

Aufgabe 11.a) Statische Bestimmtheit

$$a = 3 + 0 - 3 \cdot 1 = 0 \quad \checkmark \quad \text{statisch bestimmt}$$

Für $\alpha = 0$ ist das System verschieblich und somit nicht mehr statisch bestimmt.

2.a) Auflagerreaktionen

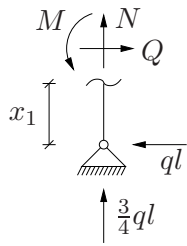
$$\uparrow \Sigma V : \quad 0 = -F \cdot \sin(\beta) + B_V = 0 \quad B_V = \frac{F}{\sqrt{2}}$$

$$\begin{aligned} \curvearrowright \Sigma M_A : \quad 0 &= B_V \cdot l + \left(l + \frac{2}{3}l\right) \cdot \tan(\alpha) \cdot C_H \\ &\quad - F \cdot \cos(\beta) \cdot \left(l + \frac{1}{3}l\right) \cdot \tan(\alpha) \\ &\quad - F \cdot \sin(\alpha) \cdot \left(l + \frac{1}{3}l\right) \\ C_H \cdot \frac{5}{3} \cdot \frac{1}{\sqrt{3}} &= -\frac{F}{\sqrt{2}} + \frac{F}{\sqrt{2}} \frac{4}{3} \left(1 + \frac{1}{\sqrt{3}}\right) \\ C_H &= \frac{3\sqrt{3}}{5\sqrt{2}} F \left(\frac{4}{3\sqrt{3}} + \frac{1}{3}\right) = C_H = \frac{F}{5\sqrt{2}} (4 + \sqrt{3}) \\ \rightarrow \Sigma H : \quad 0 &= -C_H + F \cdot \cos(\beta) + A_H \\ A_H &= \frac{F}{5\sqrt{2}} (4 + \sqrt{3}) - \frac{F}{\sqrt{2}} = \frac{F}{5\sqrt{2}} (\sqrt{3} - 1) \end{aligned}$$

3.b) Statische Bestimmtheit

$$a = 4 + 2 - 3 \cdot 2 = 0 \quad \checkmark \quad \text{statisch bestimmt}$$

4.b) Schnittkräfte



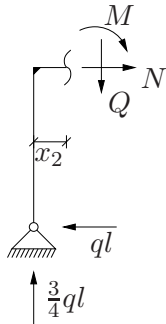
$$N(x_1) = -\frac{3}{4}ql$$

$$Q(x_1) = ql$$

$$M(x_1) = qlx_1$$

$$\rightarrow M(0) = 0$$

$$\rightarrow M(l) = ql^2$$



$$N(x_2) = ql$$

$$Q(x_2) = \frac{3}{4}ql - qx_2$$

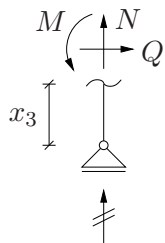
$$\rightarrow Q(0) = \frac{3}{4}ql$$

$$\rightarrow Q(l) = -\frac{1}{4}ql$$

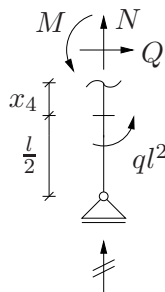
$$M(x_2) = ql^2 + \frac{3}{4}qlx_2 - \frac{1}{2}qx_2^2$$

$$\rightarrow M(0) = ql^2$$

$$\rightarrow M(l) = \frac{5}{4}ql^2$$

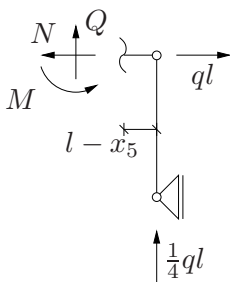


$$N(x_3) = Q(x_3) = M(x_3) = 0$$



$$N(x_4) = Q(x_4) = 0$$

$$M(x_4) = -ql^2$$



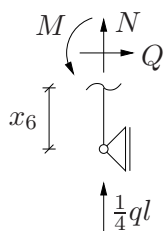
$$N(x_5) = ql$$

$$Q(x_5) = -\frac{1}{4}ql$$

$$M(x_5) = \frac{1}{4}ql(l - x_5)$$

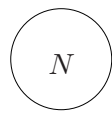
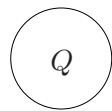
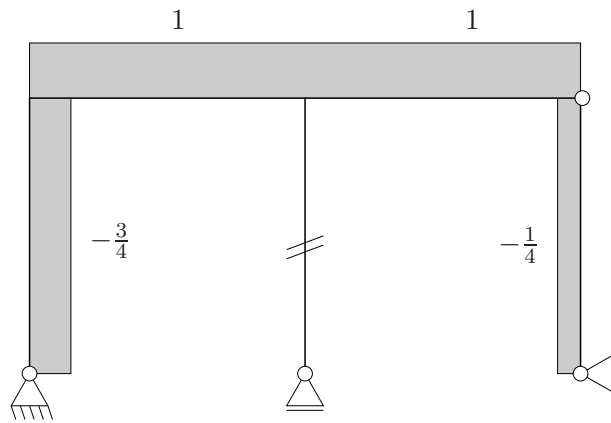
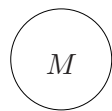
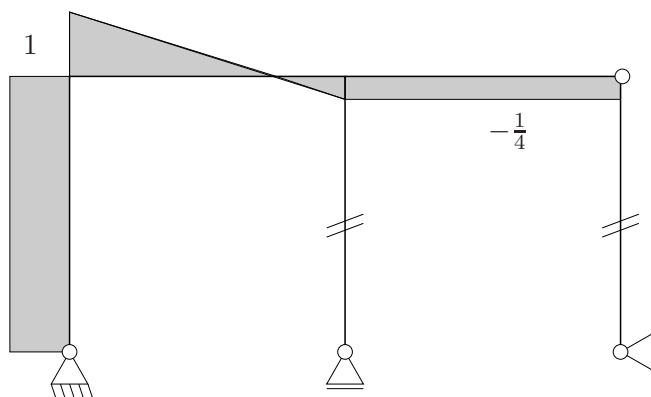
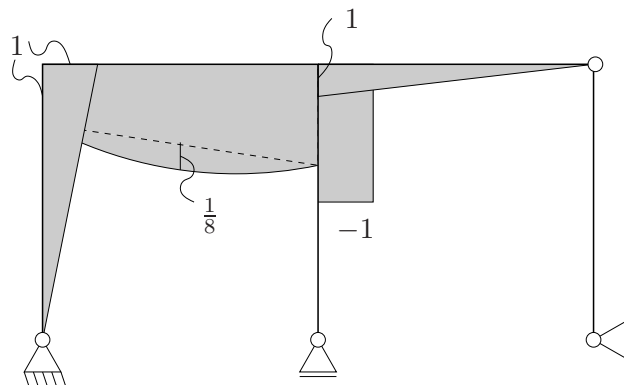
$$\rightarrow M(0) = \frac{1}{4}ql^2$$

$$\rightarrow M(l) = 0$$

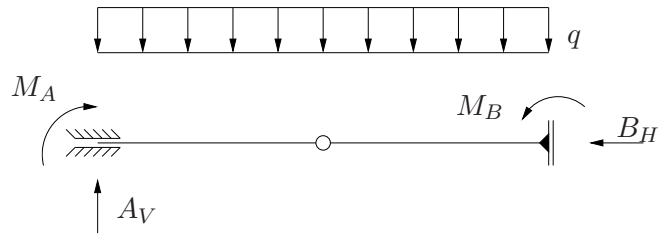


$$N(x_6) = -\frac{1}{4}ql$$

$$Q(x_6) = M(x_6) = 0$$


 $[ql]$

 $[ql]$

 $[ql^2]$


5.c) Auflagerreaktionen und Schnittkräfte



$$\rightarrow \Sigma H : 0 = -B_H$$

$$B_H = 0$$

$$\uparrow \Sigma V : 0 = -2ql + A_V$$

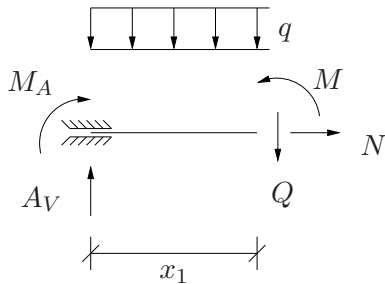
$$A_V = 2ql$$

$$\circlearrowleft \Sigma M_{Gelenk} : 0 = -M_B - \frac{ql^2}{2}$$

$$M_B = \frac{1}{2}ql^2$$

$$\circlearrowleft \Sigma M_A : 0 = -M_A + M_B - 2ql^2 = 0$$

$$M_A = -\frac{3}{2}ql^2$$



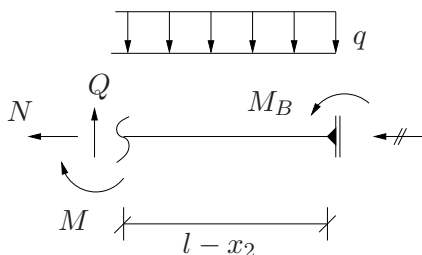
$$N(x_1) = 0$$

$$Q(x_1) = 2ql - qx_1$$

$$M(x_1) = -\frac{3}{2}ql^2 + 2qlx_1 - \frac{1}{2}qx_1^2$$

$$\rightarrow M(0) = -\frac{3}{2}ql^2$$

$$\rightarrow M(l) = 0$$



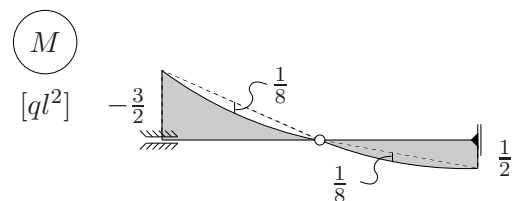
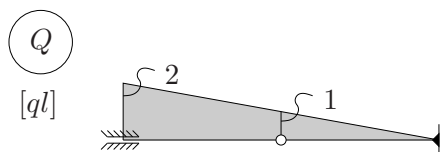
$$N(x_2) = 0$$

$$Q(x_2) = q(l - x_2)$$

$$M(x_2) = \frac{ql^2}{2} - \frac{1}{2}q(l - x_2)^2$$

$$\rightarrow M(0) = 0$$

$$\rightarrow M(l) = \frac{ql^2}{2}$$



Lösung:**Fachwerkaufgabe****1. statische Bestimmtheit**

1. äußere statische Bestimmtheit

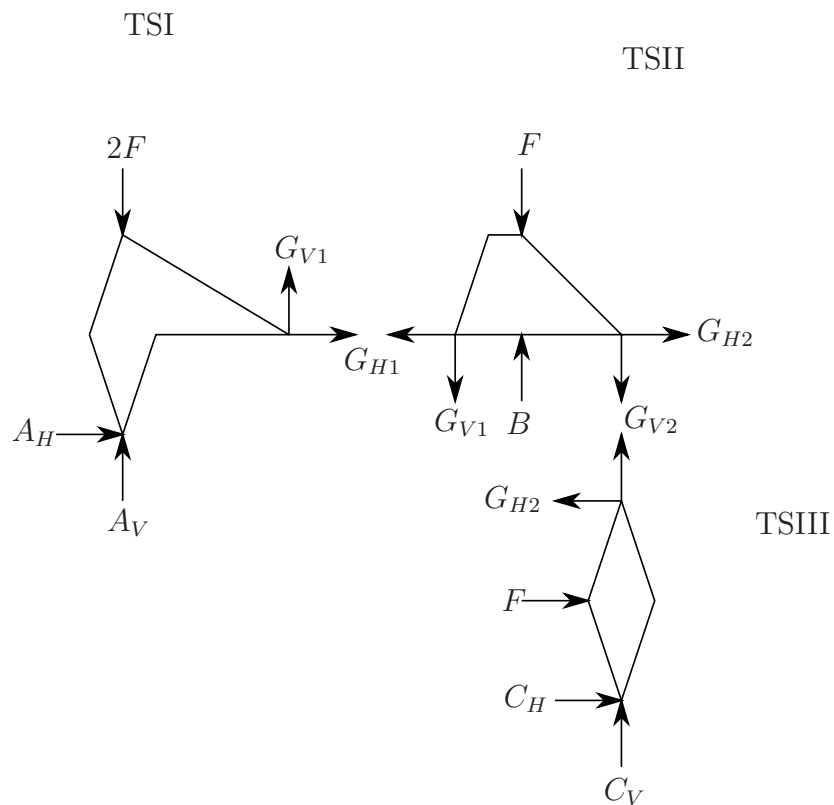
$$\begin{aligned}
 f &= 3 * n - (a + z) \\
 &= 3 * 3 - (2 + 2 + 1 + 4) = 0
 \end{aligned}
 \tag{1}$$

2. innere statische Bestimmtheit

$$\begin{aligned}
 f &= 2k - (a + s) \\
 &= 2 * 12 - (5 + 19) = 0
 \end{aligned}
 \tag{2}$$

2. Auflagerreaktion

1. Freischnitt:

**Abbildung 1:** Freischnitt

2. Gleichgewicht:

$$TS3: \quad \Sigma M_{G2}^{\circlearrowleft}: \quad F * 3l + C_H * 6l = 0 \quad \rightarrow C_H = -\frac{1}{2}F$$

$$GS: \quad \Sigma H: \quad A_H * F + C_H = 0 \quad \rightarrow A_H = -\frac{1}{2}F$$

$$TS1: \quad \Sigma M_{G1}^{\circlearrowleft}: \quad A_H * 3l - A_V * 5l + 2F * 5l = 0$$

$$A_V = 2F - \frac{1}{10}3F \quad \rightarrow A_V = \frac{17}{10}F$$

$$GS: \quad M_C^{\circlearrowleft}: \quad -A_V * 10l - A_H * 3l + 2F * 10l + F * 3l - B * 3l - F * 3l = 0$$

$$-17F * \frac{3}{2}F + 20F = 3B \quad \rightarrow B = \frac{3}{2}F$$

$$GS: \quad \Sigma V: \quad A_V - 2F + B - F + C_V = 0$$

$$\frac{17}{10}F - 2F + \frac{3}{2}F - F + C_V = 0$$

$$C_V = (-\frac{32}{10} + 3)F \quad \rightarrow C_V = -\frac{1}{5}F$$

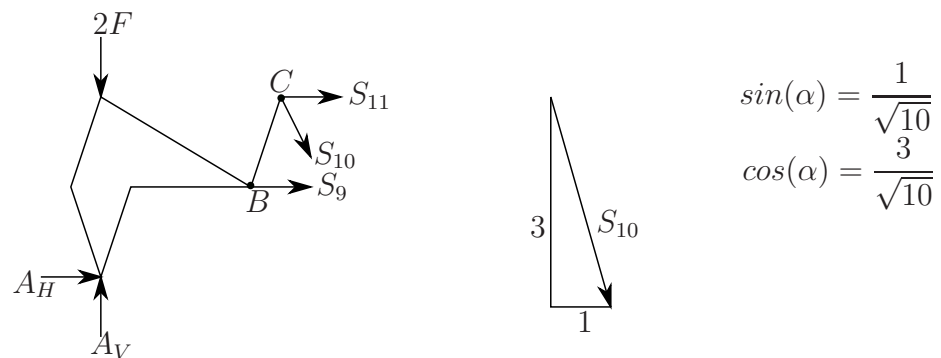
3. Ritterschnitt für S_9, S_{10}, S_{11} 

Abbildung 2: Ritterschnitt und Kraftzerlegung

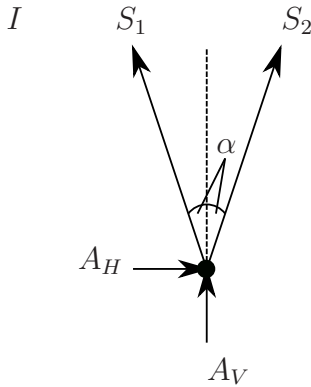
$$\Sigma M_C^{\circlearrowleft}: \quad A_H * 6l - A_V * 6l + 2F * 6l + S_9 * 3l = 0$$

$$S_9 = 2A_V - 2A_H - 4F = 2 * \frac{17}{10}F + 2 * \frac{1}{2}F - 4F \quad \rightarrow S_9 = -\frac{2}{5}F$$

$$\Sigma V: \quad A_V - 2F - \cos(\alpha) * S_{10} = 0$$

$$S_{10} = \frac{1}{\cos(\alpha)} * (A_V - 2F) = \frac{\sqrt{10}}{3} * (\frac{17}{10}F - 2F) \quad \rightarrow S_{10} = -\frac{1}{\sqrt{10}}F$$

4. Stäbe 1-6

Abbildung 3: Freischnitt S_1 und S_2

$$\Sigma H : -\sin(\alpha)S_1 + \sin(\alpha)S_2 + A_H = 0$$

$$S_1 = S_2 + \frac{1}{\sin(\alpha)}A_H \quad (1)$$

$$\Sigma V : \cos(\alpha)S_1 + \cos(\alpha)S_2 + A_V = 0 \quad (2)$$

$$(1) \rightarrow (2) : \cos(\alpha)S_2 + \frac{\cos(\alpha)}{\sin(\alpha)}A_H + \cos(\alpha)S_2 + A_V = 0$$

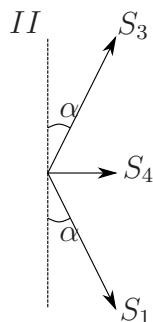
$$S_2 = -\frac{1}{2} * \left(\frac{1}{\sin(\alpha)}A_H + \frac{1}{\cos(\alpha)}A_V \right)$$

$$S_2 = -\frac{1}{2} \left(-\frac{15\sqrt{10}}{30}F + \frac{17\sqrt{10}}{30}F \right)$$

$$S_2 = -\frac{1}{3\sqrt{10}} \quad (3)$$

$$(3) \rightarrow (1) : S_1 = -\frac{1}{3\sqrt{10}}F + \sqrt{10}\left(-\frac{1}{2}\right)F$$

$$S_1 = -\frac{16}{3\sqrt{10}}$$

Abbildung 4: Freischnitt S_1, S_3 und S_4

$$\Sigma V : \cos(\alpha)S_3 - \cos(\alpha)S_1 = 0$$

$$S_3 = S_1 = -\frac{16}{3\sqrt{10}}$$

$$\Sigma H : S_4 + \sin(\alpha)S_3 + \sin(\alpha)S_1 = 0$$

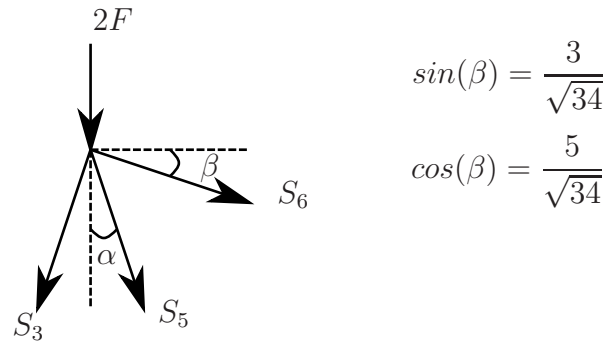
$$S_4 = -\sin(\alpha)S_3 - \sin(\alpha)S_1 = -\frac{2}{\sqrt{10}}\left(-\frac{16}{3\sqrt{10}}F\right)$$

$$S_4 = \frac{16}{15}F$$

$$\sin(\beta) = \frac{3}{\sqrt{34}}; S_3 = -\frac{16}{3\sqrt{10}}F$$

$$\cos(\beta) = \frac{5}{\sqrt{34}}$$

III

Abbildung 5: Freischnitt S_3, S_5 und S_6

$$\downarrow \Sigma V : \frac{3}{\sqrt{10}}S_3 + \frac{3}{\sqrt{10}}S_5 + \frac{3}{\sqrt{34}}S_6 + 2F = 0$$

$$\frac{S_3}{\sqrt{10}} + \frac{S_5}{\sqrt{10}} + \frac{S_6}{\sqrt{34}} + \frac{2}{3}F = 0$$

$$-\frac{16}{30}F + \frac{20}{30}F + \frac{S_5}{\sqrt{10}} + \frac{S_6}{\sqrt{34}} = 0$$

$$S_5 = \sqrt{10} * \left(-\frac{S_6}{\sqrt{34}} - \frac{2}{15}F\right) \quad (1)$$

$$\rightarrow \Sigma H : -\frac{1}{\sqrt{10}}S_3 + \frac{1}{\sqrt{10}}S_5 + \frac{5}{\sqrt{34}}S_6 = 0$$

$$\frac{16}{30}F + \frac{1}{\sqrt{10}}S_5 + \frac{5}{\sqrt{34}}S_6 = 0 \quad (2)$$

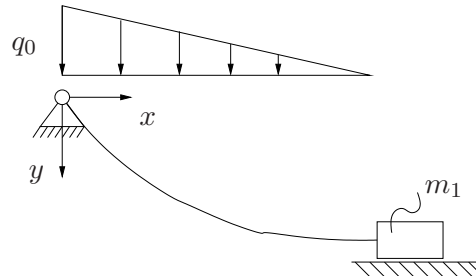
$$(1) \rightarrow (2) \frac{16}{30}F - \frac{S_6}{\sqrt{34}} - \frac{2}{15}F + \frac{5}{\sqrt{34}}S_6 = 0$$

$$\frac{4}{\sqrt{34}}S_6 = -\frac{12}{30}F = -\frac{4}{10}F$$

$$S_6 = -\frac{\sqrt{34}}{10} \quad (3)$$

$$(3) \rightarrow (1) S_5 = \sqrt{10} \left(\frac{1}{10}F - \frac{2}{15}F\right)$$

$$S_5 = -\frac{1}{3\sqrt{10}}F$$

Aufgabe 31. Seilkurve

$$y''(x) = -\frac{1}{H}q(x), \quad q(x) = \frac{q_0}{l}(-x + l)$$

$$y''(x) = -\frac{1}{H}\frac{q_0}{l}(l - x) = -\frac{1}{H}q_0 + \frac{1}{H}\frac{q_0}{l}x$$

$$y'(x) = -\frac{1}{H}q_0x + \frac{1}{2H}\frac{q_0}{l}x^2 + c_1$$

$$y(x) = -\frac{1}{2H}q_0x^2 + \frac{1}{6H}\frac{q_0}{l}x^3 + c_1x + c_2$$

$$y(x=0) = 0$$

$$c_2 = 0$$

$$y'(x=l) = -\frac{1}{H}q_0l + \frac{1}{2H}q_0 \cdot l + c_1 = 0$$

$$c_1 = \frac{1}{2H}q_0l$$

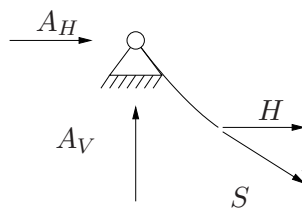
$$y(x=l) = h$$

$$= -\frac{1}{2H}q_0l^2 + \frac{1}{6H}q_0l^2 + \frac{1}{2H}q_0l^2 = h$$

$$H = \frac{q_0l^2}{6h}$$

$$y(x) = -\frac{6h}{2q_0l^2}q_0x^2 + \frac{1}{6}\frac{6h}{q_0l^2}\frac{q_0}{l}x^3 + \frac{6h}{2q_0l^2}q_0lx$$

$$y(x) = -3h\left(\frac{x}{l}\right)^2 + h\left(\frac{x}{l}\right)^3 + 3h\frac{x}{l}$$

2. Reaktionskräfte in A

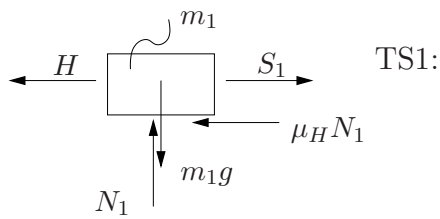
$$A_H = H$$

$$S = H\sqrt{1 + (y')^2}|_{x=0} = H\sqrt{1 + c_1^2} = H\sqrt{1 + \left(\frac{1}{2H}q_0l\right)^2}$$

$$A_V = \sqrt{S^2 - H^2} = \sqrt{H^2\left[1 + \left(\frac{1}{2H}q_0l\right)^2\right] - H^2}$$

$$A_V = \sqrt{\frac{q_0^2l^2}{4}} = \frac{q_0l}{2}$$

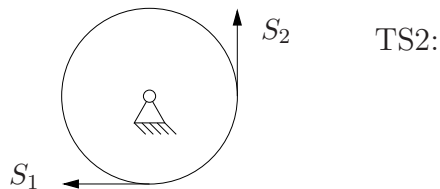
3. Minimale und maximale Masse für Ruhelage



$$\rightarrow \Sigma H = 0 : \quad H + \mu_H N_1 - S_1 = 0$$

$$\uparrow \Sigma V = 0 : \quad N_1 = m_1 g$$

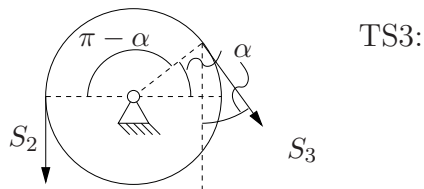
$$S_1 = H + \mu_H m_1 g$$



$$S_2 > S_1$$

$$S_2 = e^{\mu_R \frac{\pi}{2}} \cdot S_1$$

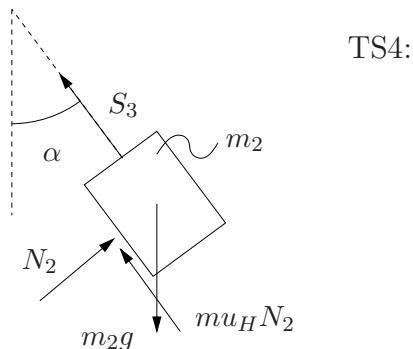
$$S_2 = e^{\mu_R \frac{\pi}{2}} (H + \mu_H m_1 g)$$



$$S_3 > S_2$$

$$S_2 = e^{\mu_R (\pi - \alpha)} \cdot S_3$$

$$S_2 = e^{\mu_R (\frac{3\pi}{2} - \alpha)} (H + \mu_H m_1 g)$$



$$\nearrow \Sigma = 0 : \quad N_2 = m_2 g \sin \alpha$$

$$\nwarrow \Sigma = 0 : \quad S_3 + \mu_H N_2 - m_2 g \cos \alpha = 0$$

$$S_3 = m_2 g \cos \alpha - \mu_H m_2 g \sin \alpha$$

$$S_3 = m_2 g (\cos \alpha - \mu_H \sin \alpha)$$

$$e^{\mu_R (\frac{3\pi}{2} - \alpha)} (H + \mu_H m_1 g) = m_2 g (\cos \alpha - \mu_H \sin \alpha)$$

$$m_2 = e^{\mu_R \frac{3\pi}{2} - \alpha} \cdot \frac{H + \mu_H m_1 g}{g (\cos \alpha - \mu_H \sin \alpha)}$$

$$TS1 : \quad S_1 = H - \mu_H m_1 g$$

$$TS2 : \quad S_1 > S_2$$

$$S_1 = e^{\mu_R \frac{\pi}{2}} \cdot S_2 \quad \rightarrow \quad S_2 = e^{-\mu_R \frac{\pi}{2}} (H - \mu_H m_1 g)$$

$$TS3 : \quad S_2 > S_3$$

$$S_2 = e^{\mu_R (\pi - \alpha)} S_3 \quad \rightarrow \quad S_3 = e^{-\mu_R (\pi - \alpha)} S_2$$

$$S_3 = e^{-\mu_R (\frac{3\pi}{2} - \alpha)} (H - \mu_H m_1 g)$$

$$TS4 : \quad S_3 = m_2 g (\cos \alpha + \mu_H \sin \alpha)$$

$$m_2 = e^{-\mu_R (\frac{3\pi}{2} - \alpha)} \frac{H - \mu_H m_1 g}{g (\cos \alpha + \mu_H \sin \alpha)}$$