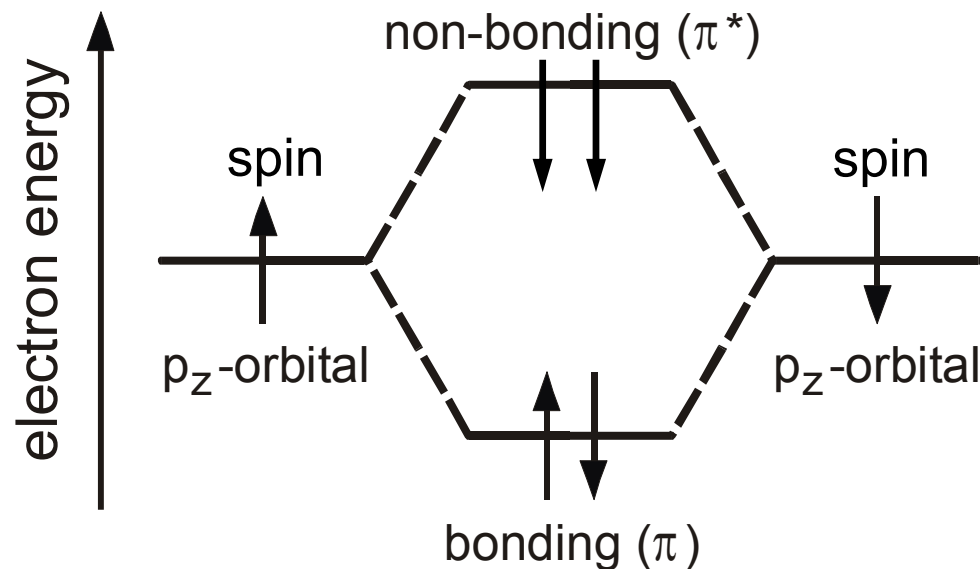
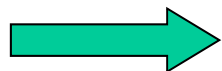


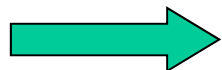
Figure 5.25: bonding (π) and a non-bonding (π^*) state lead to two energy states within the molecule

- Each of the two carbon atoms delivers one electron for the chemical π -bond.
- Energy of the electrons in the bonding state π is lower than energy of separated electrons. In the bonding state the electrons must have opposite spin.
- Energy of non-bonding state is higher than the energy of separated electrons, with the consequence, that the chemical bond will break. However: Due to the σ -bond, the molecule cannot brake up and so also the π -bond with its two energy states keeps existing.

If the distance between the carbon double bonds within polymer chains is not too far, the π - and π^* -states overlap and the electrons are delocalized along the whole molecule.



Bonding π -states form the valence band



Non-bonding π^* -states form the conduction band

5.5.3 Homo / Lumo

Homo = highest occupied molecule orbital
= π -bonds
= valence band

Lumo = lowest unoccupied molecule orbital
= π^* -bonds
= conduction band

5.5.4 Organic semiconducting materials

a) PPV and modifications

PPV = Poly-Phenylene-Vinylene

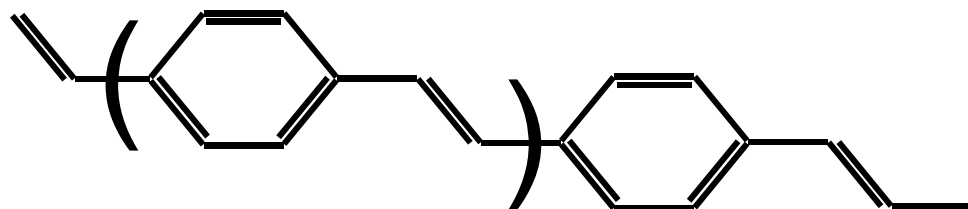


Figure 5.26: Structure and molecular unit of Poly-Phenylene-Vinylene.

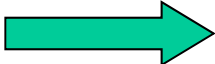


©JHW

The band gap E_g of PPV is

$E_g = 2.5 \text{ eV}$  Green light

By modifying PPV, the band gap E_g and color can be tuned.

Polythiophene		Red
PPV		Green
Polyfluorene		Blue

Current transport takes place along the molecules.

b) Small molecules

Single separated molecules (no polymers)

Example:

Tris-(8-hydroxyquinoline)-aluminium = Alq₃

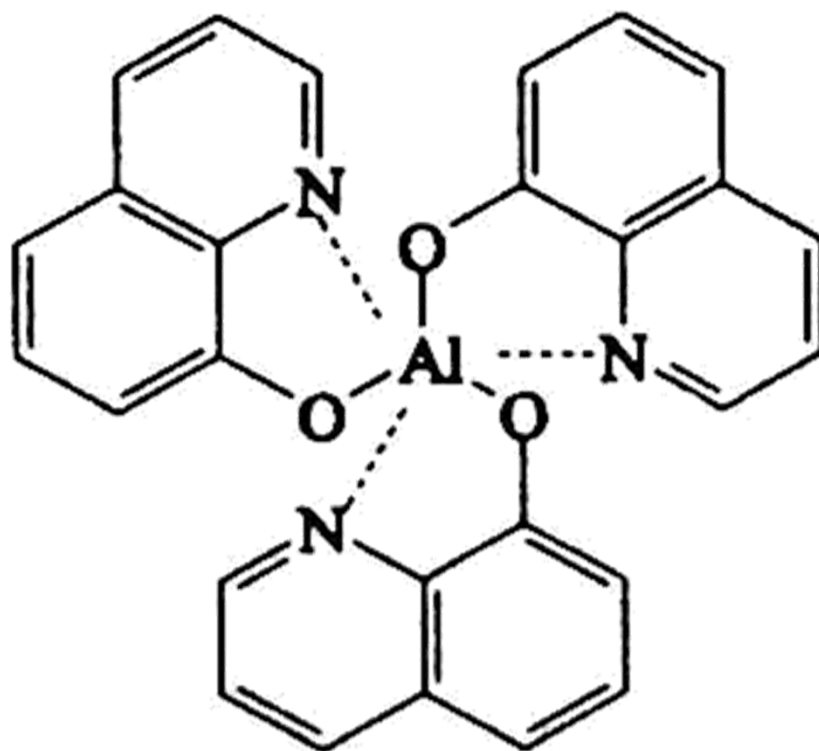


Figure 5.26: Structure of Alq_3 ; melting temperature $T_m = 414 \text{ K}$, formula $\text{C}_{27}\text{H}_{18}\text{AlN}_3\text{O}_3$, molecular mass $M_{mol} = 459.44$.

5.5.5 Operating principle of OLEDs

Generally different to LEDs made from inorganic semiconductors!

- Inorganic LEDs use pn-junction with space charge region.
- Minorities (electrons, holes) are injected from the reservoirs of majorities (n-, p-type doped regions) into the space charge region. Large diffusion length L due to large diffusion constants $D = \mu kT/q$ (high mobility $\mu = 1000 \text{ cm}^2/\text{Vs}$) leads to recombination of electrons and holes within the whole space charge region of width $w \cong 1 \text{ }\mu\text{m}$; it holds $L > w$.

Organic LEDs operate without pn-junction!

- The low mobilities μ_n , μ_p of electrons and holes in organic semiconductors lead to significantly lower diffusion length.
- $\mu_n = 10^{-7} - 10^{-5} \text{ cm}^2/\text{Vs}$
- $\mu_p = 10^{-5} - 10^{-3} \text{ cm}^2/\text{Vs}$
- Charge carriers cannot diffuse far, but are tied to a particular molecule for a long time.
- Electrons and holes recombine if they enter the same molecule.
- Radiant recombination occurs if no defects are present within the diffusion lengths of electrons and holes.
- Low diffusion length results in a relatively high acceptable defect concentration.
- As long as the diffusion length is lower than the average distance between defects, electrons and holes rather see each other and recombine by emitting a photon, than recombining at a defect.

5.5.6 OLED technology

Electric contacts for [5.9]

- Electrons to conduction band (LUMO) with low work function Φ_a , e.g. Ca , $\Phi_a(\text{Ca}) = 2.8 \text{ eV}$
- Holes to valence band (HOMO) with high work function Φ_a , e.g. ITO , $\Phi_a(\text{ITO}) = 4.8 \text{ eV}$

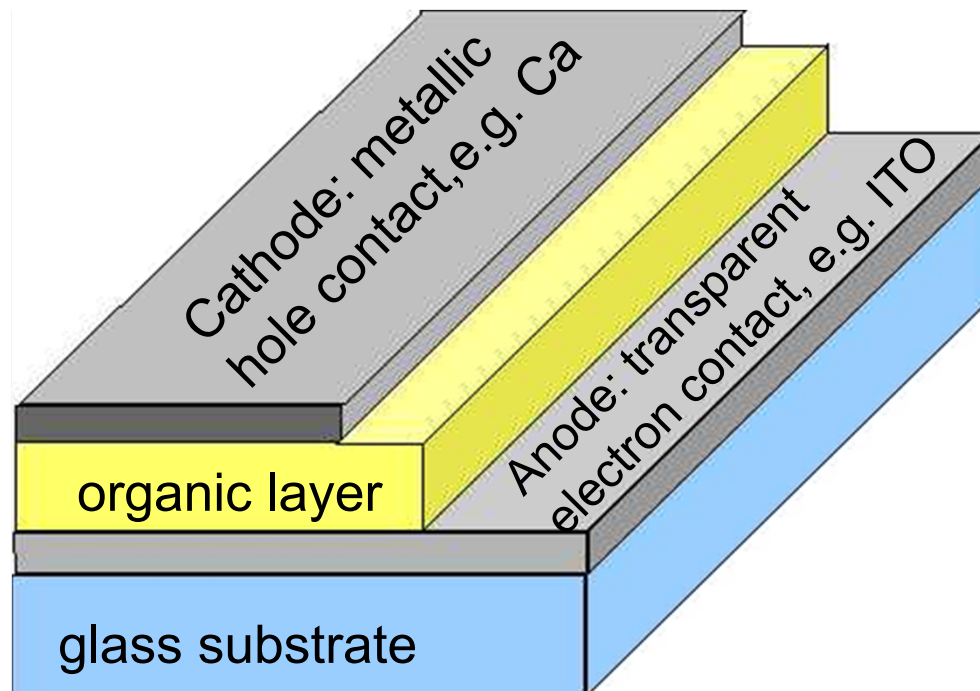


Figure 5.27: The transparent contact for electrons (transparent conductive oxide TCO) is directly deposited on the glass substrate. Organic layer is de-positied via spin coating or evaporation in vacuum. Metallic cathode layer is deposited by evaporation in vacuum.

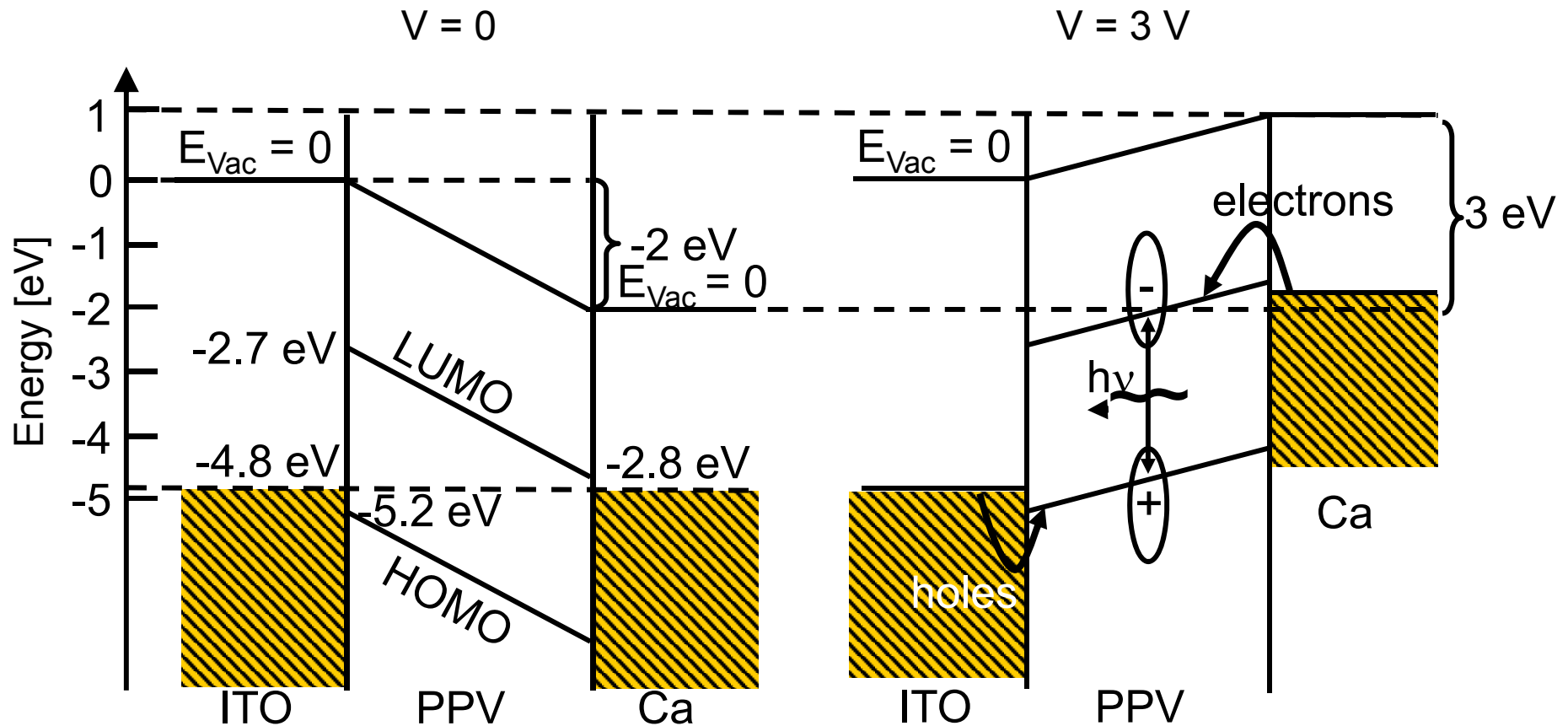


Figure 5.28: Electron affinities $q\chi_{PPV} = 2.7 \text{ eV}$; $q\chi_{ITO} = 4.8 \text{ eV}$; $q\chi_{Ca} = 2.8 \text{ eV}$; band gap $E_g(PPV) = 2.5 \text{ eV}$. Applied voltage $V = 3 \text{ V}$ leads to injection of electrons from the Ca-contact and holes from the transparent ITO contact. Electrons have to surmount a barrier of $E_B = 0.1 \text{ eV}$ and holes a barrier of $E_B = 0.4 \text{ eV}$.

5.5.7 Luminous efficacy of OLEDs

Luminous efficacy of OLEDs comparable to incandescent lamps and conventional LEDs.

Best efficacy $\eta_V = 31 \text{ lm/W}$

5.5.8 Applications

- Flexible substrates
- Large area displays
- Each kind of pattern, size and shape realizable

5.5.9 Problems, research tasks

a) Lifetime and stability

PPV and other polymers, as well as Ca-contacts are extremely sensitive to humidity and oxygen. Fast degradation of devices if in-diffusion of water vapor and O_2 is not avoided.



Improved encapsulation and sealing technology to be developed

b) Large area displays (> 8 inch $\hat{=}$ 10^6 pixel)

- Active matrix pixel control using thin-film transistors (TFTs) necessary.
- In contrast to liquid crystal displays (LCDs) OLEDs require considerable dc-currents at high luminosities.

b) Large area displays (> 8 inch $\triangleq 10^6$ pixel)

- Thin film transistors with high field effect mobility required to avoid considerable energy losses (high voltage drop).
- Mobility of conventional amorphous silicon TFTs not sufficient.
- Polycrystalline silicon TFTs necessary.

c) Ohmic contacts

- High voltage losses at the contacts
- Surface roughness of ITO leads to shortcircuits

5.6 Literature, Internet Links

[5.1]: <http://www.darpa.mil/mto/suvos/presentations/Schetzina.pdf>

[5.2]: <http://www1.eere.energy.gov/buildings/ssl/efficacy.html>

[5.3]: http://led-info.de/grundlagen/led_weiled.htm

[5.4]: <http://www.iaf.fhg.de/bullelu/bericht-emrs.pdf>

[5.5]: http://www.nichia.com/specification/catalogue/200902/NICHIA_Corporation_200902_LED_Catalogue.pdf

[5.6]: Niedrig, H. (Hrsg.), *Bergmann Schäfer Lehrbuch der Experimentalphysik, Band 3*, 9. Aufl., (Walter de Gruyter, Berlin New York 1993), S. 50.

[5.7]: http://w4.siemens.de/Ful/en/archiv/zeitschrift/heft2_99/artikel08/index.html

[5.8]: <http://kottan-labs.bgsu.edu/teaching/workshop2001/chapter7.pdf>

[5.9]: [http://www.ee.ust.hk/eee/course/elec695/slides/Organic Light-emitting Diodes by Haiying.ppt](http://www.ee.ust.hk/eee/course/elec695/slides/Organic%20Light-emitting%20Diodes%20by%20Haiying.ppt)

6. Lasers

Laser

light amplification by
stimulated emission of radiation

6.1 Fundamental processes

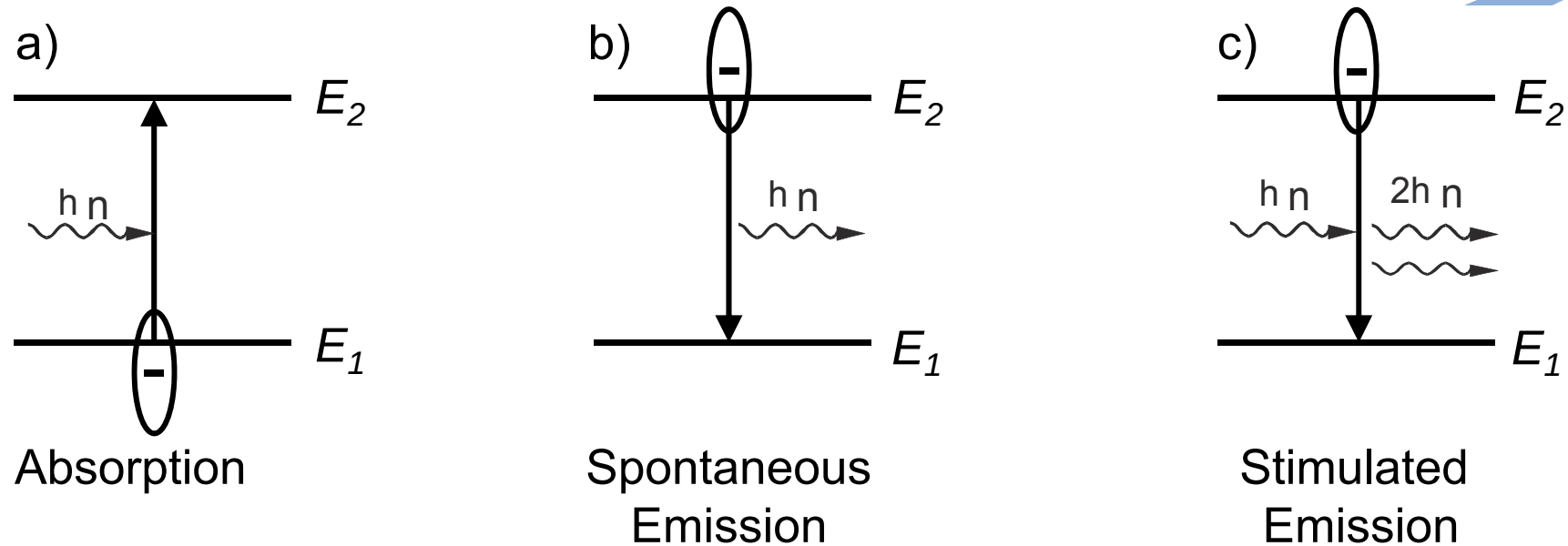


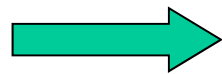
Figure 6.1: a) Atom or molecule in ground state E_1 absorbs photon of energy $h\nu = E_2 - E_1$ and is excited to energy state E_2 . b) Excited atom or molecule in energy state E_2 spontaneously returns to the ground state E_1 by emitting a photon of energy $h\nu = E_2 - E_1$. c) Photon of energy $h\nu$ forces excited atom or molecule to emit a photon with same energy, direction, and phase. The atom returns to the ground state E_1 .

6.2 Occupation probability in thermodynamic equilibrium

$$\frac{\text{Number of Electrons per cm}^3 \text{ in } E_2}{\text{Number of Electrons per cm}^3 \text{ in } E_1} = \frac{N_2}{N_1} = e^{-(E_2 - E_1)/kT}$$

Example: $E_2 - E_1 = 2.4 \text{ eV}$ green light

$$T = 300 \text{ K}$$

 $N_2/N_1 = 5 \times 10^{-41}$

Under room temperature and thermodynamic equilibrium conditions it holds:

$$N_2 \ll N_1$$

©JHW



6.3 Microscopic analysis

6.3.1 Energy density, power density

Energy density ρ	[eV/cm ³]
Photon density Φ	[Photons/cm ³]
Photon flux density Φ_j	[Photons/cm ² s]
Power density I	[eV/cm ² s] = [W/cm ²]

It holds

$$\Phi = \Phi_j / c$$

$$\rho = h\nu \Phi$$

$$I = h\nu \Phi_j$$

6.3.2 Absorption

Excitation of atoms from state E_1 into E_2 (absorption) leads to decrease of power density I and photon flux density Φ_j [6.1] according to

$$\begin{aligned} I(x) &= I_0 e^{-\alpha x}, \\ \Phi_j(x) &= \Phi_0 e^{-\alpha x}, \end{aligned} \quad (6.1)$$

with α = absorption coefficient, $\alpha = [\text{cm}^{-1}]$.

The number density N_1 , N_2 of electrons in energy states E_1 , E_2 changes according to

$$\left. \frac{dN_1}{dt} \right|_{abs} = - \left. \frac{dN_2}{dt} \right|_{abs} = \frac{d\Phi}{dt}, \quad (6.2)$$

with Φ = photon density, $\Phi = [\text{Photons}/\text{cm}^3]$.

Each absorption process consumes one photon!



From

$$\frac{d\Phi}{dt} = \frac{1}{c} \frac{d\Phi_j}{dt} = \frac{1}{\frac{dx}{dt}} \frac{d\Phi_j}{dt} = \frac{d\Phi_j}{dx}$$

and Eq. 6.2 follows

$$\left. \frac{dN_1}{dt} \right|_{abs} = \frac{d\Phi_j}{dx}. \quad (6.3)$$

Beer's absorption law results in

$$\frac{d\Phi_j}{dx} = -\alpha \Phi_j = \left. \frac{dN_1}{dt} \right|_{abs}. \quad (6.4)$$

The change of number density N_1 is given by

$$\left. \frac{dN_1}{dt} \right|_{abs} = -\sigma_{12} N_1 \Phi_j, \quad (6.5)$$

with σ_{12} = absorption cross section.



Comparing Eq. 6.4 and Eq. 6.5 leads to

$$\alpha = \sigma_{12} N_1. \quad (6.6)$$

σ_{12} may be expressed by Einstein's coefficient B_{12} , if the frequency dependence of the absorption line form function $F(\nu)$ is taken into account, according to

$$\sigma_{12} = B_{12} \frac{h\nu_{12}}{c} / \int F(\nu) d\nu \approx B_{12} \frac{h\nu}{c} / \Delta\nu \quad (6.7)$$

6.3.3 Spontaneous emission

Excited states E_2 have a limited lifetime τ [6.2]. The number density N_2 decreases with time t according to

$$N_2(t) = N_2(0)e^{-t/\tau}. \quad (6.8)$$



Differentiation leads to

$$\left. \frac{dN_2}{dt} \right|_{sp} = -\frac{N_2}{\tau} = -AN_2, \quad (6.9)$$

with A = Einstein's coefficient for absorption, $A = 1/\tau$.

6.3.4 Stimulated emission

Similar to absorption, the number density N_2 of excited state E_2 changes by *stimulated* emission [2.3]

$$\left. \frac{dN_2}{dt} \right|_{stim} = -\sigma_{21}N_2\Phi_j, \quad (6.10)$$

with σ_{12} = cross section of stimulated emission.

In contrast to absorption, stimulated emission leads to an increase of the power density I , the inversion of Beer's absorption law

$$\left. \frac{dI}{dx} \right|_{stim} = \sigma_{21}N_2I. \quad (6.11)$$



The power density I increases, in case of stimulated emission. Light is amplified. A stimulated photon has the same direction as the irradiated photon; we obtain *coherent radiation*. The electromagnetic waves have the same frequency and phase.

A quantum mechanical calculation yields

$$\sigma_{12} = \sigma_{21} \quad \text{or} \quad B_{12} = B_{21}. \quad (6.12)$$

6.3.5 Relation between stimulated and spontaneous emission

Equations 6.9 and 6.10 result in

$$\frac{\left. \frac{dN_2}{dt} \right|_{stim}}{\left. \frac{dN_2}{dt} \right|_{sp}} = \frac{\sigma_{12}}{A} \Phi_j \quad (6.13)$$

The higher the photon density Φ_j , the larger is the stimulated emission processes!

Stimulated emission is enhanced at locations within a laser, where the photon density is high. With the resonator, we create such places! Feedback of optical radiation is achieved using an optical resonator. As a result, standing waves generate locations with high and low radiation intensity (or photon density).

6.3.6 Gain factor G

Optical gain [6.4] by stimulated emission counteracts absorption. In total, the change of power density is given by

$$\frac{dI}{dx} = \left. \frac{dI}{dx} \right|_{abs} + \left. \frac{dI}{dx} \right|_{ind}$$

$$= -\alpha I + \sigma_{12} N_2 I$$

$$\quad \downarrow$$

$$= \sigma_{12} N_1$$

$$= -\sigma_{12} (N_1 - N_2) I(x)$$

©JHW



Integration leads to

$$G = \frac{I(x)}{I_0} = e^{-\sigma(N_1 - N_2)x}, \quad (6.14)$$

which is the generalization of Beer's absorption law.

Amplification of radiation, gain factor $G > 1$, requires more atoms to be in the excited state E_2 than in the ground state E_1 .

1st laser condition

$$N_2 > N_1$$

Inversion

The differential gain g is given by

$$\begin{aligned} g &= -\sigma_{12}(N_1 - N_2) = -\sigma_{12}N_1 \left(1 - \frac{N_2}{N_1}\right) \\ &= -\alpha \left(1 - \frac{N_2}{N_1}\right) = \alpha \left(\frac{N_2}{N_1} - 1\right) \end{aligned}$$

It holds

$$I(x) = I_0 e^{gx}$$

©JHW



6.4 Setup of a laser

Each laser consists of three components, as illustrated in Fig. 6.2.

- a) Optical or electrical *pumping system* generates inversion, (1st lasing condition).
- b) *Optical resonator* selects frequency or wavelength of light and enables increase of power density via optical feedback, (2nd lasing condition).

Standing waves generate locations of high and low radiation density.

- c) *Optically active medium* transfers pumping energy into laser radiation.

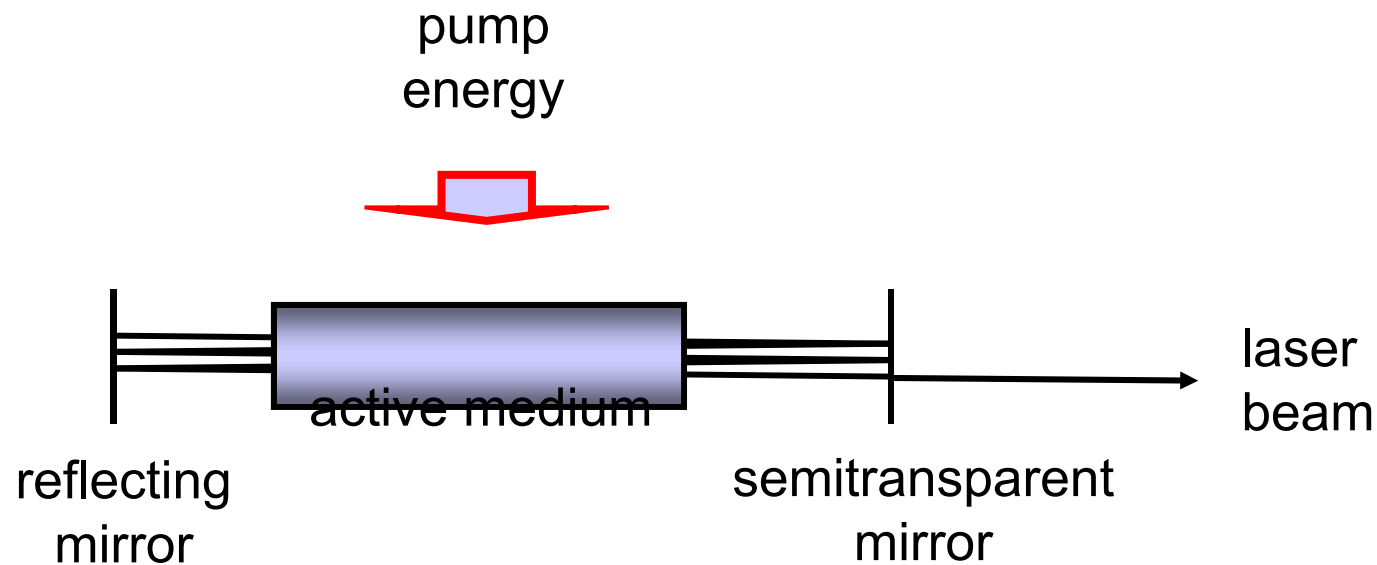


Figure 6.2: Electrical or optical pumping leads to inversion of active medium. Optical resonator increases photon flux density and thus increases stimulated emission. Resonator also defines the wavelength of emitted laser beam.

6.5 Optical resonator

- Most simple resonator consists of two in parallel arranged flat mirrors (Fabry-Perot-Resonator) [6.5].
- Resonator amplifies electromagnetic waves, which satisfy the 2nd laser condition

$$m \frac{\lambda_m}{2} = d$$

with λ_m = wavelength in active medium, and d = length of resonator, m = integer number.



m multiples of half wavelength must fit into the resonator length

- Laser mirrors generate standing wave via optical feedback.

- Distance between subsequent intensity maxima equals one half wavelength.
- Intensity maxima oscillate with frequency

$$\nu = \frac{c_m}{\lambda_m} = \frac{c}{n_r \lambda_m} = \frac{c}{n_r} \frac{m}{2d},$$

with c_m = speed of light in active medium,
 c = speed of light in vacuum
 n_r = refraction index of active medium.

- Difference $\Delta \nu$ between subsequent frequencies ν_{m+1} , ν_m (longitudinal modes) is given by

$$\Delta \nu = \nu_{m+1} - \nu_m = \frac{c / n_r}{2d}$$

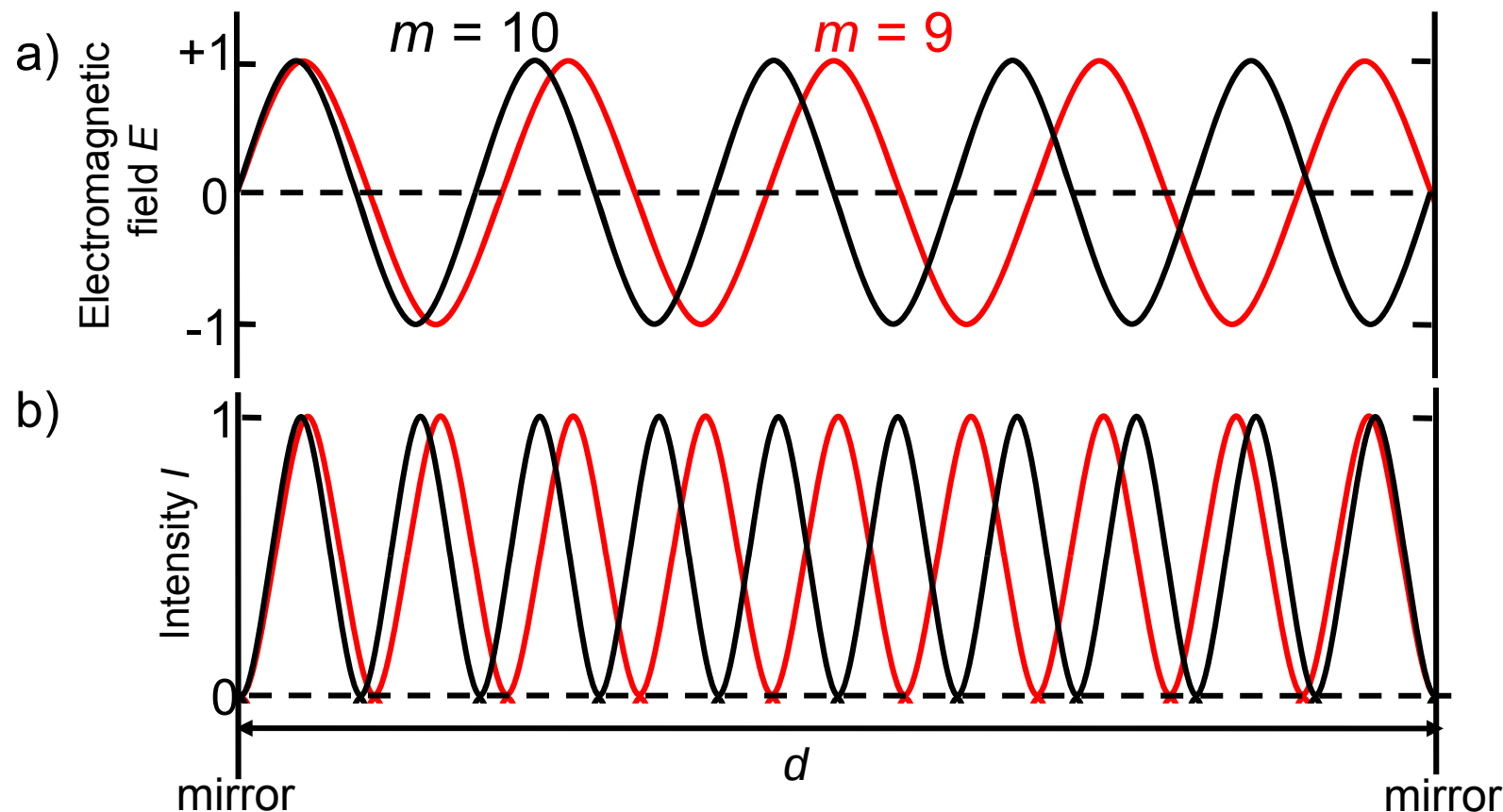


Figure 6.3: Electromagnetic waves, which satisfy the second laser condition, $m\lambda/2 = d$, with $m = 10$ (black curve) and $m = 9$ (red curve), and therefore can be amplified. a) normalized electromagnetic field strength E . b) normalized intensity $I \sim E^2$ of electromagnetic field.

6.5.1 Stable resonators

a) Fabry-Perot resonator

- Both laser mirrors must be exactly flat (deviation less than $\lambda/20$).
- Reflecting surfaces of both laser mirrors must be adjusted exactly in parallel. Small deviations result in unstable resonator.

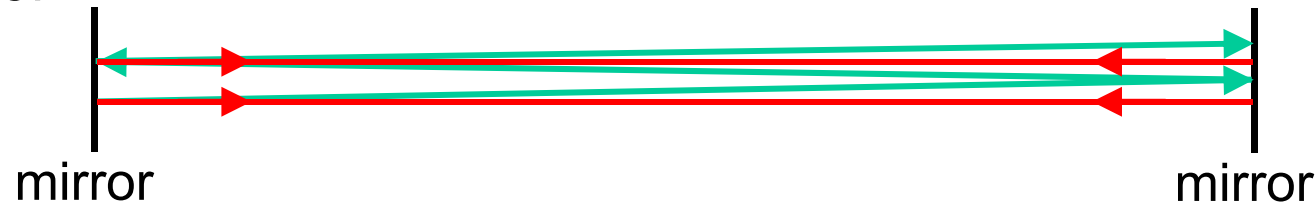


Figure 6.4: Schematic drawing of Fabry-Perot resonator. Small deviations in orientation of laser beam force the beam out of the resonator after a few round trips (green laser beam). This type of resonator is often used in lasers with high optical gain, such as pulsed lasers or semiconductor lasers. For lasers with low gain, e.g. argon-ion-lasers, its optical losses are too high.

b) Confocal resonator

- Radii R_1 , R_2 of curved mirrors equal resonator length d .
- The foci of both mirrors meet in the center of resonator. Focal lengths $f_1 + f_2 = d$.

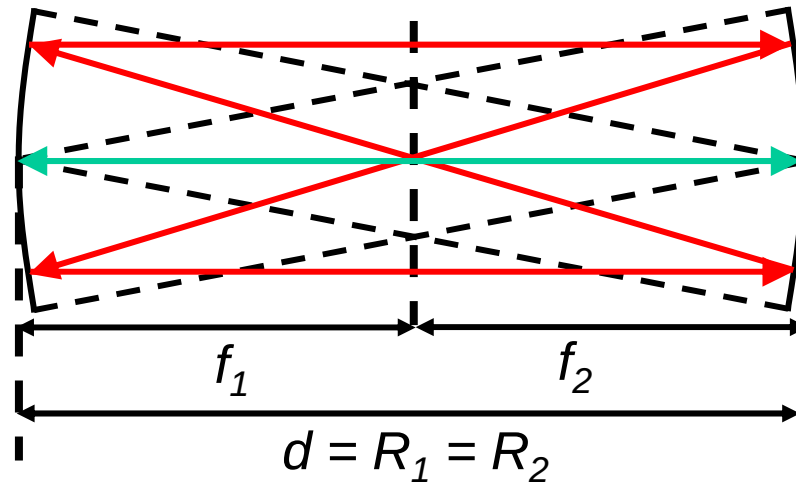


Figure 6.5: Schematic drawing of confocal resonator. Even considerably deviating beams (red beam) stay within the resonator. This type of resonator is insensitive to small deviations in adjustment. It is often used in cw-lasers like He-Ne-lasers.

Resonators with spherical (curved) mirrors having a curvature radius $L < R < \infty$ are *stable resonators* [6.6].

Stable resonators enable the formation of standing electromagnetic waves.

Solution of Maxwell's equations results in a *Gaussian distribution* of the electromagnetic field strength perpendicular (transversal) to the beam direction.

 Gaussian beam

Solution of differential equations leads to an Eigenvalue problem with the result of so-called

Transversal Electromagnetic Modes TEM_{mn} ,

with the Eigenvalues m and n .

m, n are integer numbers and define the order of the (Eigen)mode. Examples are shown in Fig. 6.6 and 6.7.

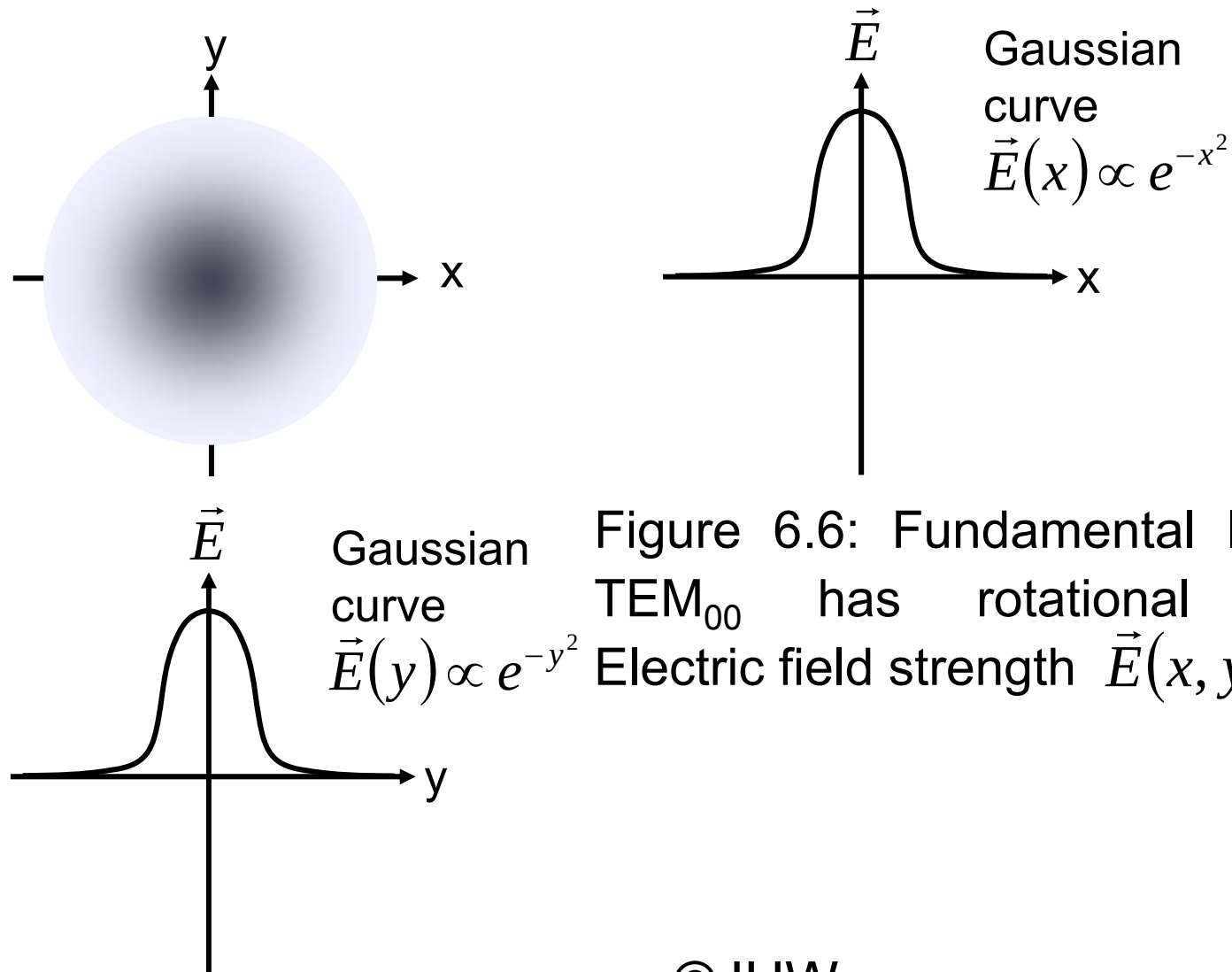


Figure 6.6: Fundamental laser mode TEM_{00} has rotational symmetry. Electric field strength $\vec{E}(x, y) \propto e^{-(x^2+y^2)}$.

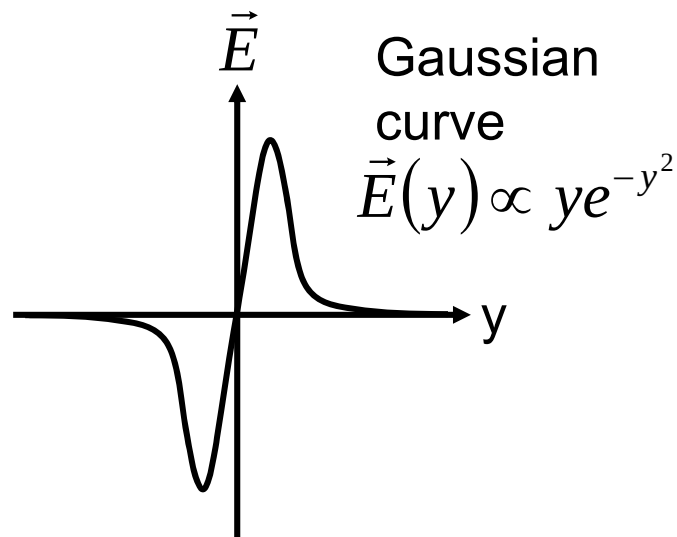
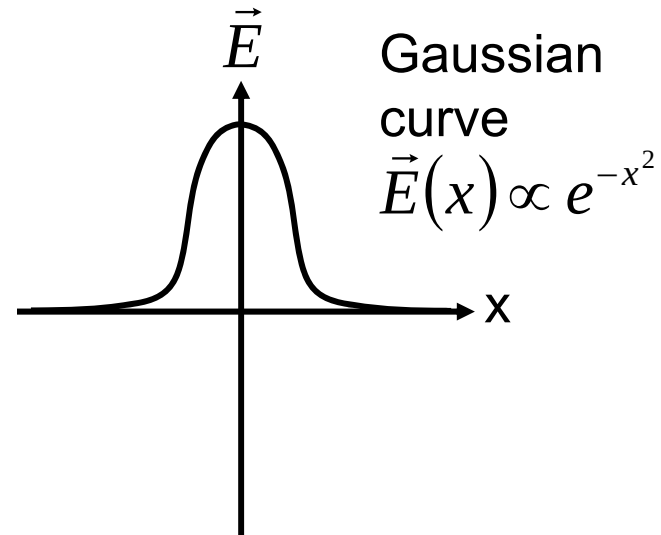
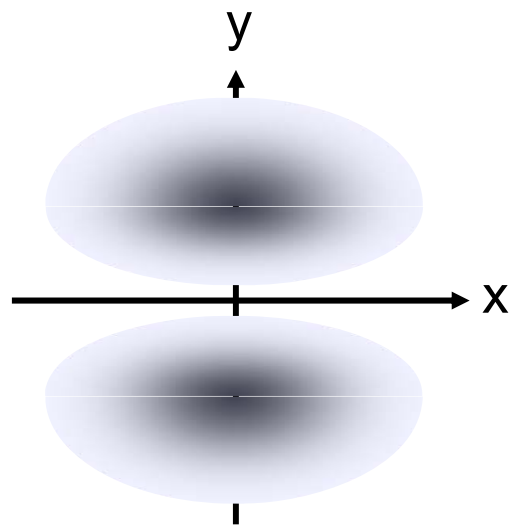


Figure 6.7: The electric field strength $\vec{E}$ of laser mode TEM_{01} has zero intensity along the x -axis and two maxima with opposite phase centered on the y -axis. $\vec{E}(x, y) \propto ye^{-(x^2+y^2)}$.

6.5.2 Unstable resonators

Are generally used in pulsed lasers or in cw-lasers with high optical gain, e.g. CO_2 -laser, and large aperture (ratio between diameter and length of the active medium) [6.7].

- Unstable resonators do not feed back part of the emitted laser radiation.
- In contrast to stable resonators, unstable resonators do not allow standing waves to develop.
- Each photon will make only a limited number of round trips in the optically active medium, before it leaves the laser.

Realization of unstable resonator is shown in Fig. 6.8.

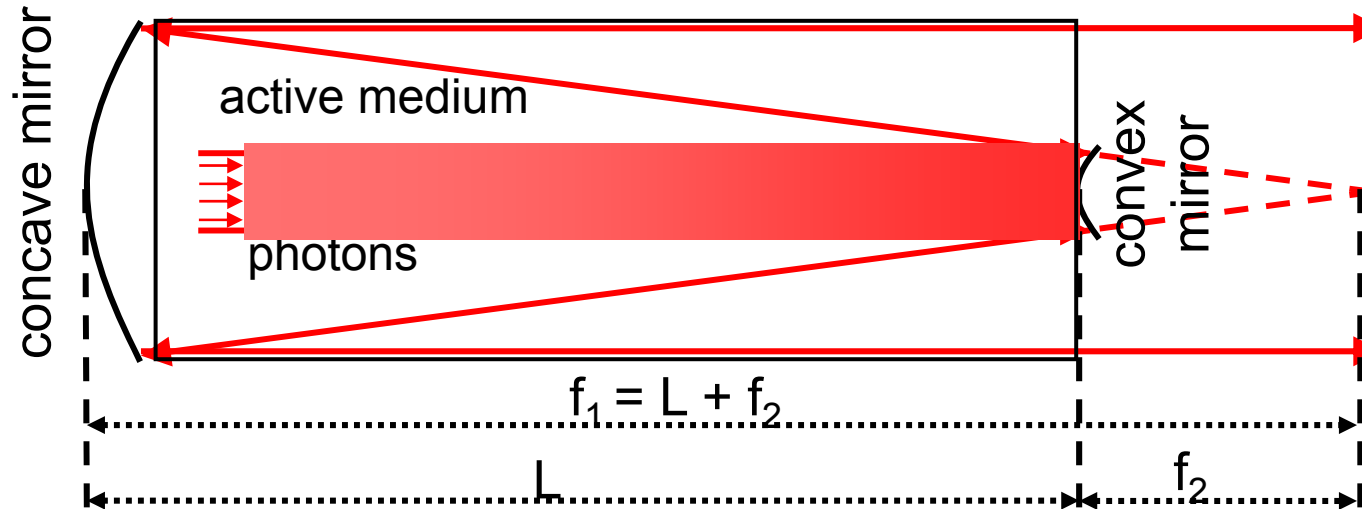


Figure 6.8: Most common unstable resonator type enables three round trips, before laser beam leaves the active medium. At the beginning of the laser pulse, only spontaneously emitted photons from the left center in the active medium directed in parallel to the center line have the chance to be amplified by stimulated emission during three round trips. All other spontaneous photons leave the active medium much earlier and are therefore amplified much less, because amplification follows an exponential law, $I(x) = I_0 e^{gx}$.

6.6 Line width

6.6.1 Natural line width [6.8]

follows from Heissenberg's uncertainty relation

$$\Delta x \Delta p \geq h, \quad (6.15)$$

with Planck's constant $h = 6.6 \times 10^{-34}$ Js, Δx the position uncertainty and Δp the momentum uncertainty.

Energy E , momentum p and mass m are related according to

$$E = p^2/(2m). \quad (6.16)$$

Differentiation of Eq. (6.16) leads to

$$\frac{dE}{dp} = \frac{p}{m} = v = \frac{dx}{dt},$$

or

$$\Delta p = \Delta E \Delta t / \Delta x.$$

Now we can write Heissenberg's uncertainty relation as

$$\Delta E \Delta t \geq h. \quad (6.17)$$

Here Δt is the average lifetime of an excited atom in energy level E_2 , see Fig. 6.9. From Heissenberg's uncertainty relation it follows that the energy level E_2 is not exactly defined. Its uncertainty ΔE amounts to

$$\Delta E \geq h / \Delta t. \quad (6.18)$$

Consequently, the frequency ν or wavelength λ of the emitted photon have limited band widths $\Delta \nu$ and $\Delta \lambda$ according to

$$\Delta \nu = \Delta E / h, \quad (6.19)$$

$$\nu = c / \lambda \quad \longrightarrow \quad |\Delta \lambda| = \frac{c}{\nu^2} \Delta \nu.$$

©JHW

(6.20)

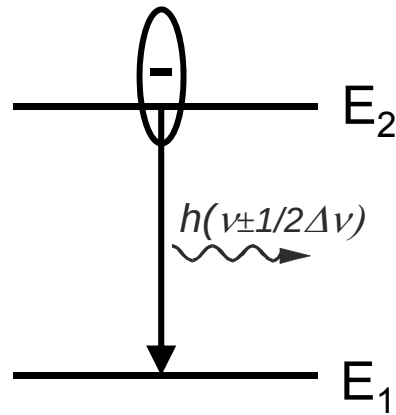



Figure 6.9: Average lifetime Δt of excited atom in energy state E_2 results in indistinct frequency $\nu \pm \Delta \nu$ of the emitted photon $h\nu$ according to Heisenberg's uncertainty relation $\Delta E \Delta t \geq h$. Typical natural lifetime Δt of excited state is $\Delta t = 100$ ns. The resulting band width $\Delta \nu$ of frequency ν is $\Delta \nu = 1/\Delta t = 10^7$ Hz.

If the wavelength λ of emitted photon is $\lambda = 550$ nm, the band width $\Delta \lambda = 10^{-3}$ nm.

6.6.2 Doppler broadening [6.9]

results from the thermal velocity of gas atoms in case of gas lasers. High temperature results in high velocity. If the velocity of the atom is directed towards the observer, the wavelength of the emitted photon appears shorter (blue shifted). If the velocity is directed away from the observer, the wavelength is red shifted.

Frequency shift $\Delta\nu_D$ due to Doppler effect is given by

$$\frac{\Delta\nu_D}{2} = \pm\nu \frac{v}{c}, \quad (6.21)$$

with $\pm v$ = velocity component of atom directed to (+) or away from (-) observer, c = speed of light, ν = frequency of photon.

Kinetic energy E_{kin} of atom is given by

$$E_{kin} \approx kT = \frac{m}{2} v^2, \quad (6.22)$$

with k = Boltzmann's constant, T = Temperature, m = mass of atom.

It follows

$$\Delta\nu_D \approx \frac{2\nu}{c} \sqrt{\frac{2kT}{m}}. \quad (6.23)$$

Exact thermodynamic derivation of Doppler broadening leads to

$$\Delta\nu_D = \frac{2\nu}{c} \sqrt{\frac{2kT \ln 2}{m}} \quad (6.24)$$

Example: excited Neon (Ne) gas, having an atomic mass $m_a = 20$ proton masses, at a temperature T of $T = 300$ K emits radiation at a wavelength $\lambda = 633$ nm. Here, Doppler broadening yields a line width $\Delta\nu_D$ of

$$\Delta\nu_D \approx 1.5 \times 10^9 \text{ Hz.}$$

6.6.3 Collision broadening [6.10]

Collisions between gas atoms shorten the lifetime of excited states.

Shorter lifetime  Increased line width

©JHW



Average time between two collisions determines line width $\Delta\nu_C$ according to

$$\Delta\nu_C = \sqrt{\frac{3}{4mkT}} d^2 p,$$

with d = diameter of atoms and p = gas pressure.

Important: Increasing temperature at constant pressure reduces number density of atoms and thus reduces collision probability!

Typical conditions in *He-Ne-Laser* (Ne , $T = 300 \text{ K}$, $p = 0.1 \text{ mbar}$) result in a negligible collision broadening if compared with Doppler broadening. Collision broadening in this case is

$$\Delta\nu_C = 10^8 \text{ Hz}$$

Line width of lasing transition determines the number of longitudinal laser modes, which are amplified by the resonator.

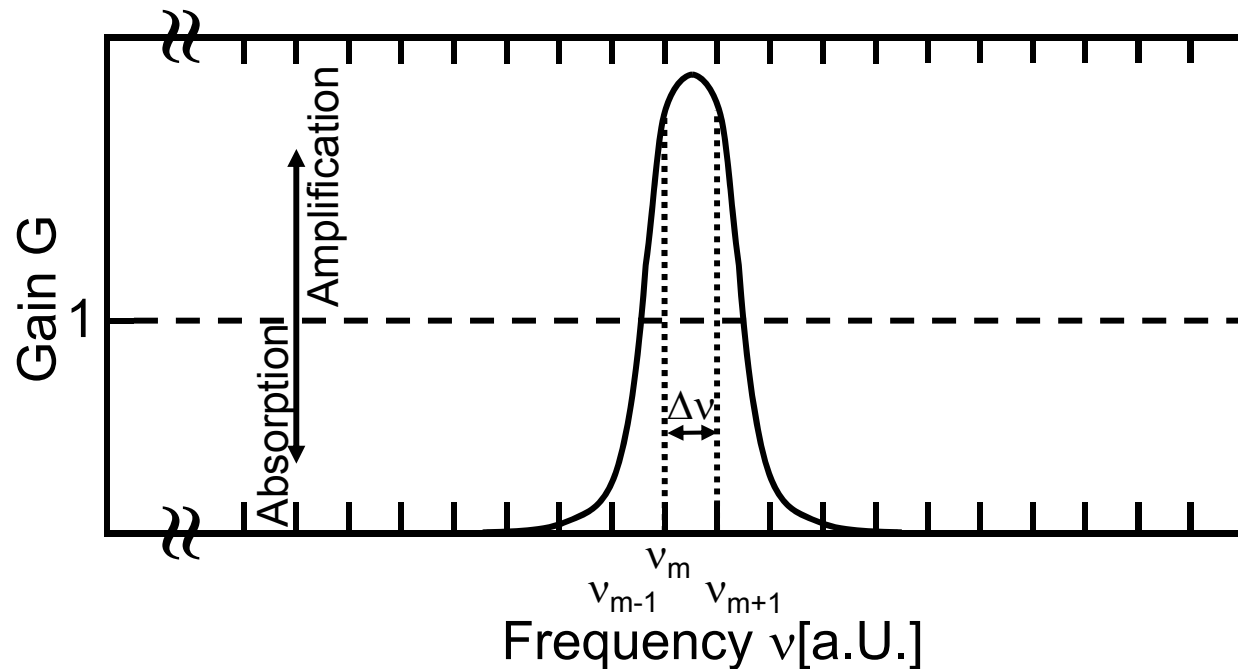


Figure 6.10: Frequency difference between resonator modes $\Delta\nu = \nu_{m+1} - \nu_m = c/(n_r 2d)$ determines the number of possible longitudinal laser modes. Laser active medium amplifies those resonator modes, where the gain factor G of the emission spectrum exceeds 1, $G > 1$.

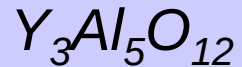
6.7 Laser types

6.7.1 Solid state lasers, Nd:YAG-Laser

Granat

Nd:YAG = Neodymium doped Yttrium-Aluminum-Garnet crystal

Chemical formula



Approximately 1% of the crystals Y^{3+} ions are replaced by Nd^{3+} ions, which are responsible for lasing.

4 energy states involved in lasing process: 4-level-laser [6.11]

- Absorption bands must have short lifetime for rapid energy transfer to upper laser level.
- Lifetimes of upper/lower laser levels must be long/short to enable continuous population inversion and continuous operation.

Figure 6.11 explains the involved energy levels of the Nd:YAG crystal.

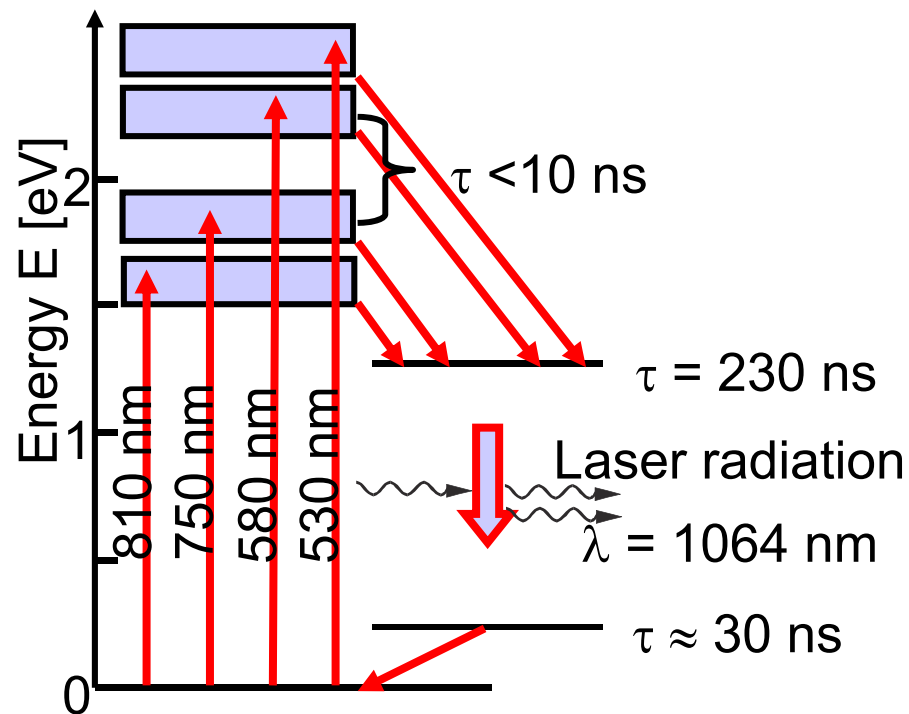


Figure 6.11: Optical pumping is possible via 4 absorption bands at 810 nm, 750 nm, 580 nm or 530 nm. From absorption bands energy is rapidly transferred to upper laser level (lifetime τ of absorption bands is shorter than 10 ns).

Upper laser level has considerably longer lifetime ($t = 230$ ns). Consequently, the pumping energy is accumulated in upper laser level and population inversion is achieved. Stimulated emission induces transitions from upper laser level to lower laser level. Lifetime $t \approx 30$ ns of lower laser level is sufficiently short, with consequence of low population density and high population inversion.

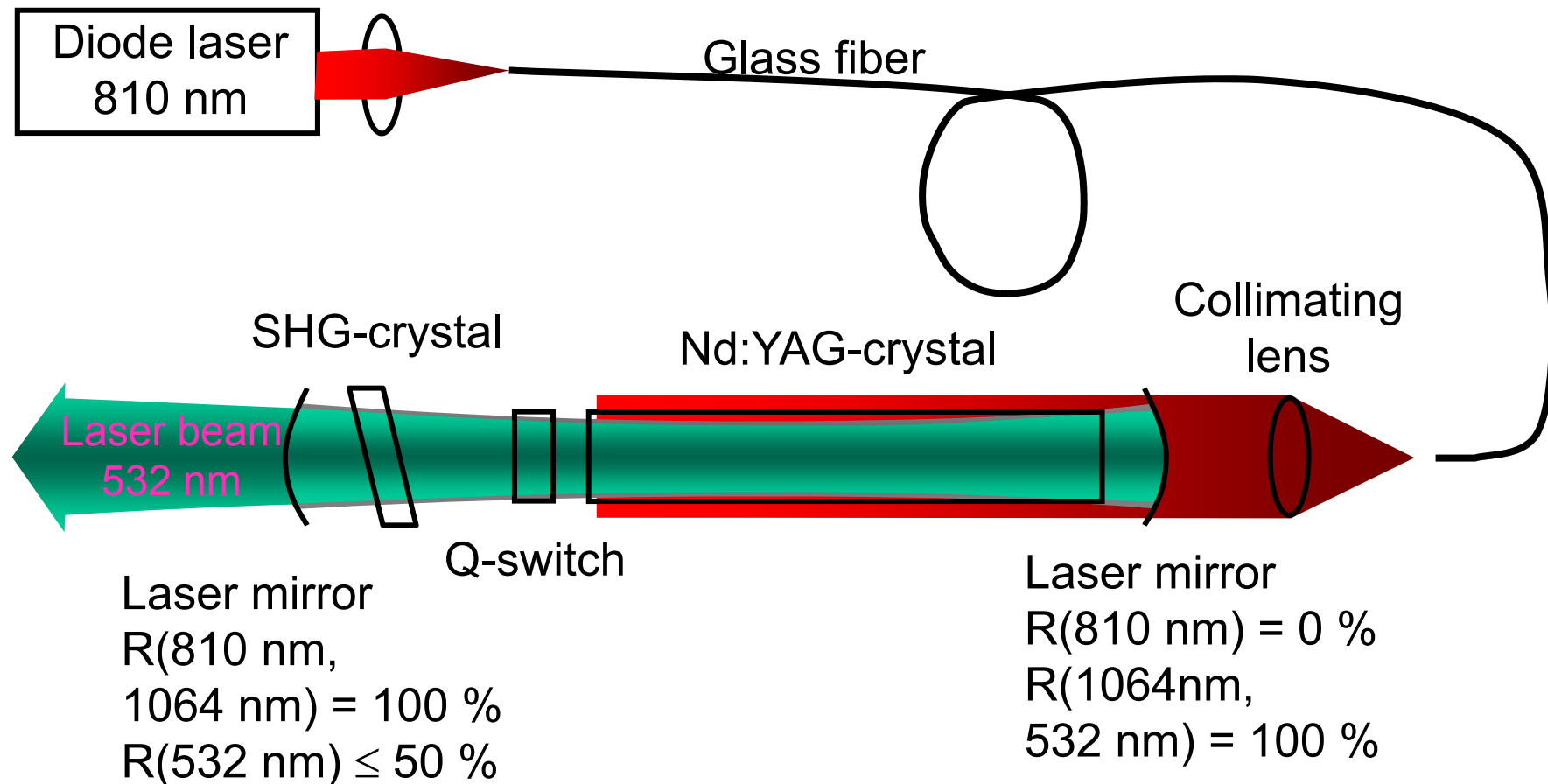


Figure 6.12: Technical realization of a diode laser pumped pulsed Nd:YAG-Laser.

An SHG-crystal doubles the frequency (Second Harmonic Generation) of the laser pulse, and generates a green laser pulse with wavelength $\lambda = 532$ nm.

Principle of frequency doubling [6.12]

Electric field E of laser beam polarizes medium.
Polarization P is nonlinear function of E .

$$\vec{P} = \vec{P}(\vec{E})$$

Assume laser beam directed in z-direction, polarized in x-direction.

$$\vec{E} = \begin{pmatrix} E_x \\ 0 \\ 0 \end{pmatrix}$$



$$P_x = P_x(E_x)$$

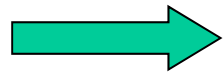
Taylor expansion yields

$$P_x(E_x) = 0 + \frac{1}{1!} \frac{\partial P_x}{\partial E_x} E_x + \frac{1}{2!} \frac{\partial^2 P_x}{\partial E_x^2} E_x^2 + \dots$$

©JHW



In case low intensity or electric field polarization depends linearly on electric field E



Linear or classical optics

$$P_x \propto E_x;$$

$$P_x(E_x) = \varepsilon_0 \chi_1 E_x,$$

with χ_1 the first order susceptibility.

In case of high intensity (laser beam) third term in Taylor expansion becomes relevant

$$P_x(E_x) = \varepsilon_0 (\chi_1 E_x + \chi_2 E_x^2),$$

with χ_2 = second order susceptibility.

Electric field E_x is time dependent according to

$$E_x(t) = E_{x0} e^{i\omega t}$$

©JHW





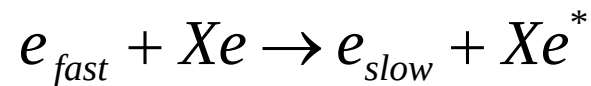
$$P_x(t) = \varepsilon_0 (\chi_1 E_{x0} e^{i\omega t} + \chi_2 E_{x0}^2 e^{i2\omega t}).$$

second term oscillates with the second harmonic 2ω of fundamental frequency ω and generates electric field of same frequency 2ω .

If phase velocities of fundamental and second harmonic electric field in SHG-crystal are the same, a coherent laser beam with doubled frequency is generated.

6.7.2 Gas lasers, Excimer laser

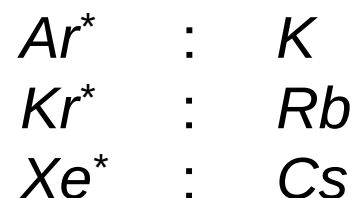
Fast electrons e_{fast} generated by a pulsed gas discharge collide with rare gas atoms and form excited rare gas atoms, e.g. Xe^* [6.13].



Rare gas atoms have completely filled electron shells and therefore are chemically stable. They do not react with other atoms or molecules.

If they are in the lowest excited state, one of the electrons of the outer shell occupies the next higher shell.

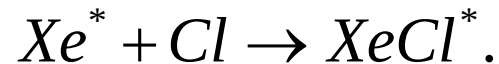
In this case the excited (*) rare gas atom behaves similar to an atom of the first group in the periodic table of elements.



Those excited rare gas atoms are extremely reactive and form excited molecules together with halogen atoms (7th group in periodic table of elements).

F (Fluorine)
Cl (Chlorine)
Br (Bromine)
I (Iodine)

Chemical reaction of excited rare gas atoms and halogen atoms,
e.g. Xenon and Chlorine

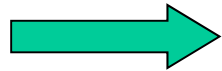
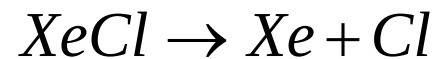


Result is an excited XeCl molecule, which is chemically stable in the excited state *. It is called an excited dimer or Excimer.

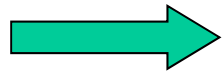
The excited dimer returns to its ground state by emitting a photon of energy $h\nu$.



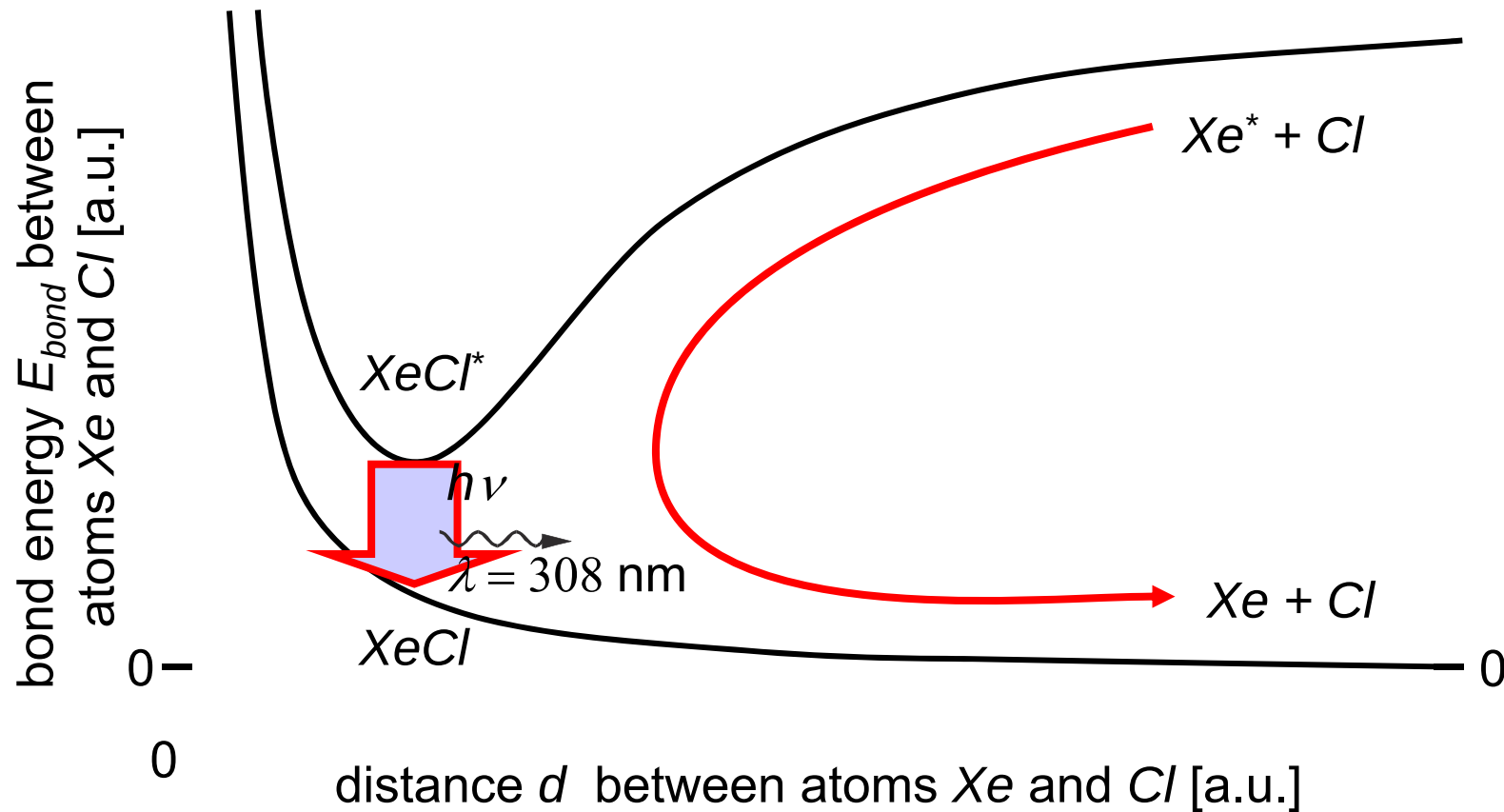
Ground state is chemically unstable. Molecule dissociates instantaneously into a Chlorine atom and a (not excited) Xenon atom.



Molecules in ground state do not exist, ground state is not occupied.



First laser condition, $N_2 > N_1$, is fulfilled.



6.13: Excited Xenon atom Xe^* reacts with Chlorine atom Cl and forms excited dimer XeCl^* . Excited dimer returns to ground state by emitting a photon $h\nu$ of wavelength $\lambda = 308 \text{ nm}$. Ground state is *repulsive* and dissociates immediately into single atoms Xe and Cl .

abstoßend

Table 6.1 Typical data of commercial excimer lasers

	<i>ArF</i>	<i>KrF</i>	<i>XeCl</i>	<i>XeF</i>
Wavelength [nm]	193	248	308	351
Pulse energy [mJ]	800	1200	1000	800
Rep. rate [Hz]	100	250	250	200
Average power [W]	50	250	200	100
Pulse duration [ns]	15	20	20	20
Pulse power [MW]	50	60	50	40

6.8 Semiconductor lasers

6.8.1 The first lasing condition for semiconductor lasers

For semiconductor lasers, the request for occupation inversion results in the request that the quasi-Fermi levels of electrons and holes have to be separated by more than the band gap at the sites where recombination takes place, i.e.

$$E_F^n - E_F^p \geq E_g \approx h\nu.$$

This equation requires that at least *one* of the two quasi-Fermi levels is within a band.

Note: Under thermodynamical equilibrium, both Fermi levels are equal. In order to get lasing, one needs therefore non-equilibrium conditions by injection of carriers. In order to ensure lasing, one dopes usually not only *one* but both sides of the semiconductor so high that both Fermi levels are within a band (degenerate semiconductors). Figure 6.14 compares LED and laser.

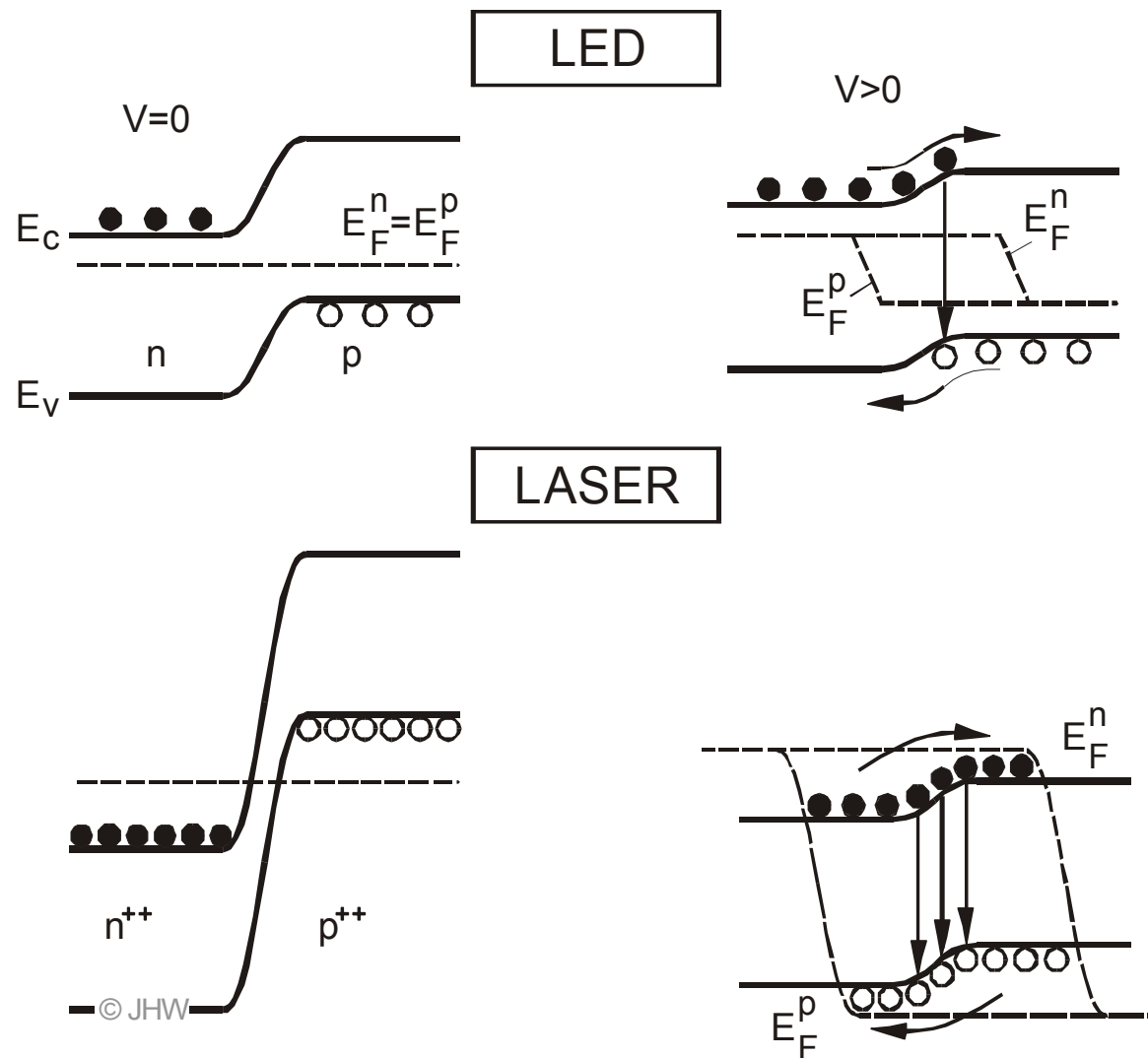


Fig. 6.14: LED and semiconductor laser. In a laser, the quasi-Fermi levels have to be separated by at least the band gap value.

6.8.2 The gain in a semiconductor laser

In order to obtain lasing, the optical gain g has to exceed optical losses. The gain in a semiconductor laser depends on the population of the conduction and valence band.

A positive differential gain $g > 0$ requires

$$h\nu < E_f^n - E_f^p = \Delta E_f .$$

Figure 6.15 illustrates the condition of positive gain.

A detailed derivation of this equation will be given next semester in the lecture

Optoelectronic Devices and Circuits I



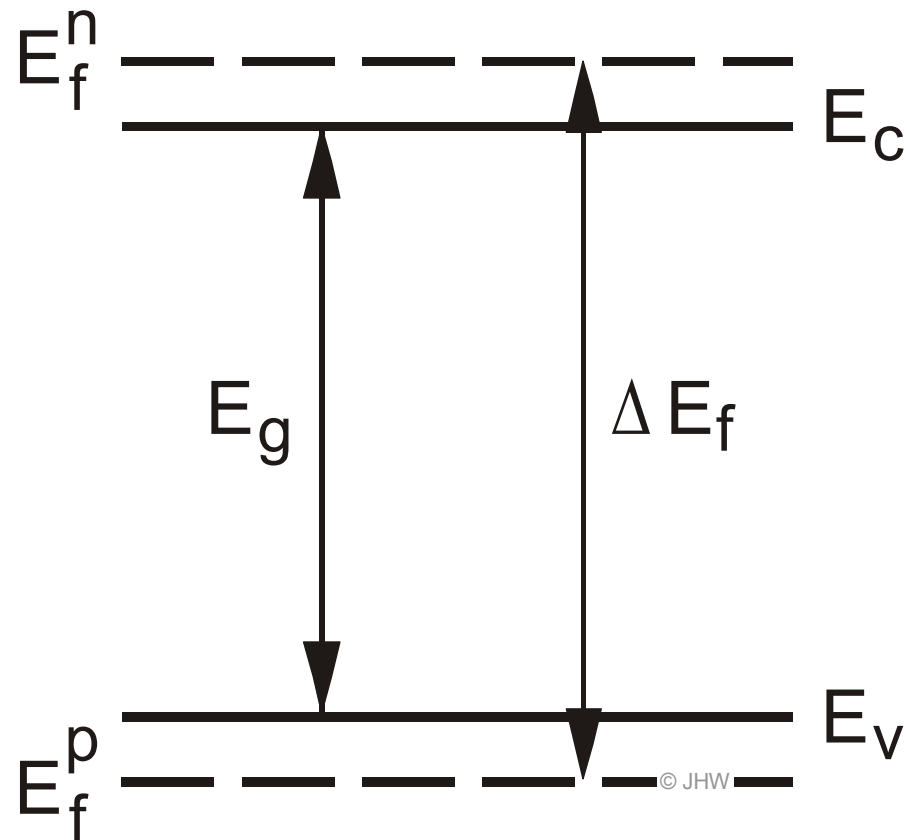


Fig. 6.15: Only electron levels in the energy regime between ΔE_F and E_g yield a positive gain g .

As a consequence, only light with energies $h\nu$ below the distance of the quasi-Fermi levels is amplified. The light has an energy $h\nu$ that is larger than the band gap E_g . As a consequence, it holds for amplification

$$E_g < h\nu < \Delta E_f .$$

For $h\nu = E_g$ and $h\nu = \Delta E_F$ the gain g is zero. Between these two values, we expect a peak.

Figure 6.16 shows a calculated gain curve for a laser that contains a GaAs-layer, highly p⁺-doped with $p^+ = 1 \times 10^{19} \text{ cm}^{-3}$, which corresponds to a position of the hole Fermi level of $E_v - E_f^p = 12 \text{ meV}$ below the valence band edge E_v . Into this layer, electrons of different concentrations are injected by the application of a bias voltage. From a certain electron concentration n on, the gain g becomes positive for energies between the band gap E_g and the separation ΔE_f of the quasi-Fermi levels.

Table 6.2: Values for Fig. 6.16. For too low an injection, the gain g is negative (absorption) because the distance ΔE_f of the quasi-Fermi levels for electrons and holes is smaller than the band gap E_g .

electr. conc. [cm ⁻³]	$E_f^n - E_c$ [meV]	$\Delta E_f - E_g$ [meV]
$n_1 = 2.2 \times 10^{17}$	-15	-3
$n_2 = 4.0 \times 10^{17}$	5	17
$n_3 = 5.6 \times 10^{17}$	15	27
$n_4 = 8.2 \times 10^{17}$	30	42

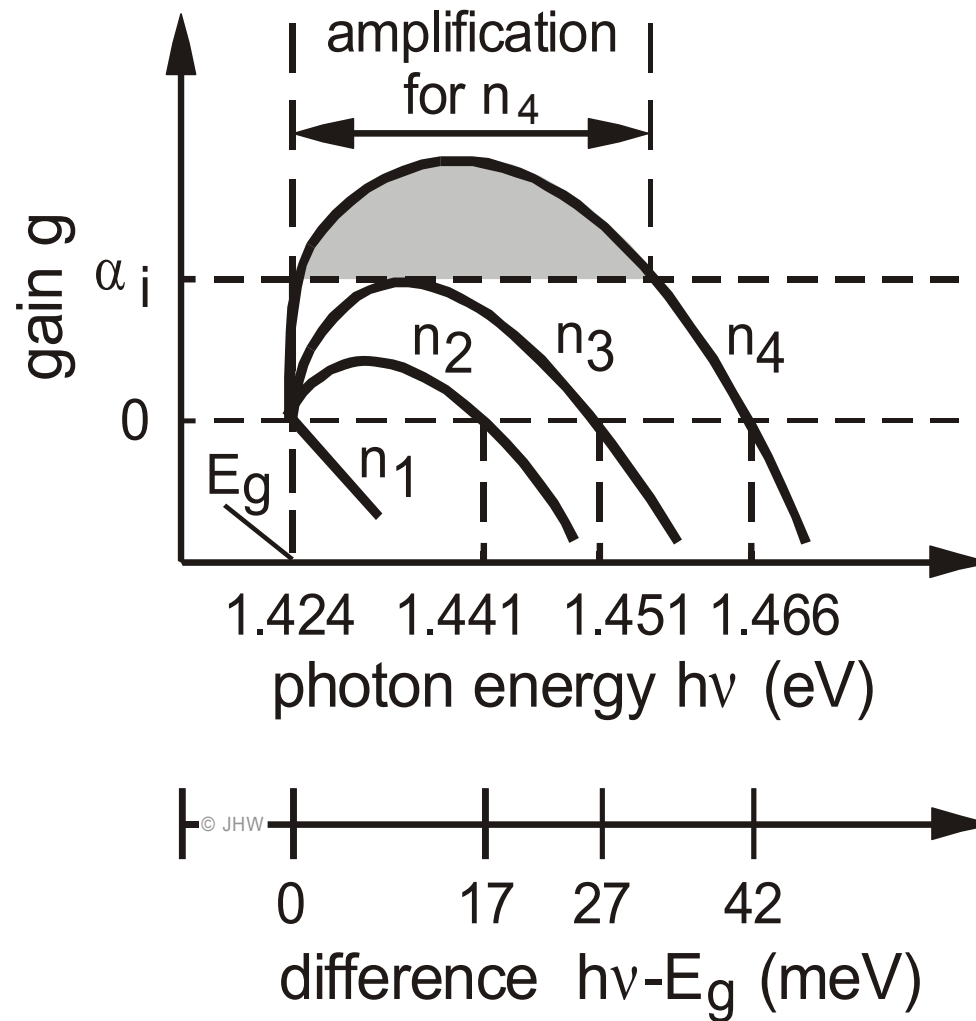


Fig. 6.16: Gain curves for a GaAs laser with a band gap $E_g = 1.424$ eV.

The gain g in Fig. 6.16 depends strongly on the injected carrier (electron) concentration n . For amplification it is not only necessary that the gain g is positive, but the gain g has to exceed a threshold value α_i in order to compensate intrinsic losses that are not contained in the fundamental absorption constant α . Absorption by free carriers is, for example, not included. Such losses are characterized by an additional absorption coefficient α_j .

Above threshold (i.e. for $g > \alpha_i$), the emission lines defined by the resonator (second laser condition) are amplified with the gain curve from Fig. 6.16. The higher the current and, therefore, the injected carrier density n , the less of the competing laser lines survive. Figure 6.17 shows the light output versus current density and Fig. 6.18 shows an example for an emission spectrum.

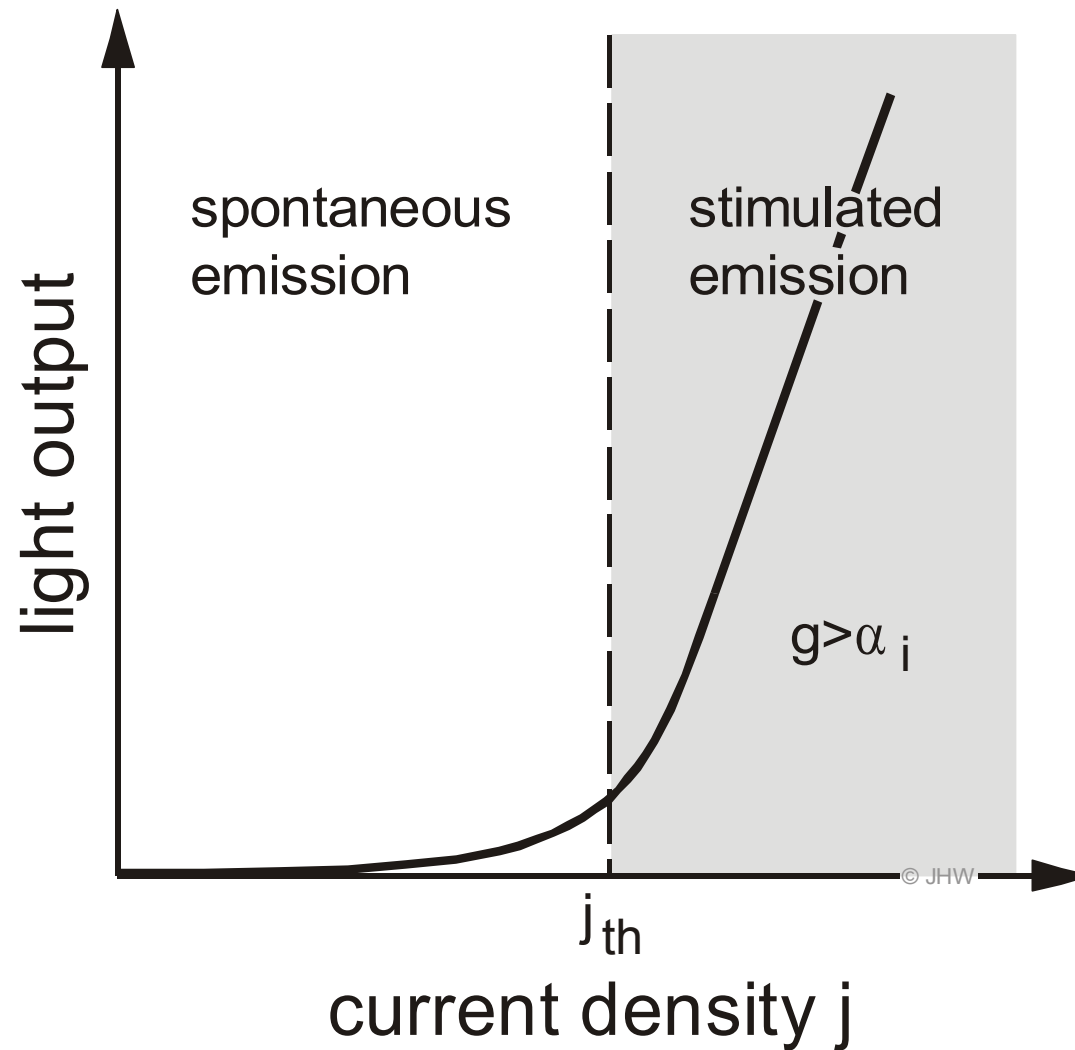


Fig. 6.17: Above the threshold current density j_{th} , the gain g_i exceeds the threshold value α_i .

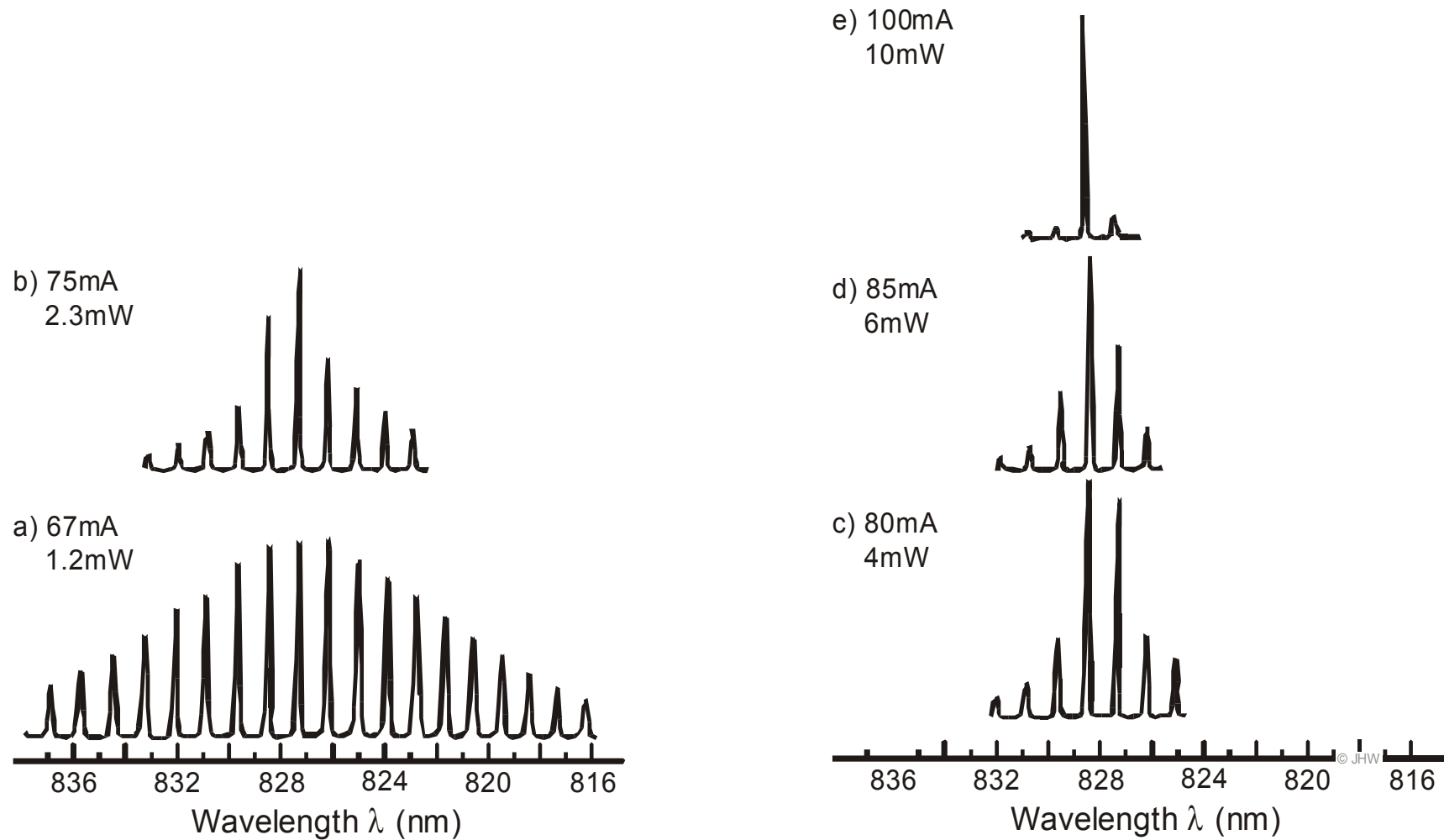


Fig. 6.18: Modes of an $AlGaAs/GaAs$ double heterostructure laser of length $d = 250 \mu m$, width $w = 12 \mu m$ for various currents at 300 K.

6.8.3 Light guiding in semiconductor lasers

a) Principle of light guiding [6.14]

The light in a laser is confined to a narrow region in order to increase the ratio of stimulated to spontaneous emission as much as possible. In order to steer the emission of photons into a certain direction, the light is *guided* within the laser. We use two methods: *Gain* guiding and *index* guiding. Both methods base on total reflection of the light when it impinges from the side with a higher refraction index onto a material with lower refraction index.

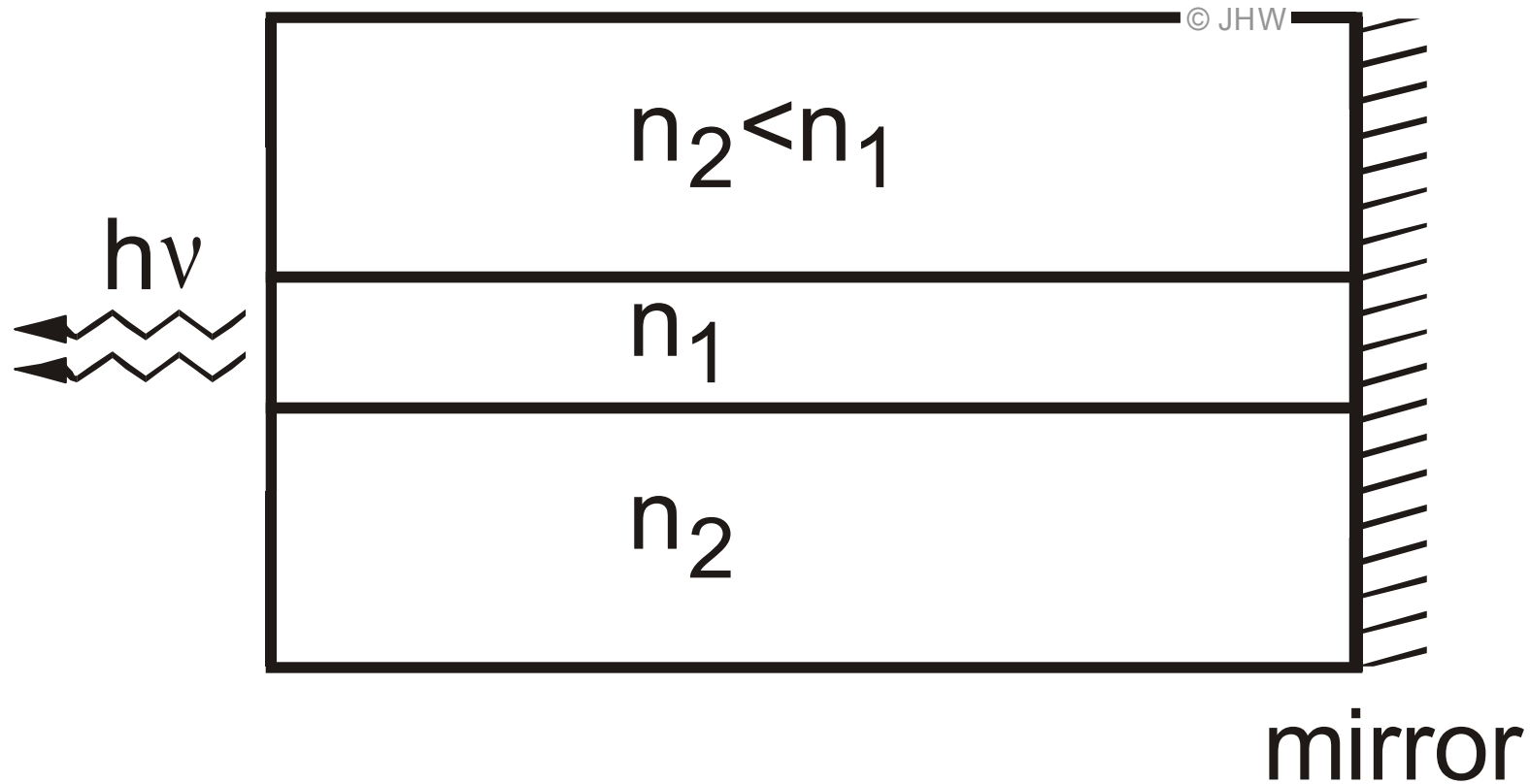


Fig. 6.19: Top (!) view of semiconductor laser, the relation $n_2 < n_1$ must hold for the refraction index n_2 of the cladding layer.

Figure 6.19 (this is a top view!!) shows the principle. The design of the laser aims at achieving a zone with low refraction index n_2 (cladding layer) with methods which are as simple as possible. In principle, the center zone with the high refractive index n_1 should be as narrow as possible, in order to avoid (transversal) optical modes. However, if the the inner zone is too narrow, the light enters into the cladding layer. This effect limits the *minimum* width of the center zone layer to a width of several μm .

b) Gain guiding (active guiding)

This effect occurs in *all* semiconductor lasers: At those sites where the carrier concentration (due to higher current) is larger, the refraction index is higher.

c) Index guiding (passive guiding)

This effect bases on a step in the refraction index which is built into the structure even when the laser is not under operation. The step-like behavior of the refraction index n_r (Moss' law $n_r^4 E_g = \text{const.}$) allows the confinement of light.

Index guiding in addition to gain guiding results in a lower threshold current density j_{th} . Figures 6.23 a, b show two hetero-structure lasers with gain guiding and index guiding.

6.8.4 The stripe contact laser

a) Carrier confinement [6.15]

Figure 6.21 shows this most simple semiconductor laser. The small (some μm wide) contact stripe confines the carriers to a narrow zone of the pn-junction. The Fabry-Perot is made by splitting the wafer along preferential crystallographic directions, usually along [110]. The (111)-split planes are then extremely flat. This type of laser emits the light at the edges (“edge emitting” laser)!

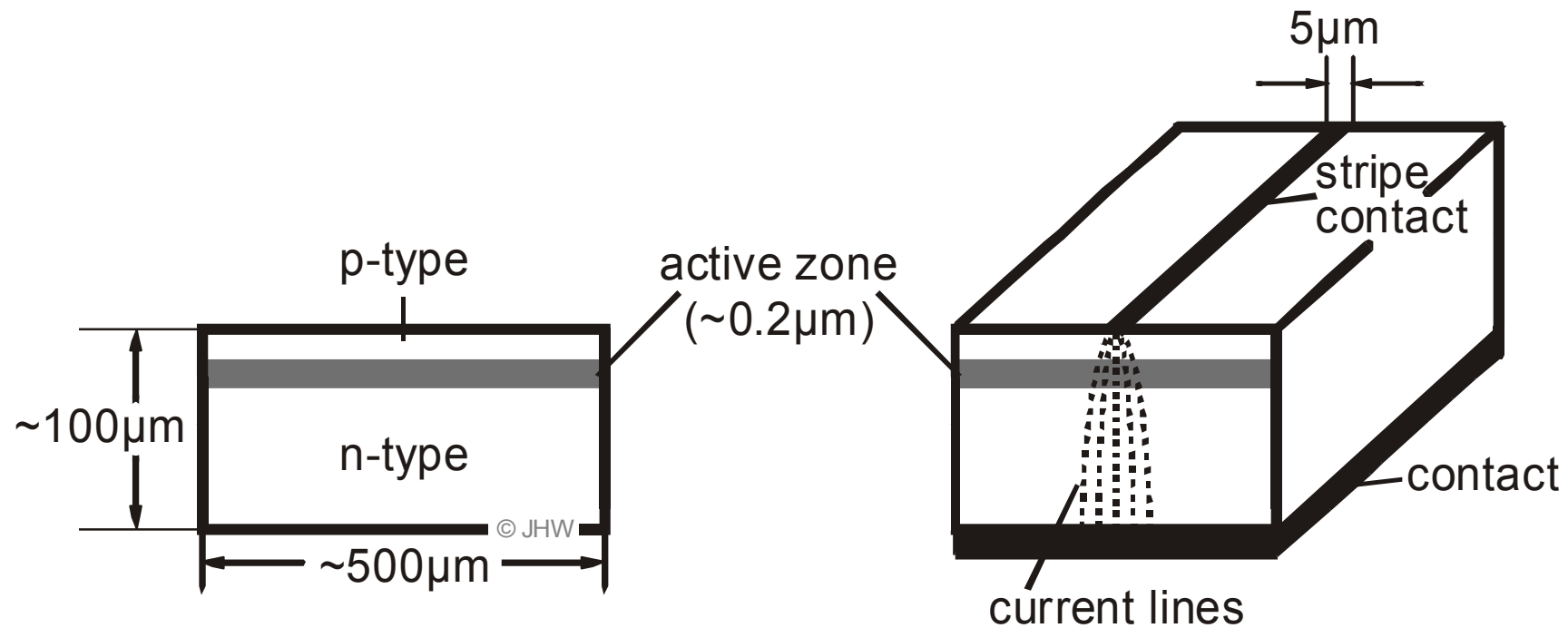


Fig. 6.20: Side view and cross section of stripe contact laser.

Confinement of the current (carriers) in the stripe contact laser is improved by either using a V-groove, hydrogen implantation to form semi-insulating *GaAs*, or by an oxide layer as shown in Fig. 6.21. The oxide with a refractive index $n_r \approx 1.5$ results also in index guiding of the light.

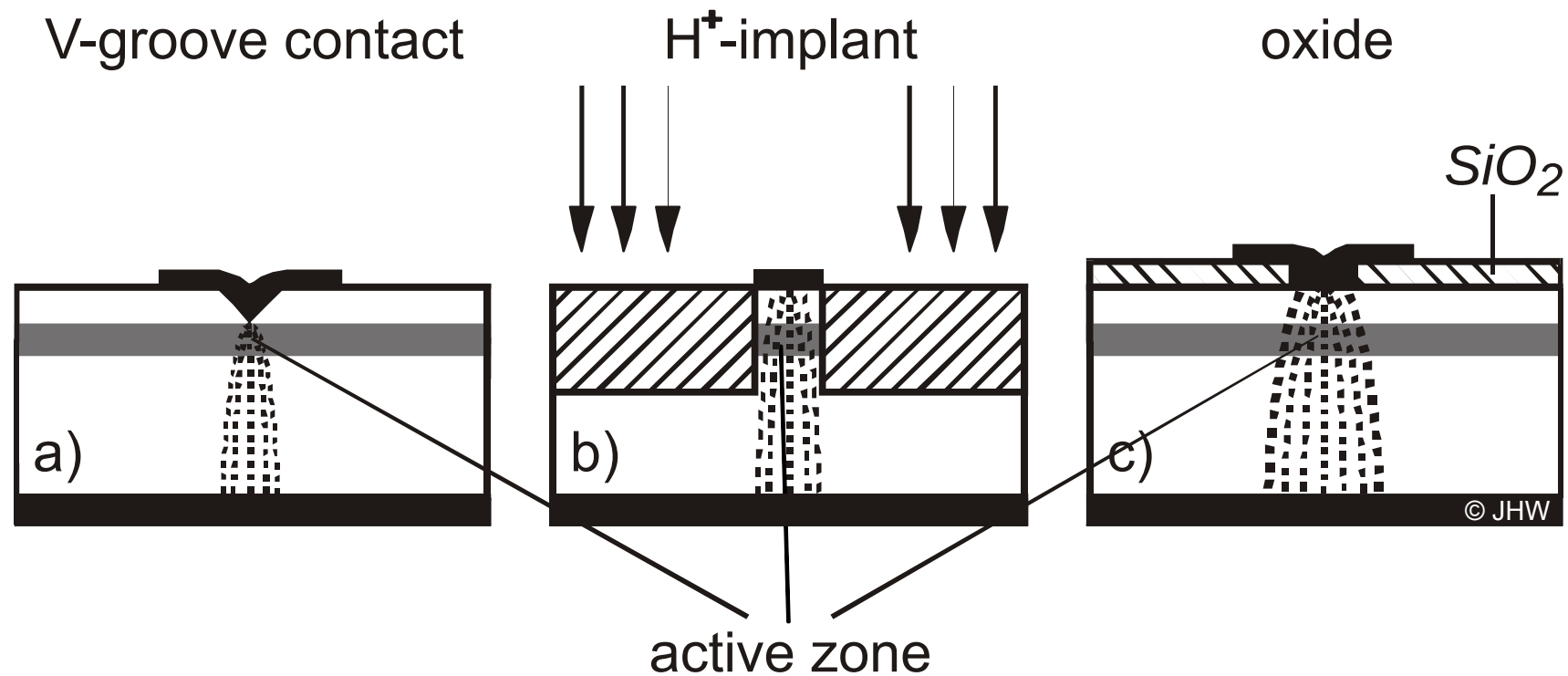


Fig. 6.21: Confinement of current lines in a GaAs stripe contact laser by
 a) V-groove contact,
 b) hydrogen implantation,
 c) oxide layer.

b) Optical confinement

is based on *gain guiding*. The center zone with the high refractive index n_1 should be as narrow as possible, in order to suppress (transversal) optical modes.

6.8.5 Modern semiconductor lasers, the double heterostructure laser

Figure 6.22 shows a double heterostructure laser. Inversion in the center layer is obtained by injection from the n-type and p-type doped layers with the high band gap [6.16].

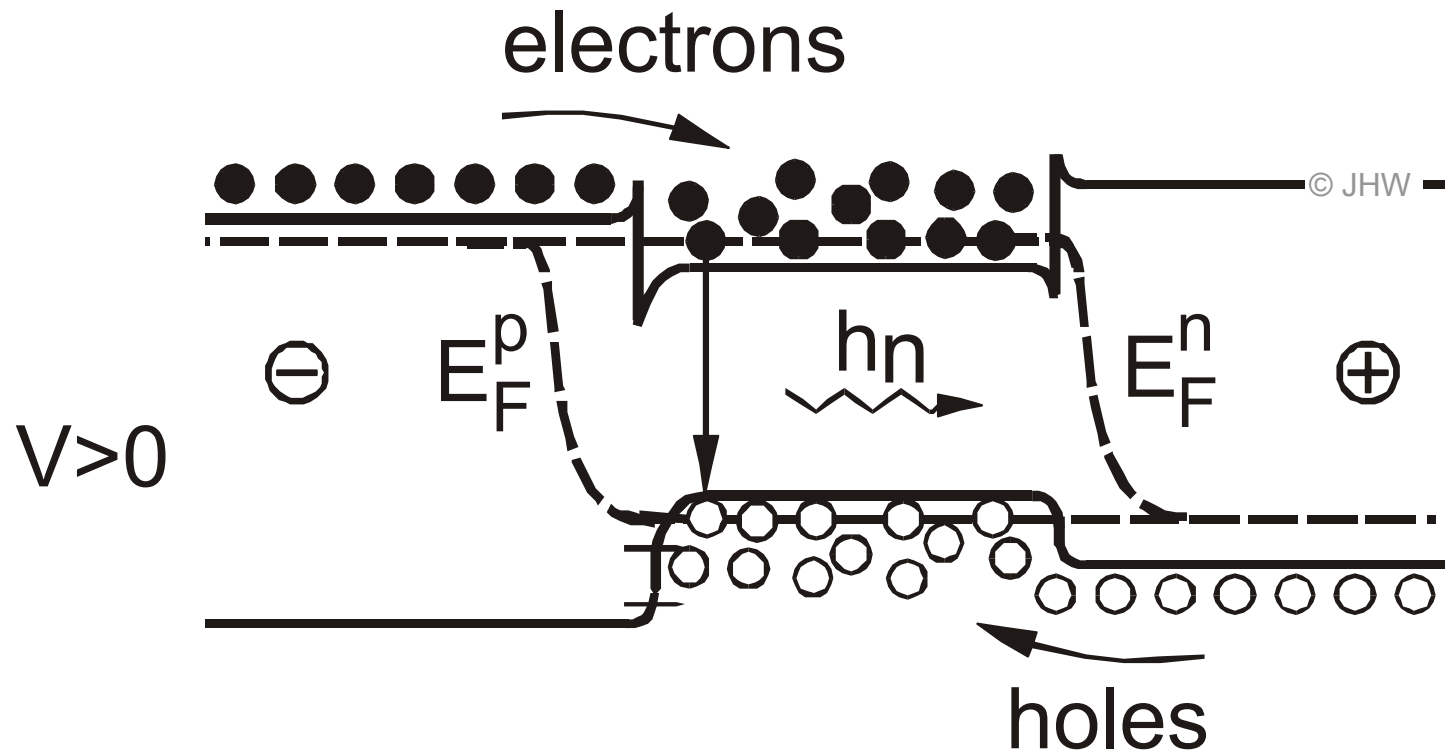


Fig. 6.22: Double heterostructure laser with bias voltage $V > 0$, consisting of an $\text{AlGaAs}/\text{GaAs}/\text{AlGaAs}$ stack.

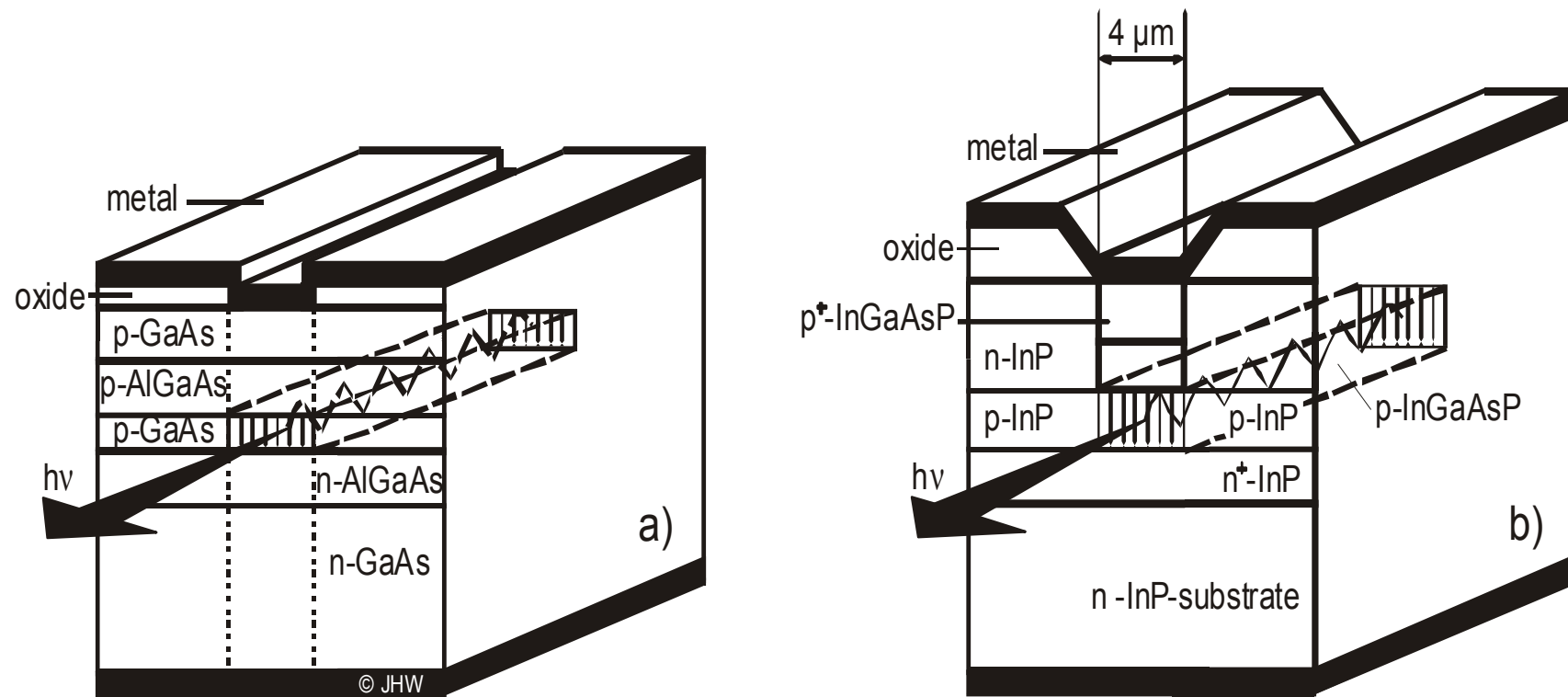


Fig. 6.20: a) gain guided *AlGaAs/GaAs*-laser,
b) index guided *InGaAsP* laser with *InP* cladding
layers.

6.9 Literature

[6.1]: Herziger, G., Weber, H. (Hrsg.), J. Eichler H. J. Eichler, *Laser: Bauformen, Strahlführung, Anwendung*, 3. Aufl., (Springer-Verlag, Berlin Heidelberg New York 1998), S. 23.

[6.2]: Herziger, G., Weber, H. (Hrsg.), J. Eichler H. J. Eichler, *Laser: Bauformen, Strahlführung, Anwendung*, 3. Aufl., (Springer-Verlag, Berlin Heidelberg New York 1998), S. 25.

[6.3]: Herziger, G., Weber, H. (Hrsg.), J. Eichler H. J. Eichler, *Laser: Bauformen, Strahlführung, Anwendung*, 3. Aufl., (Springer-Verlag, Berlin Heidelberg New York 1998), S. 27.

[6.4]: Herziger, G., Weber, H. (Hrsg.), J. Eichler H. J. Eichler, *Laser: Bauformen, Strahlführung, Anwendung*, 3. Aufl., (Springer-Verlag, Berlin Heidelberg New York 1998), S. 27, 36.

[6.5]: Herziger, G., Weber, H. (Hrsg.), J. Eichler H. J. Eichler, *Laser: Bauformen, Strahlführung, Anwendung*, 3. Aufl., (Springer-Verlag, Berlin Heidelberg New York 1998), S. 223.

[6.6]: Herziger, G., Weber, H. (Hrsg.), J. Eichler H. J. Eichler, *Laser: Bauformen, Strahlführung, Anwendung*, 3. Aufl., (Springer-Verlag, Berlin Heidelberg New York 1998), S. 232.

[6.7]: Herziger, G., Weber, H. (Hrsg.), J. Eichler H. J. Eichler, *Laser: Bauformen, Strahlführung, Anwendung*, 3. Aufl., (Springer-Verlag, Berlin Heidelberg New York 1998), S. 240.

[6.8]: Herziger, G., Weber, H. (Hrsg.), J. Eichler H. J. Eichler, *Laser: Bauformen, Strahlführung, Anwendung*, 3. Aufl., (Springer-Verlag, Berlin Heidelberg New York 1998), S. 29.

[6.9]: Herziger, G., Weber, H. (Hrsg.), J. Eichler H. J. Eichler, *Laser: Bauformen, Strahlführung, Anwendung*, 3. Aufl., (Springer-Verlag, Berlin Heidelberg New York 1998), S. 32.

[6.10]: Herziger, G., Weber, H. (Hrsg.), J. Eichler H. J. Eichler, *Laser: Bauformen, Strahlführung, Anwendung*, 3. Aufl., (Springer-Verlag, Berlin Heidelberg New York 1998), S. 30.

[6.11]: Herziger, G., Weber, H. (Hrsg.), J. Eichler H. J. Eichler, *Laser: Bauformen, Strahlführung, Anwendung*, 3. Aufl., (Springer-Verlag, Berlin Heidelberg New York 1998), S. 128.

[6.12]: Herziger, G., Weber, H. (Hrsg.), J. Eichler H. J. Eichler, *Laser: Bauformen, Strahlführung, Anwendung*, 3. Aufl., (Springer-Verlag, Berlin Heidelberg New York 1998), S. 315.

[6.13]: Herziger, G., Weber, H. (Hrsg.), J. Eichler H. J. Eichler, *Laser: Bauformen, Strahlführung, Anwendung*, 3. Aufl., (Springer-Verlag, Berlin Heidelberg New York 1998), S. 116.

[6.14]: Herziger, G., Weber, H. (Hrsg.), J. Eichler H. J. Eichler, *Laser: Bauformen, Strahlführung, Anwendung*, 3. Aufl., (Springer-Verlag, Berlin Heidelberg New York 1998), S. 218.



[6.15]: Herziger, G., Weber, H. (Hrsg.), J. Eichler H. J. Eichler,
Laser: Bauformen, Strahlführung, Anwendung, 3. Aufl.,
(Springer-Verlag, Berlin Heidelberg New York 1998), S. 171.

[6.16]: Herziger, G., Weber, H. (Hrsg.), J. Eichler H. J. Eichler,
Laser: Bauformen, Strahlführung, Anwendung, 3. Aufl.,
(Springer-Verlag, Berlin Heidelberg New York 1998), S. 170.

7. Laser Processes

7.1 Laser Doping

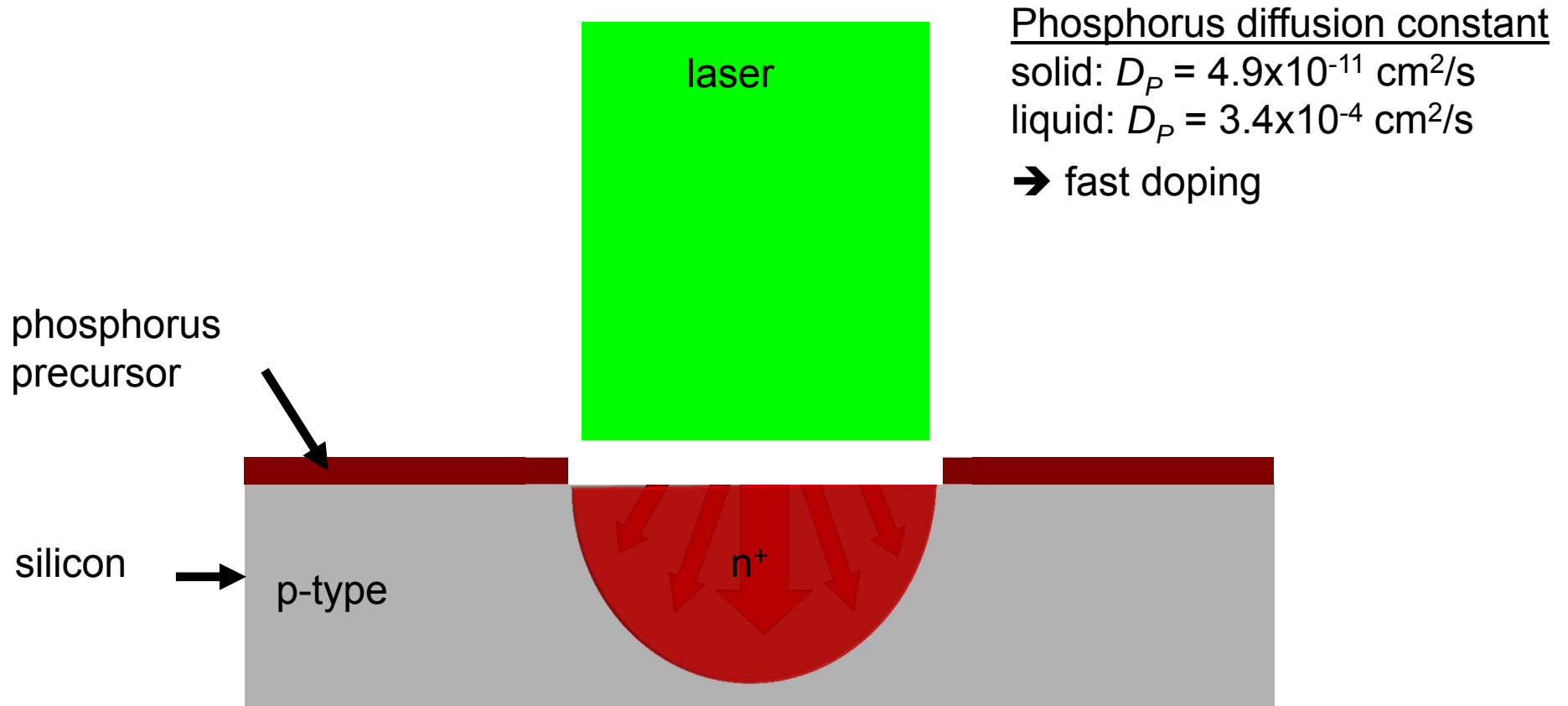
Laser Process and International Patent

developed at *ipv*

Laser Processing Group

U.S. Patent No. 20080026550A1

7.1.1 What is laser doping?



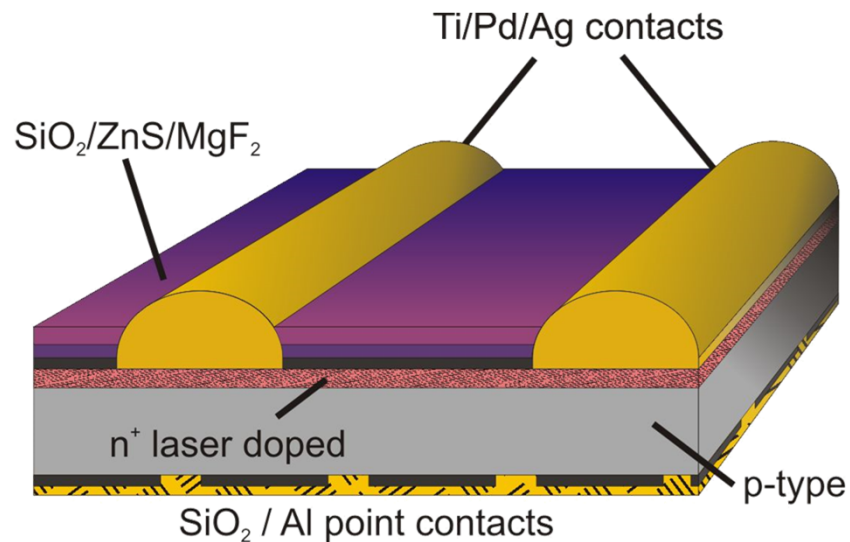
7.1.2 Laser doping results

High solar cell efficiencies

full area emitter process

$\eta = 18.9\%$ @ $A = 4 \text{ cm}^2$

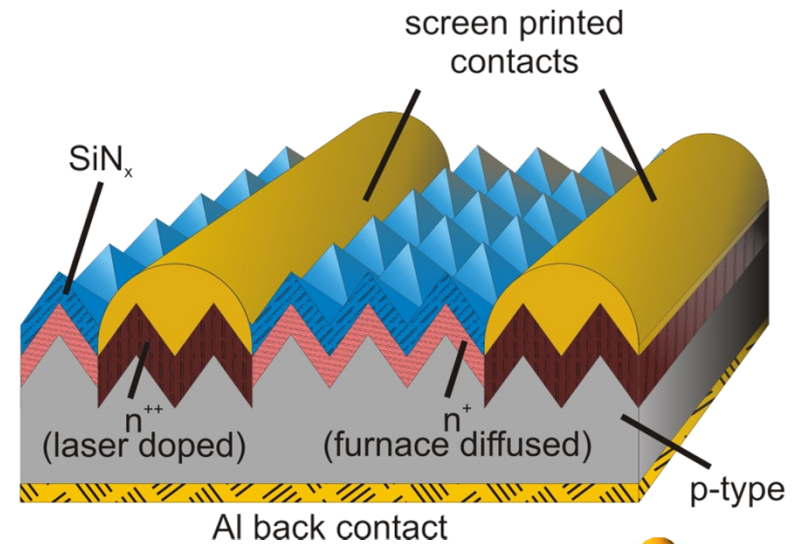
→ record efficiency



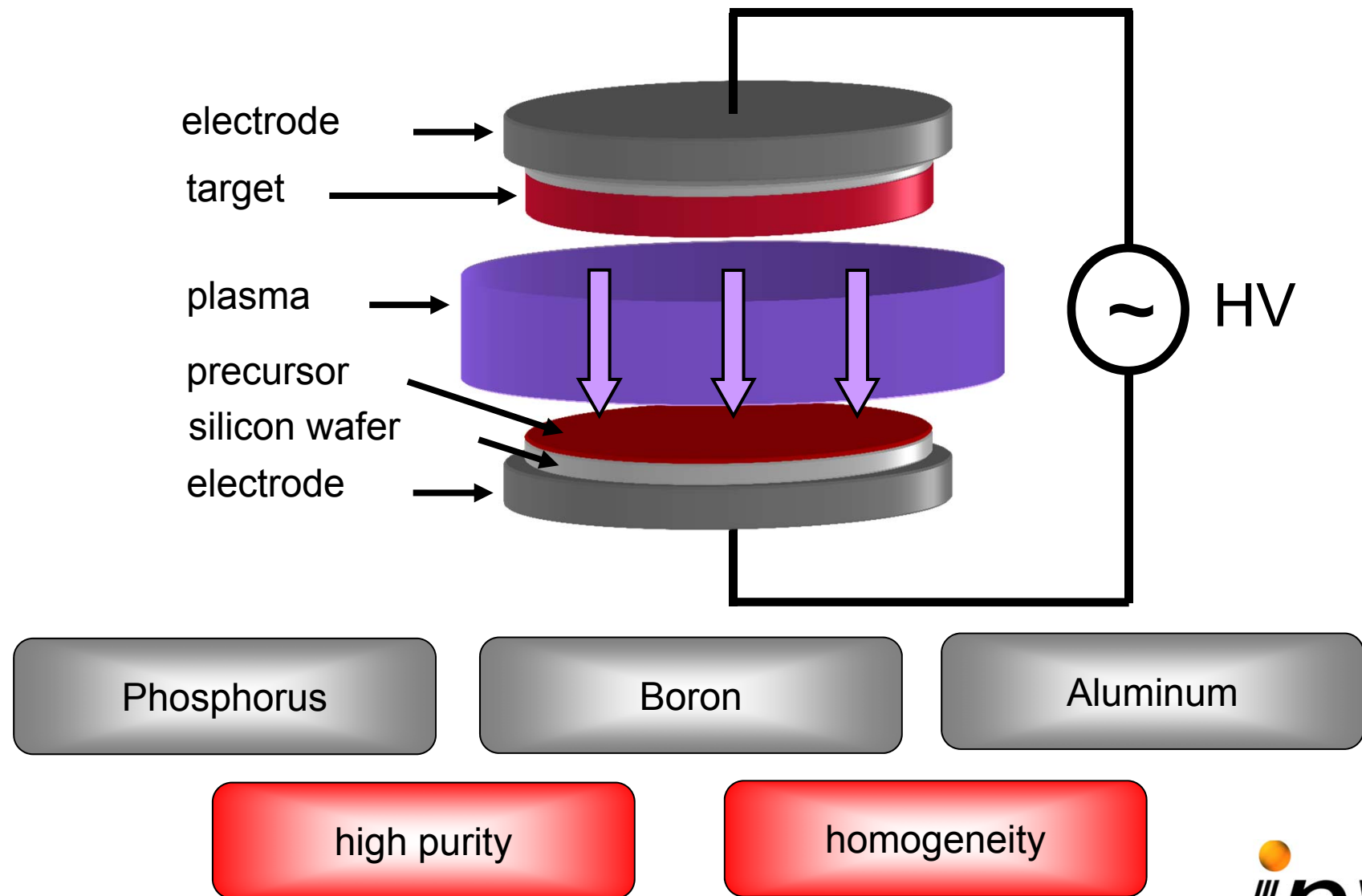
selective emitter process

@ $A = 12.5 \times 12.5 \text{ cm}^2$

→ $\Delta\eta = +0.5\%$ abs



7.1.3 Precursor deposition



7.1.4 Laser doping setup

- Pulsed Nd:YAG ($\lambda = 532 \text{ nm}$)
- Line focus
- Scanning motion
- Homogeneous doping
- ➔ pulse overlap
- Pulse energy density

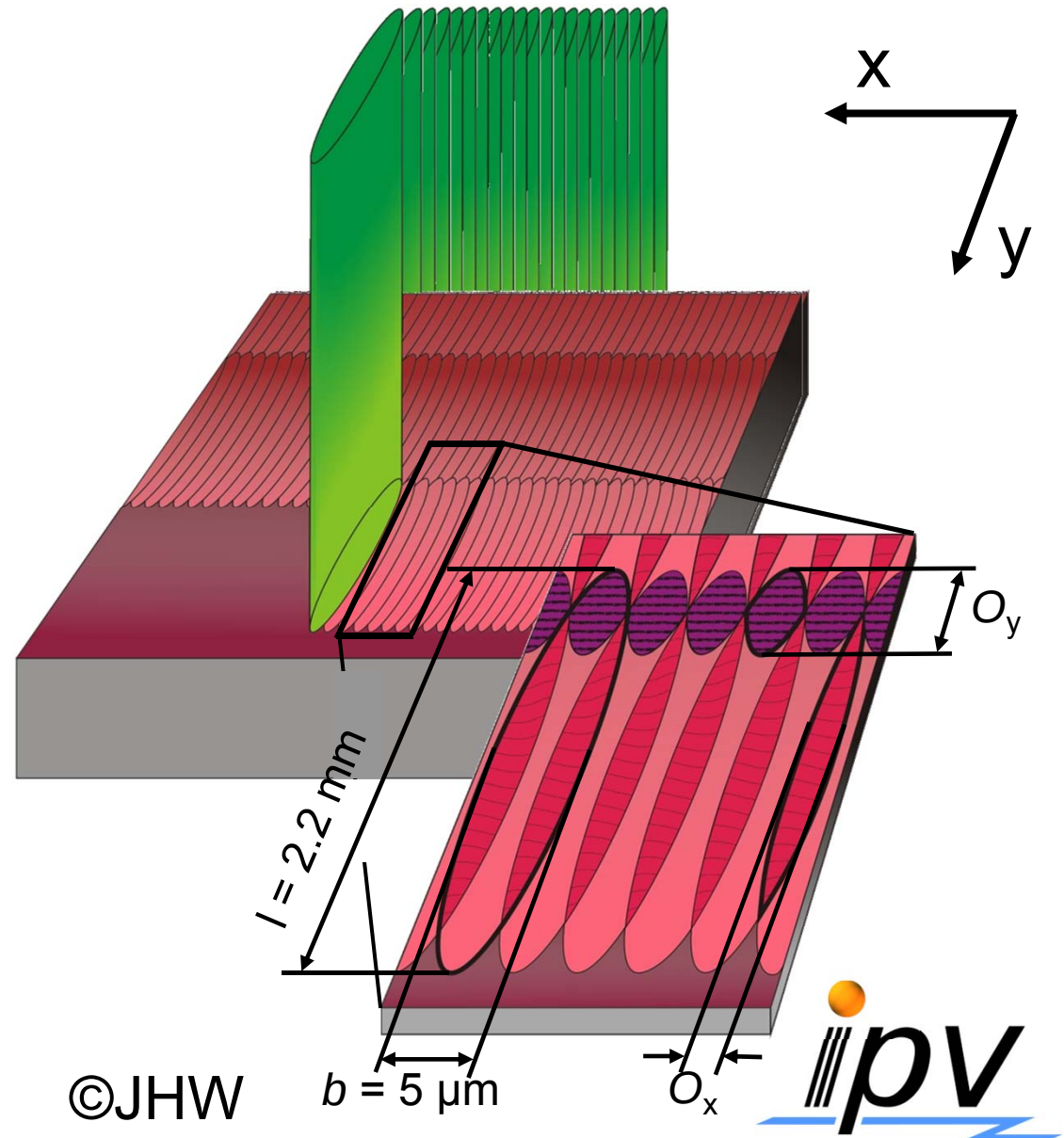
$$E_p = \frac{P}{f A} [\text{J}/\text{cm}^2]$$

P : power [W]

f : frequency [Hz]

A : Area [cm^2]

* U.S. Patent No. 20080026550A1

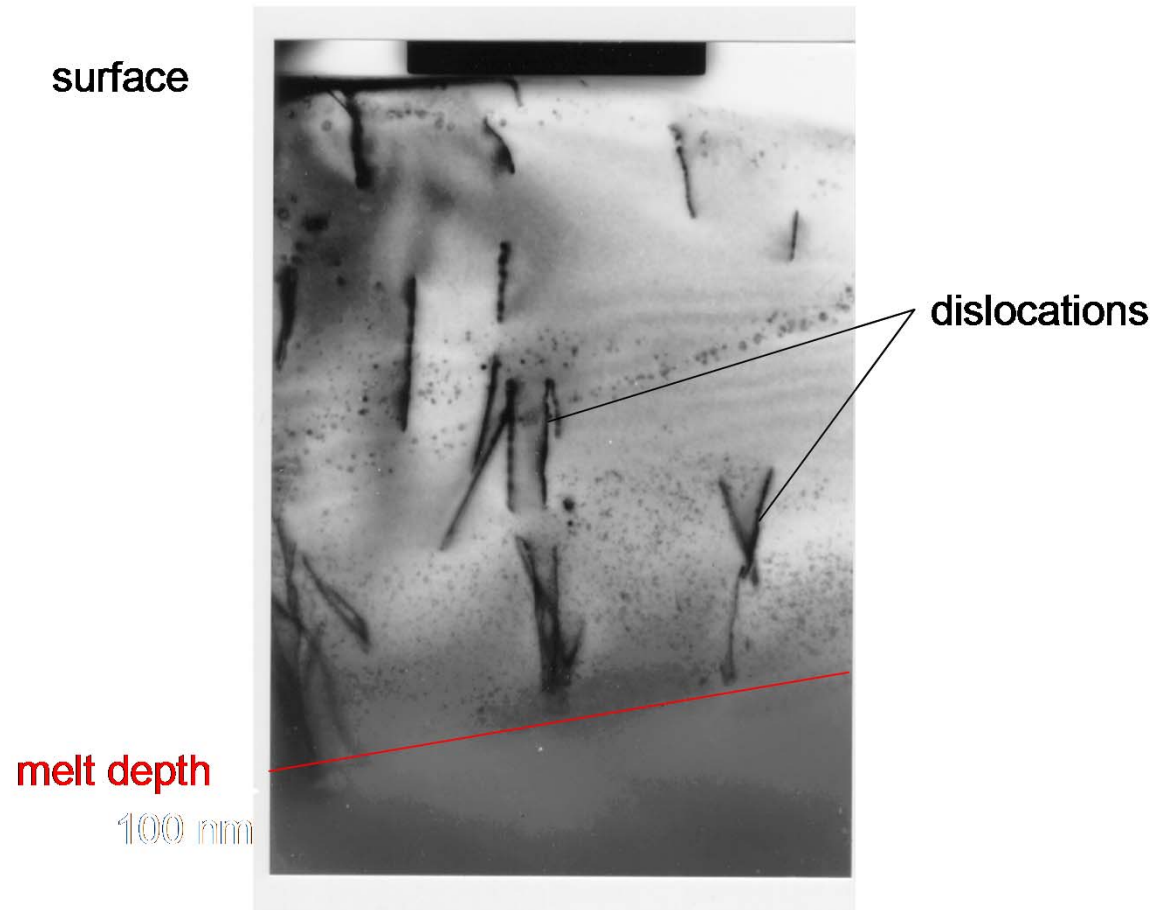


©JHW

Why Line Focus?

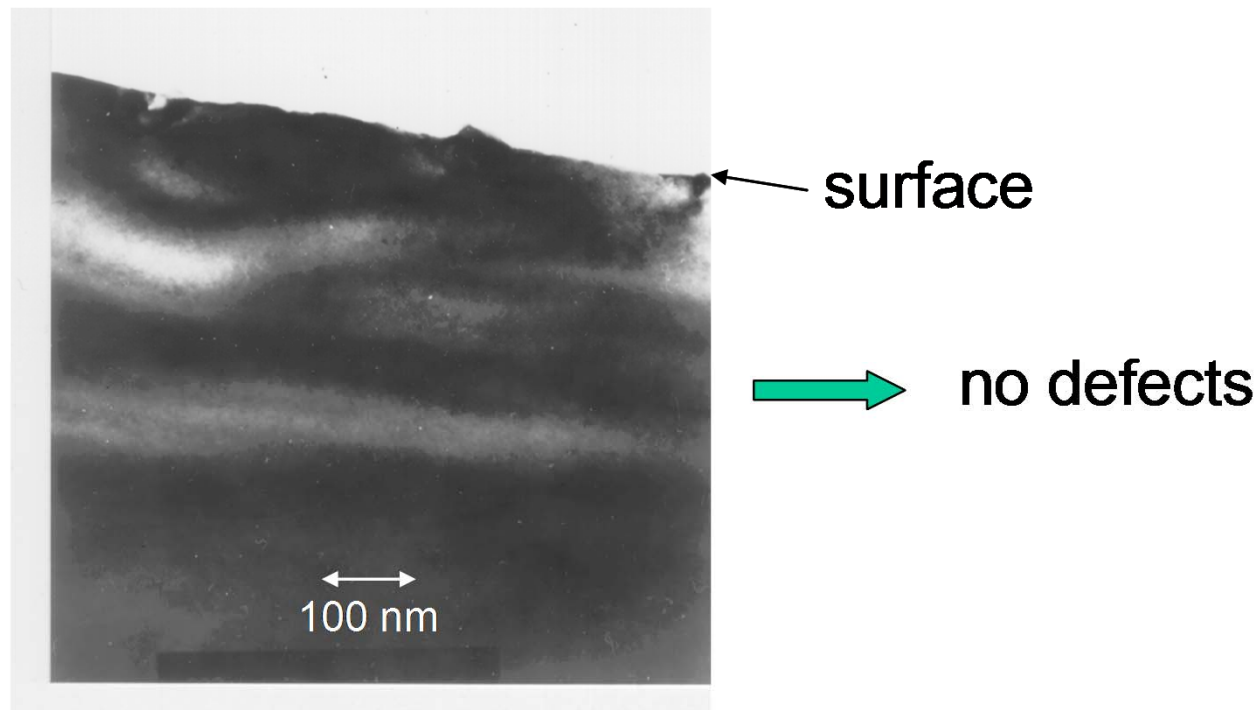
Transmission Electron Microscopy

circular laser focus:



Transmission Electron Microscopy

line laser focus:



7.1.5 Simple Model

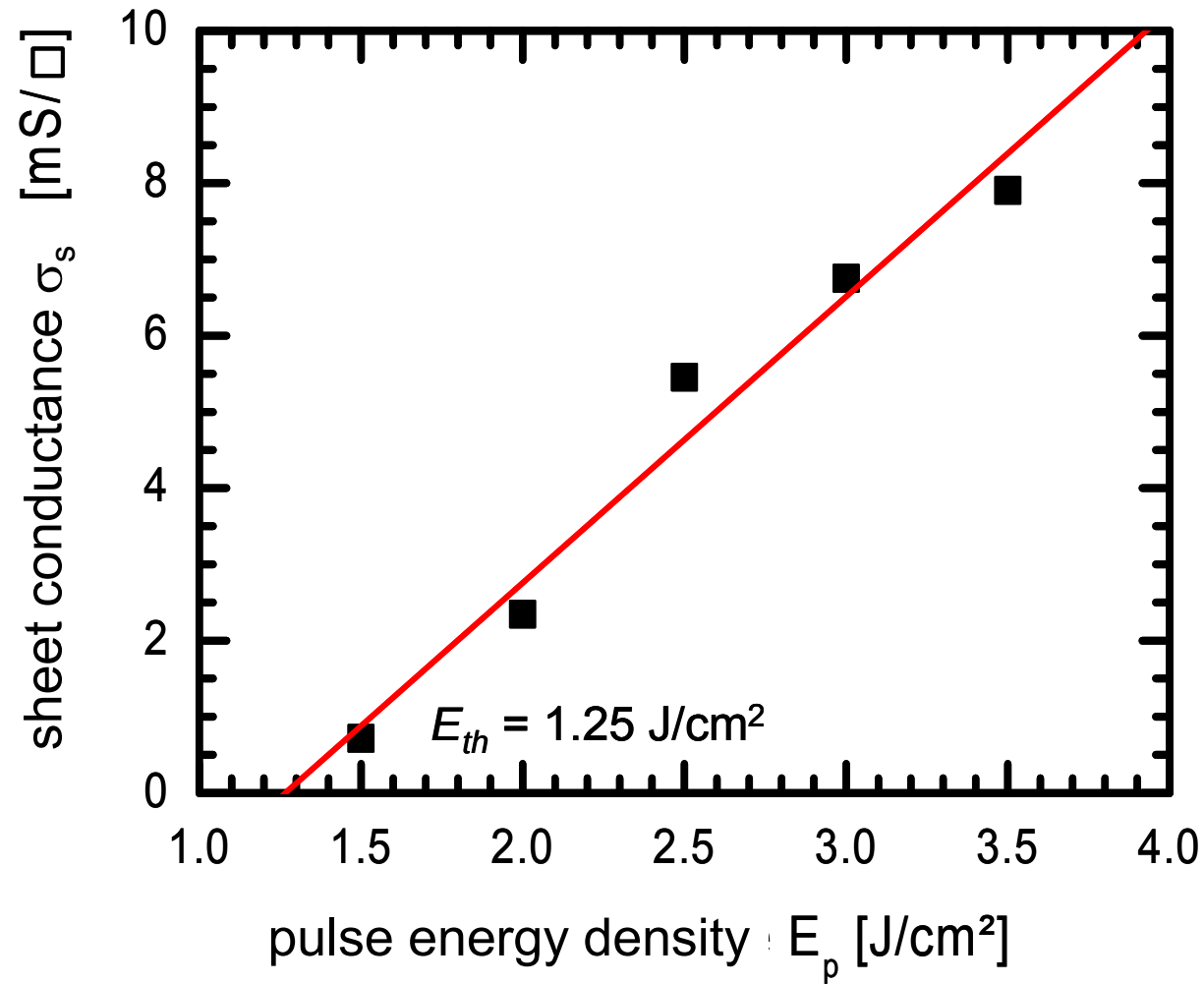
- doping requires melting of silicon

➡ threshold energy E_{th}

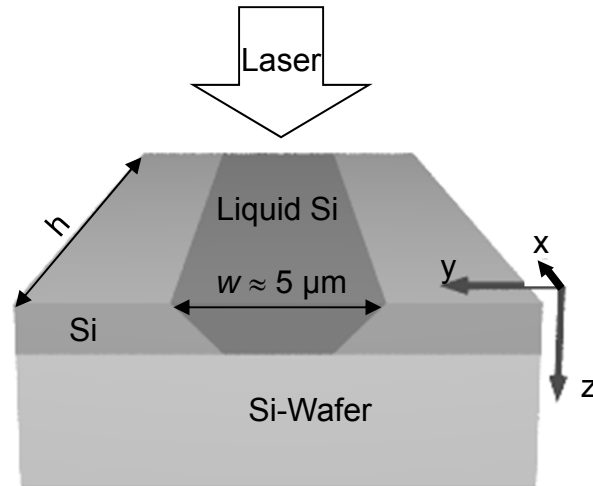
- melting depth $\sim$ (pulse energy density – threshold energy)
- constant doping concentration

➡ sheet resistance $\rho_s^{-1} \sim E - E_{th}$

Sheet Conductance



7.1.6 Numerical Model



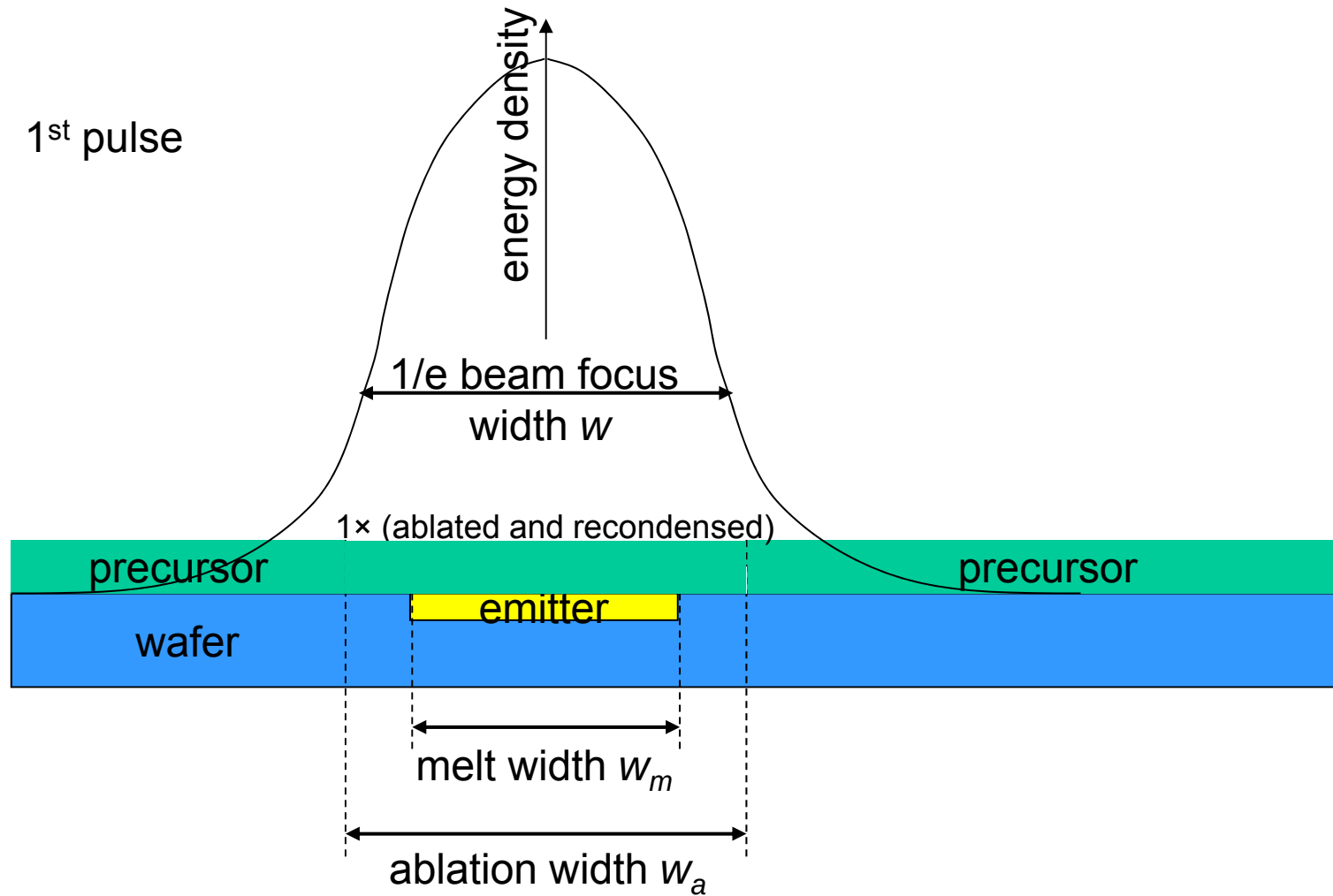
2-D heat transport equation:

$$\frac{\partial T(y, z, t)}{\partial t} = \frac{1}{c_p(T)\rho(T)} [\text{div}(\lambda(T)\text{grad}T(y, z, t)) + \alpha(T)I(y, z, t)]$$

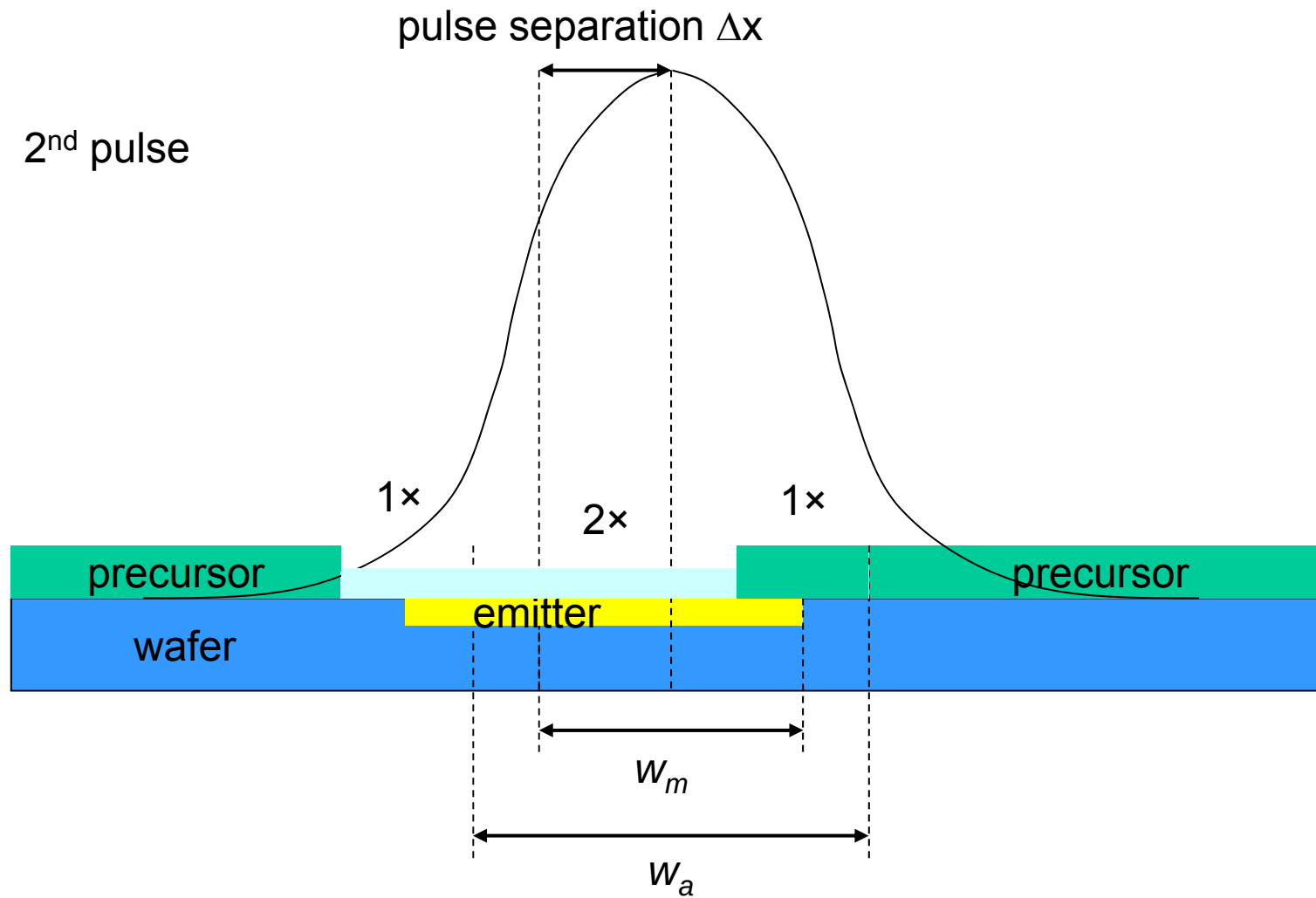
2-D diffusion equation:

$$\frac{\partial C(y, z, t)}{\partial t} = D(T, y, z, t)\Delta C(y, z, t) + Q(y, z, t)$$

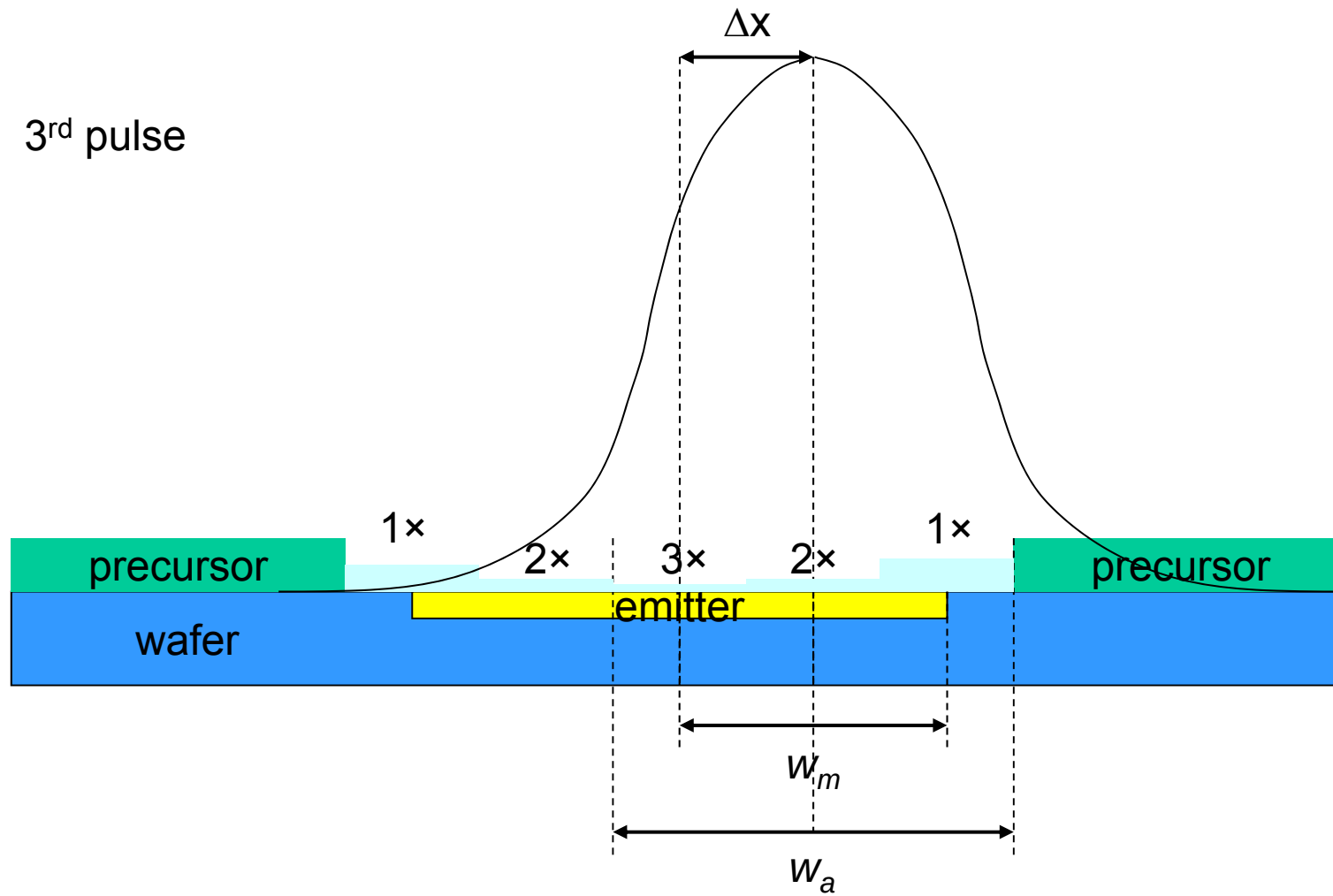
Simulation Principle



Simulation Principle



Simulation Principle



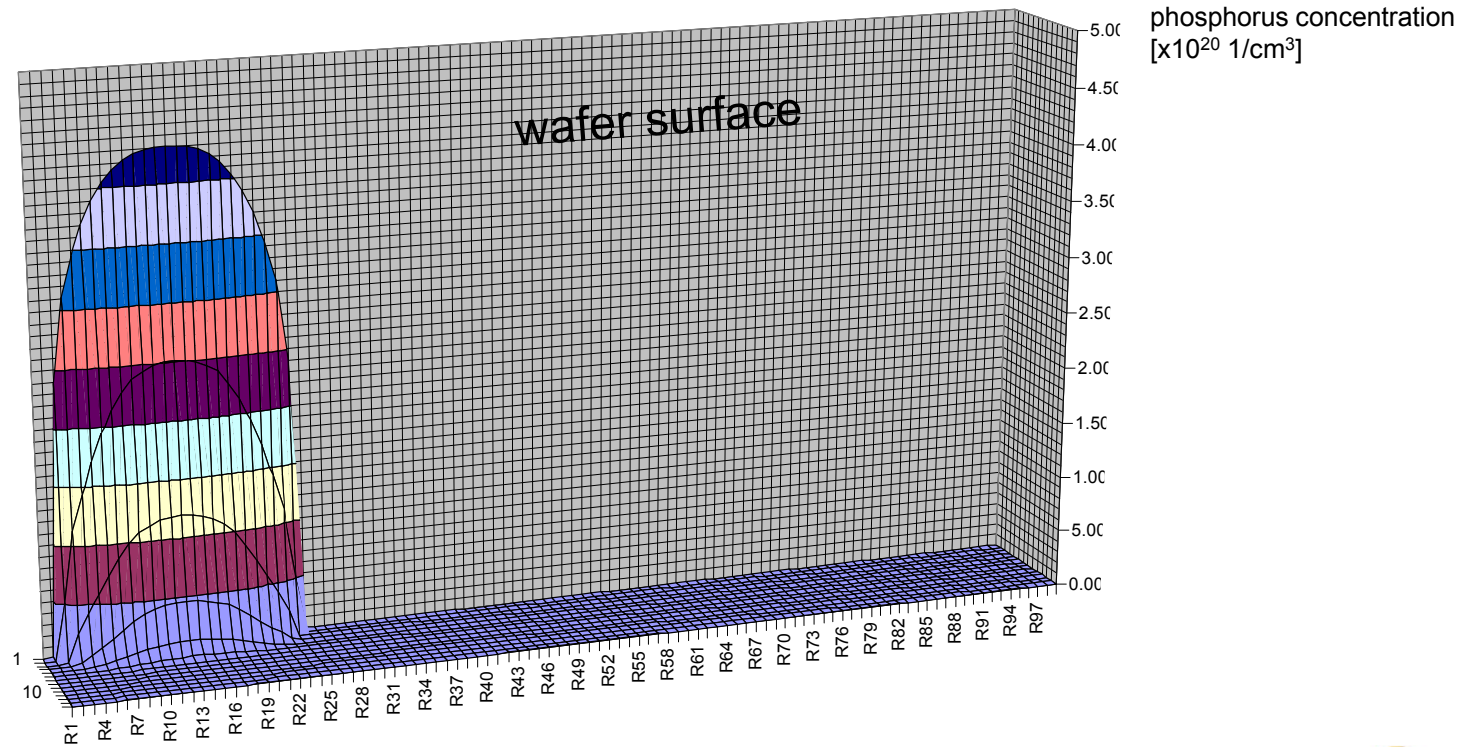
Simulation Results

10 ns laser pulses

$\Delta t = 2$ ns

Constant surface source concentration = $6.5 \times 10^{21} \text{ cm}^{-3}$

Out-diffusion coefficient = 4 cm/s



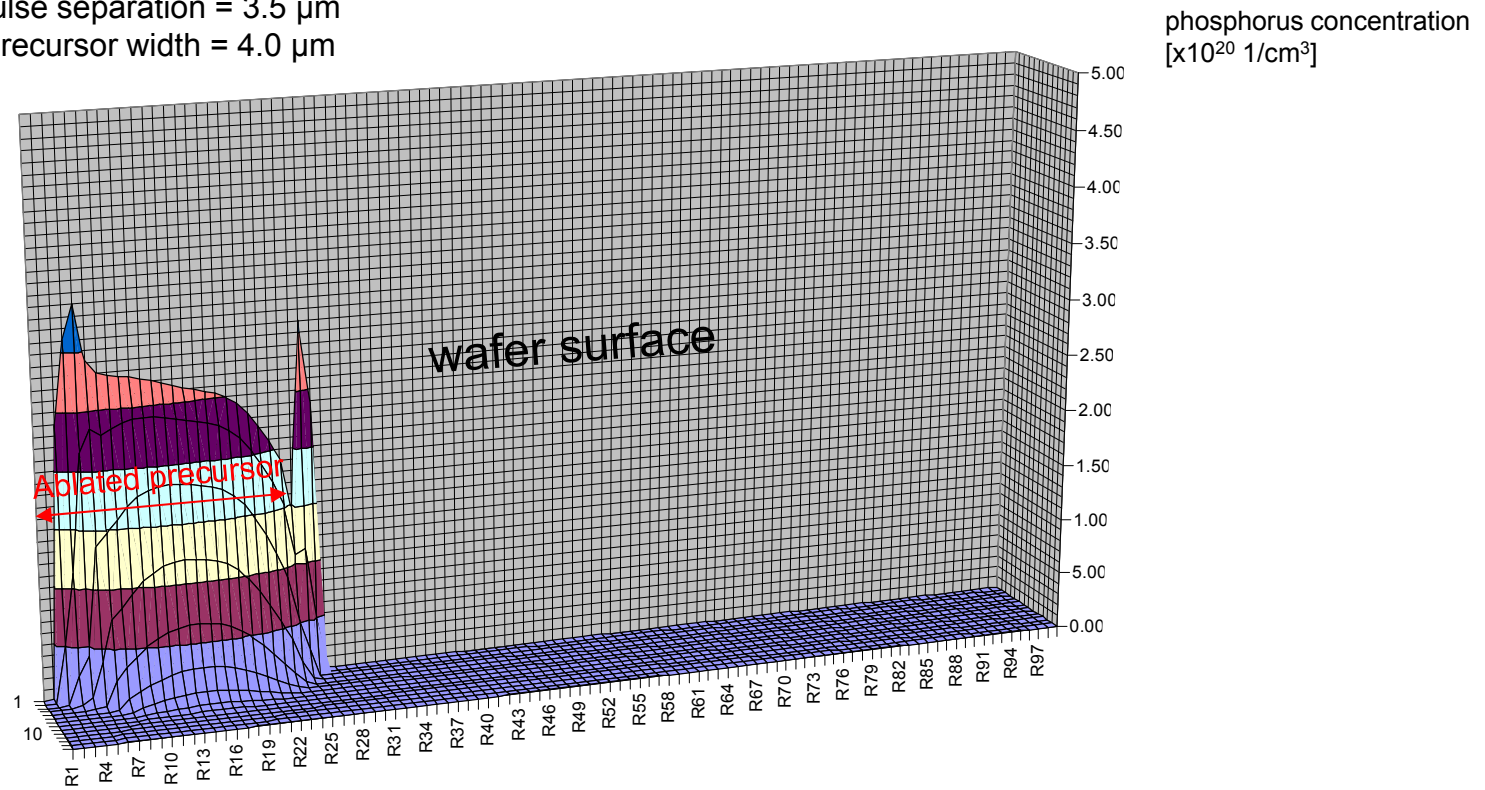
End of 1st laser pulse

©JHW



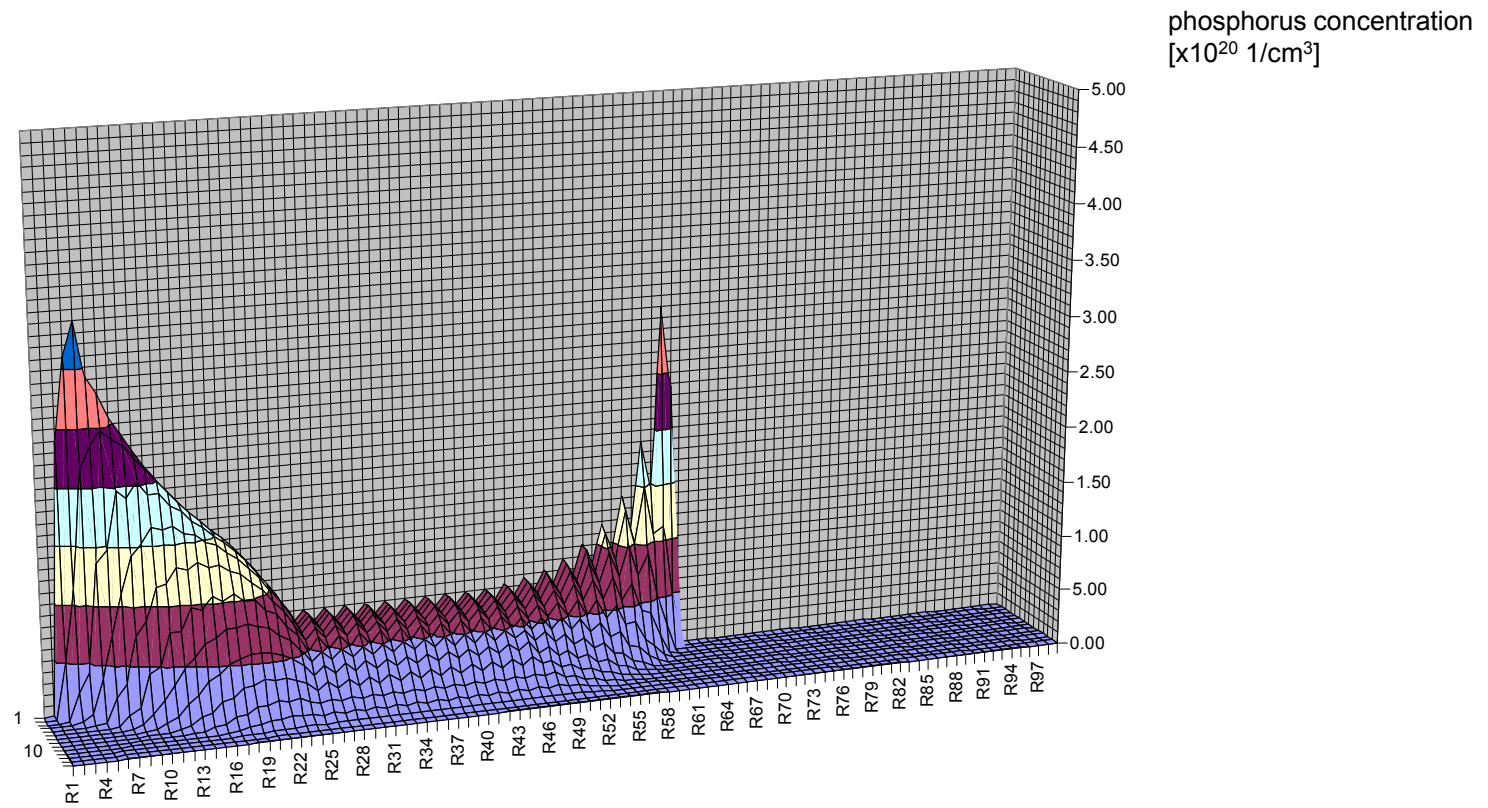
Simulation Results

Lateral pulse separation = $3.5\text{ }\mu\text{m}$
Ablated precursor width = $4.0\text{ }\mu\text{m}$



End of 2nd laser pulse

Simulation Results



End of 2nd, 3rd, 4th....., 20th laser pulse

7.1.7 Processing and Simulation Parameters

Pulse energy density: E_p [J/cm²]

Pulse to pulse separation: Δx [μm]

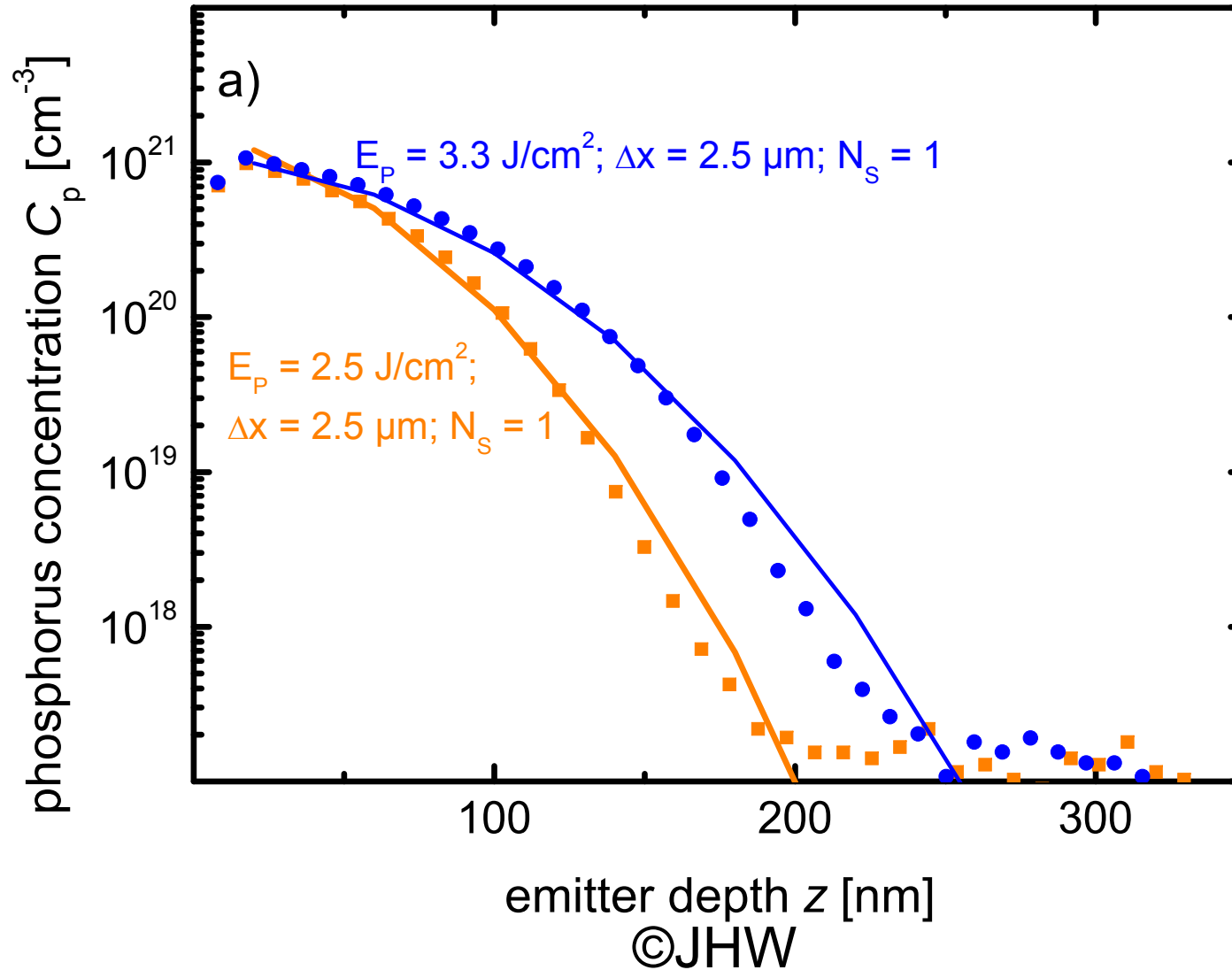
Number of laser scans: N_s [1]

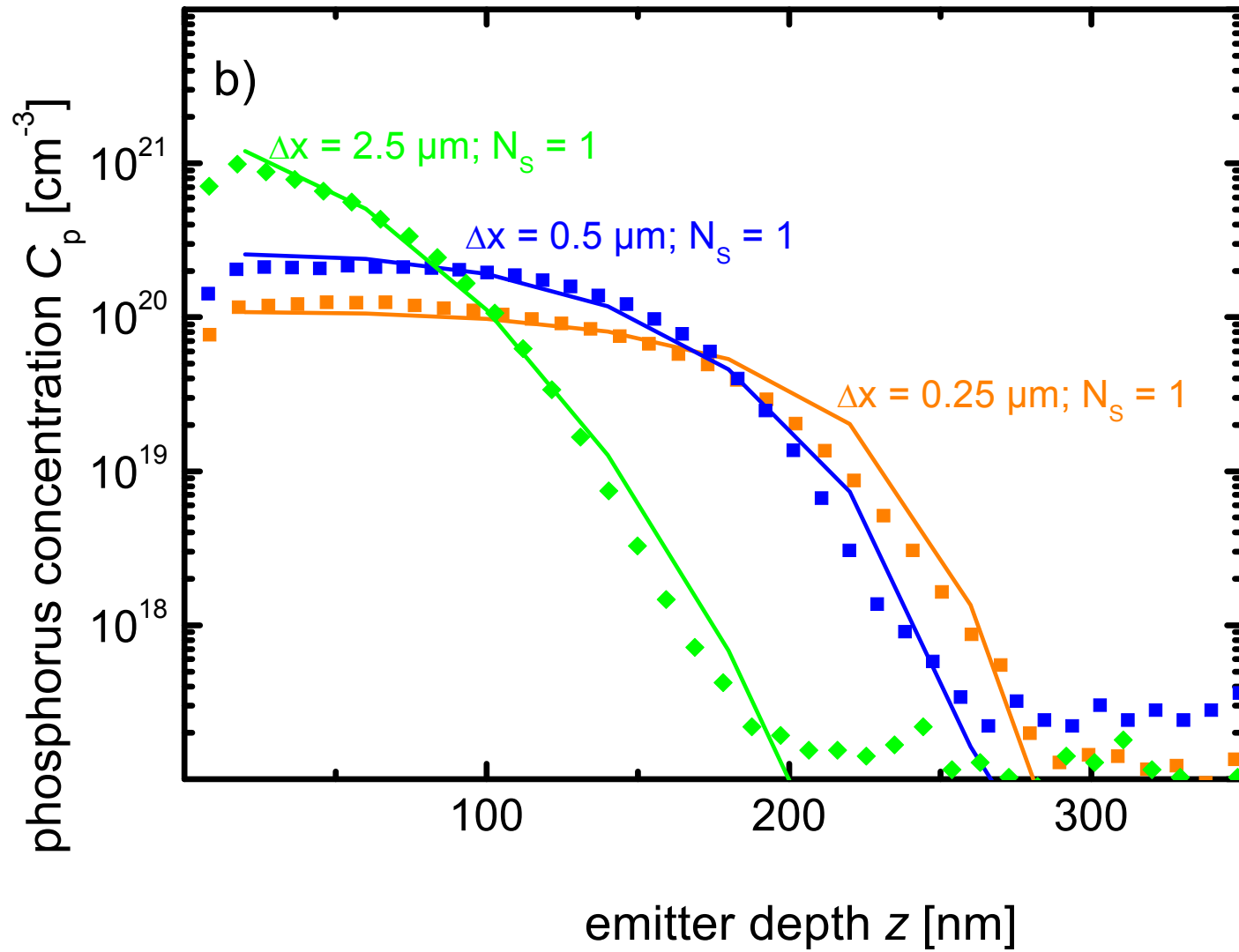
Ablation fraction: d_a [%]

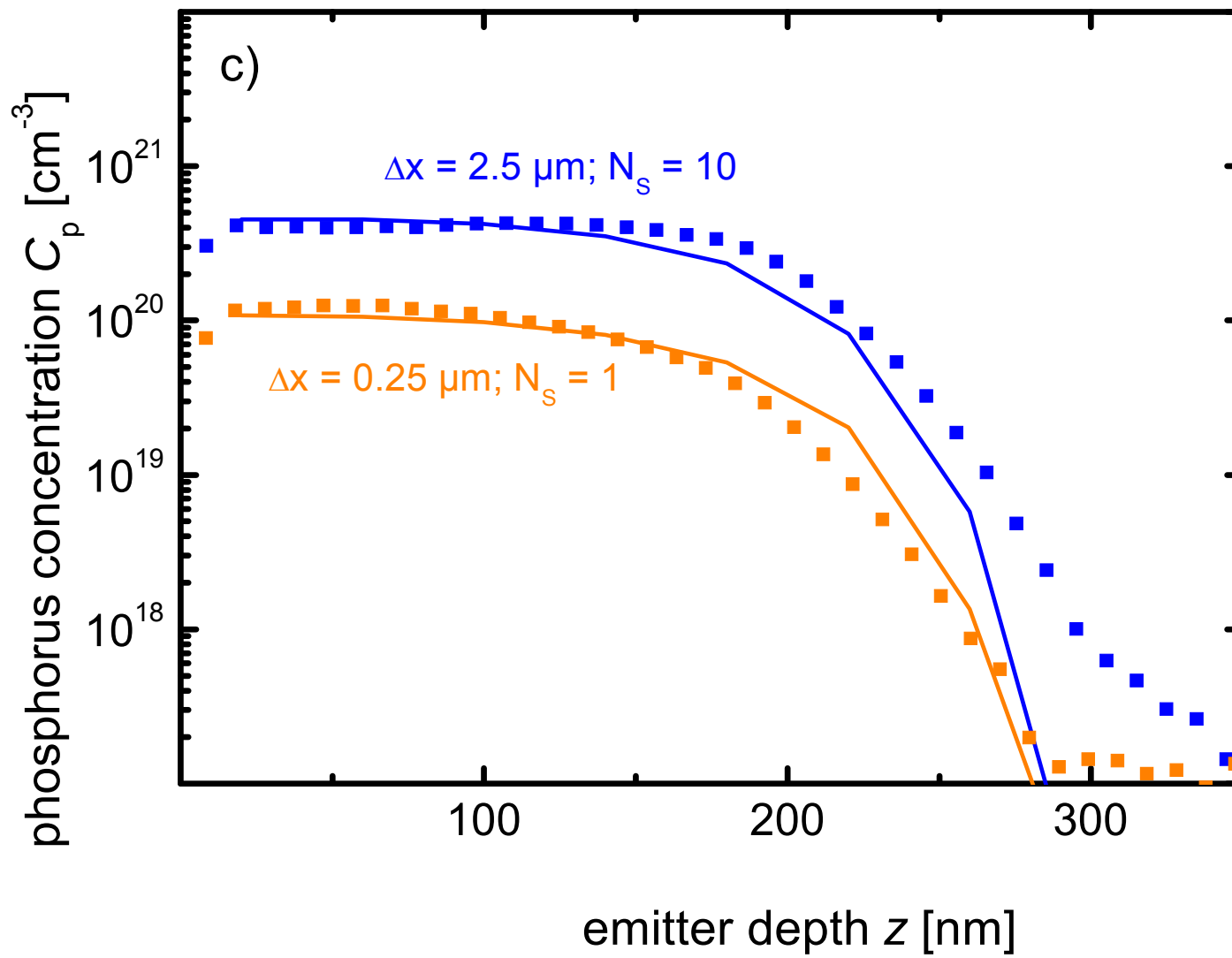
Ablation width: w_a [μm]

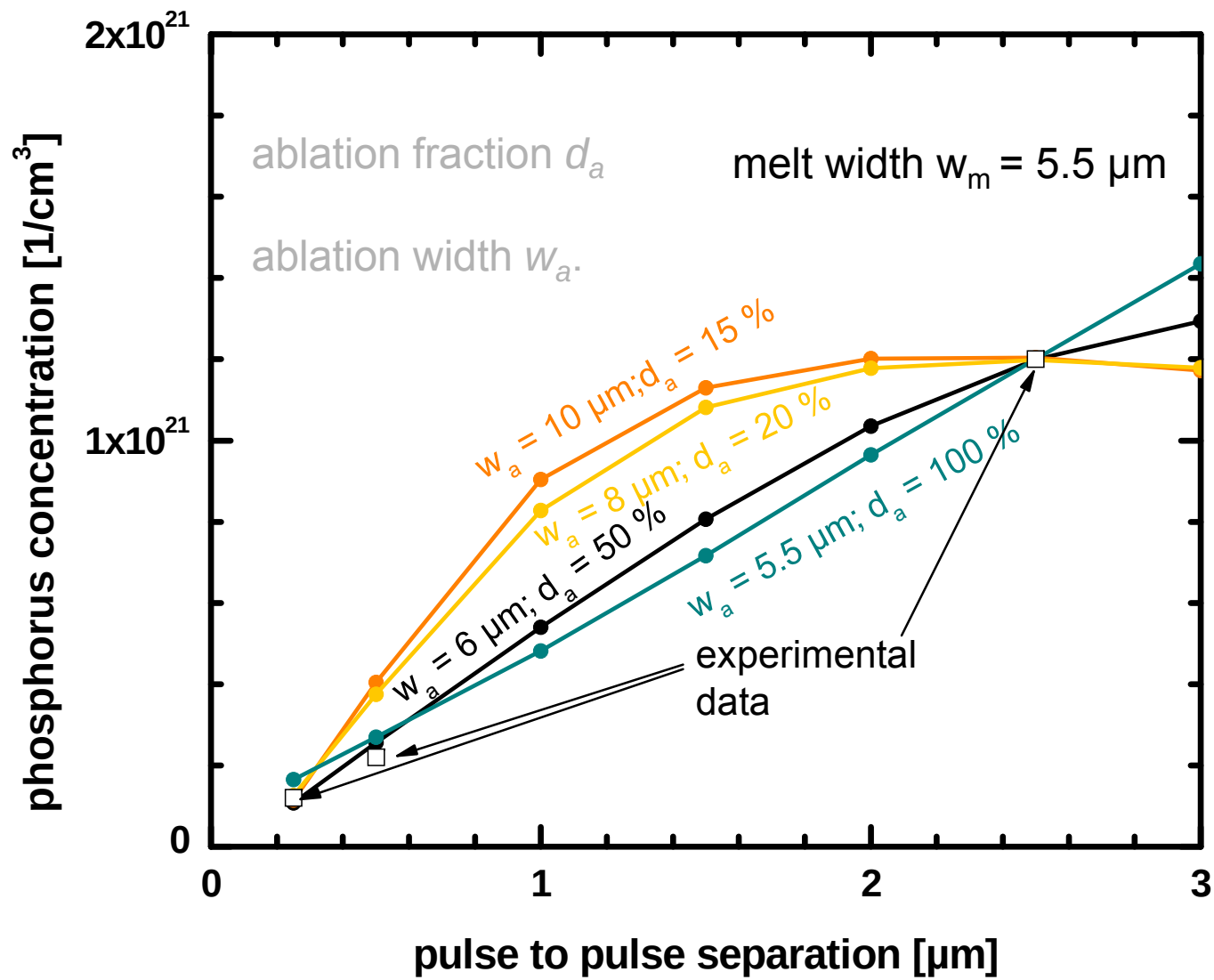
Melt width: w_m [μm]

7.1.8 Experiment and Simulation









7.1.9 Conclusions

increasing ablation fraction	→	decreasing surface concentration
increasing ablation width	→	decreasing surface concentration
increasing overlap/ decreasing pulse separation	→	decreasing surface concentration increasing fillfactor
increasing number of scans	→	decreasing surface concentration increasing fillfactor
increasing pulse energy density	→	constant surface concentration increasing dopand diffusion depth

Numerical model quantitatively explains experiments

Numerical model

- predicts dopand concentration profiles for different precursor layers
- useable to optimize laser processing parameters