

Technische Mechanik I
Bachelorprüfung WS 2014/2015

Dienstag, 17.03.2015

Rechenteil: 08:45 - 10:15 Uhr

Erlaubte Hilfsmittel: Alle, außer Kommunikationsgeräte

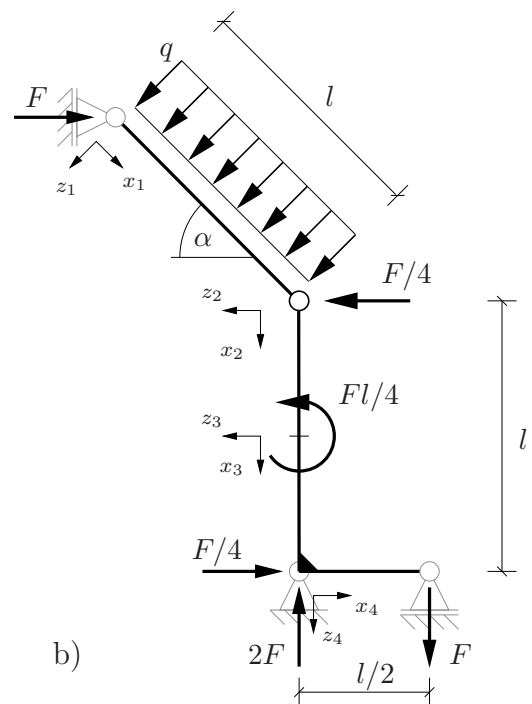
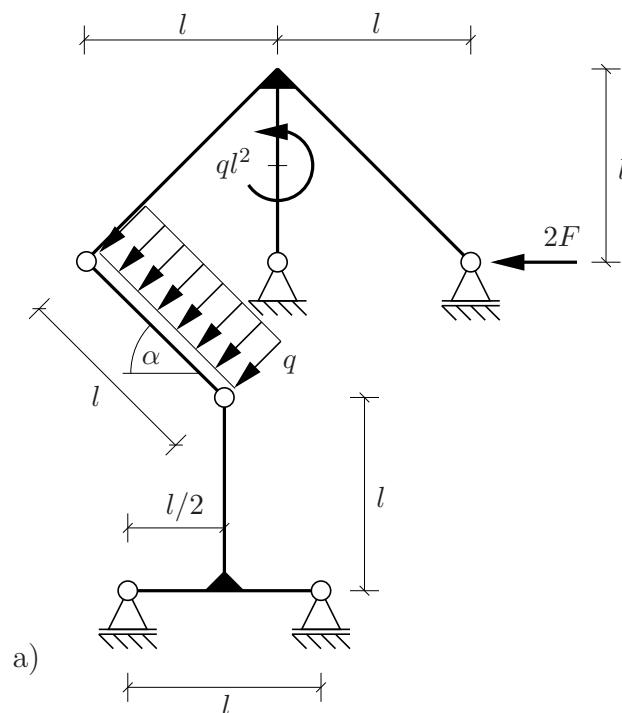
Aufgabe 1 (25 Punkte)

Das in a) dargestellte Tragwerk wird durch eine Einzellast, durch ein Einzelmoment sowie durch eine konstante Streckenlast beansprucht.

1. Überprüfen Sie das Tragwerk auf statische Bestimmtheit.
2. Ermitteln Sie alle Auflagerreaktionen.

Das in b) dargestellte System wird durch ein Einzelmoment, eine Einzelkraft und eine Streckenlast beansprucht. Alle Auflagerreaktionen sind gegeben.

3. Bestimmen Sie sämtliche Schnittgrößen in dem Tragwerk und stellen Sie diese unter Angabe aller Rand- und Extremwerte grafisch dar.



Geg.: $l, q, F = \frac{\sqrt{2}}{2}ql, \alpha = 45^\circ$.

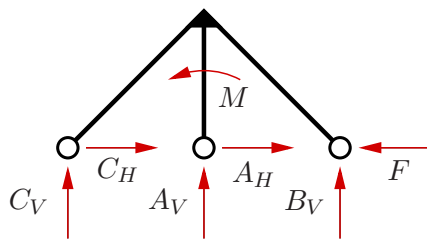
Lösung:

1. Statische Bestimmtheit

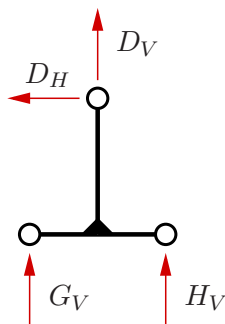
$$f = 3 \cdot 3 - (5 + 4) = 0$$

$\Rightarrow$ Das System ist statisch bestimmt

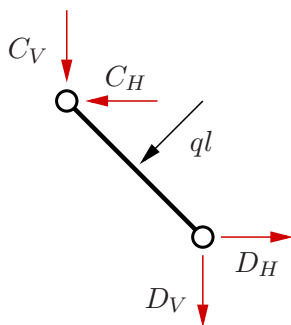
2. Auflagerkräfte



$$\begin{aligned} \uparrow \Sigma V : A_V + B_V + C_V &= 0 & \Rightarrow A_V &= -B_V - C_V \\ \rightarrow \Sigma H : C_H + A_H - F &= 0 & \Rightarrow A_H &= -C_H + F \\ \curvearrowright \Sigma M_C : M + B_V \cdot 2l + A_V \cdot l &= 0 & \Rightarrow B_V &= -\frac{1}{2} \left(\frac{M}{l} + A_V \right) \end{aligned}$$



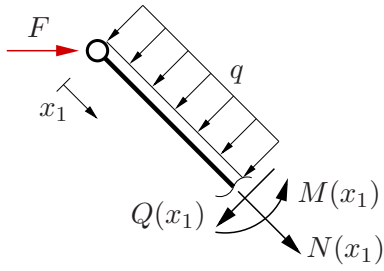
$$\begin{aligned} \uparrow \Sigma V : G_V + H_V + D_V &= 0 \\ \rightarrow \Sigma H : D_H &= 0 \\ \curvearrowright \Sigma M_D : H_V \cdot \frac{l}{2} - G_V \cdot \frac{l}{2} &= 0 \Rightarrow H_V = G_V \end{aligned}$$



$$\begin{aligned} \curvearrowright \Sigma M_C : D_V \cdot \frac{\sqrt{2}}{2}l - ql \cdot \frac{l}{2} &= 0 & \Rightarrow D_V &= -\frac{\sqrt{2}}{2}ql \\ \uparrow \Sigma V : -D_V - C_V - \frac{\sqrt{2}}{2}ql &= 0 & \Rightarrow C_V &= -D_V - \frac{\sqrt{2}}{2}ql = 0 \\ \rightarrow \Sigma H : D_H - C_H - \frac{\sqrt{2}}{2}ql &= 0 & \Rightarrow C_H &= -\frac{\sqrt{2}}{2}ql \end{aligned}$$

$$\begin{aligned} &\Rightarrow A_H = \frac{\sqrt{2}}{2}ql + F \\ A_V &= \frac{1}{2} \left(\frac{M}{l} + A_V \right) - C_V \Rightarrow A_V = \frac{M}{l} \\ &\Rightarrow B_V = -\frac{1}{2} \left(\frac{M}{l} + \frac{M}{l} \right) = -\frac{M}{l} \\ 2G_V - \frac{\sqrt{2}}{2}ql &= 0 \Rightarrow G_V = H_V = \frac{\sqrt{2}}{4}ql \end{aligned}$$

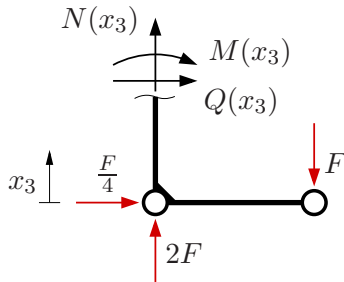
3. Schnittgrößen



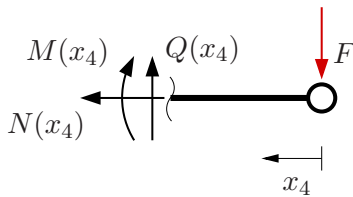
$$\begin{aligned}
 \searrow \Sigma : \quad N(x_1) + \frac{1}{\sqrt{2}} \frac{\sqrt{2}}{2} ql &= 0 \\
 \Rightarrow N(x_1) &= -\frac{1}{2} ql \\
 \swarrow \Sigma : \quad Q(x_1) + qx_1 - \frac{1}{\sqrt{2}} \frac{\sqrt{2}}{2} ql &= 0 \\
 \Rightarrow Q(x_1) &= \frac{1}{2} ql - qx_1 \\
 \curvearrowright \Sigma M_S : \quad M(x_1) + qx_1 \cdot \frac{x_1}{2} - \frac{\sqrt{2}}{2} ql \cdot \frac{1}{\sqrt{2}} x_1 &= 0 \\
 \Rightarrow M(x_1) &= \frac{1}{2} ql x_1 - \frac{1}{2} qx_1^2
 \end{aligned}$$

RW: $x_1 \in [0, l]$

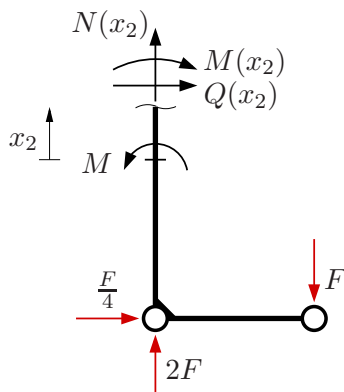
$$\begin{aligned}
 Q(x_1 = 0) &= \frac{1}{2} ql \\
 Q(x_1 = l) &= -\frac{1}{2} ql \\
 M(x_1 = 0) &= 0 \\
 M(x_1 = l) &= 0
 \end{aligned}$$



$$\begin{aligned}
 \uparrow \Sigma V : \quad N(x_3) + \sqrt{2} ql - \frac{\sqrt{2}}{2} ql &= 0 \\
 \Rightarrow N(x_3) &= -\frac{\sqrt{2}}{2} ql \\
 \rightarrow \Sigma H : \quad \Rightarrow Q(x_3) &= -\frac{\sqrt{7}}{8} ql \\
 \curvearrowright \Sigma M_S : \quad -M(x_3) - \frac{\sqrt{2}}{2} ql \cdot \frac{l}{2} + \frac{\sqrt{2}}{8} ql \cdot \left(\frac{l}{2} - x_3\right) &= 0 \\
 \Rightarrow M(x_3) &= -\frac{\sqrt{2}}{4} ql^2 + \frac{\sqrt{2}}{16} ql^2 - \frac{\sqrt{2}}{8} ql x_3
 \end{aligned}$$



$$\begin{aligned}
 \uparrow \Sigma V : \quad \Rightarrow N(x_4) &= 0 \\
 \rightarrow \Sigma H : \quad Q(x_4) - \frac{\sqrt{2}}{2} ql &= 0 \\
 Q(x_4) &= \frac{\sqrt{2}}{2} ql \\
 \curvearrowright \Sigma M_S : \quad M(x_4) + \frac{\sqrt{2}}{2} ql \left(\frac{l}{2} - x_4\right) &= 0 \\
 \Rightarrow M(x_4) &= \frac{\sqrt{2}}{2} ql x_4 - \frac{\sqrt{2}}{4} ql^2
 \end{aligned}$$



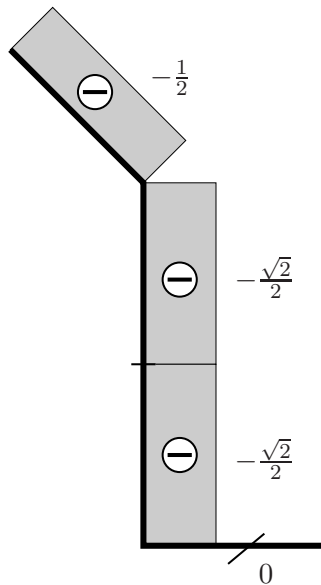
$$\begin{aligned}
 \uparrow \Sigma V : \quad N(x_2) + \sqrt{2} ql - \frac{\sqrt{2}}{2} ql &= 0 \\
 \Rightarrow N(x_2) &= -\frac{\sqrt{2}}{2} ql \\
 \rightarrow \Sigma H : \quad Q(x_2) + \frac{\sqrt{2}}{8} ql &= 0 \\
 Q(x_2) &= -\frac{\sqrt{2}}{8} ql \\
 \curvearrowright \Sigma M_S : \quad M(x_2) - M - \frac{\sqrt{2}}{8} ql \cdot \left(\frac{l}{2} + \left(\frac{l}{2} - x_2\right)\right) + \frac{\sqrt{2}}{2} ql \cdot \frac{l}{2} &= 0 \\
 \Rightarrow M(x_2) &= M - \frac{\sqrt{2}}{8} ql^2 - \frac{\sqrt{2}}{8} ql x_2
 \end{aligned}$$

RW: $x_2 \in [0, \frac{l}{2}]$

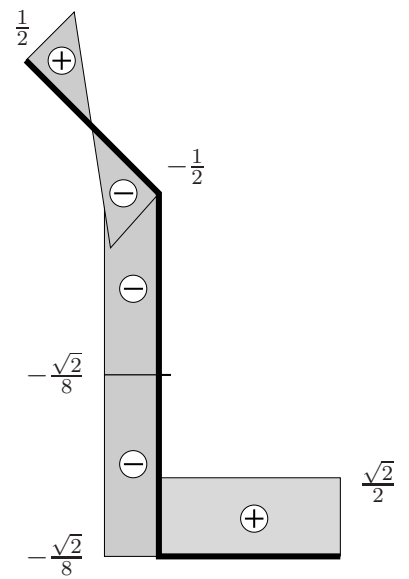
$$M(x_2 = 0) = M - \frac{\sqrt{7}}{8}ql^2 = 0$$

$$M(x_2 = \frac{l}{2}) = -\frac{\sqrt{2}}{16}ql^2$$

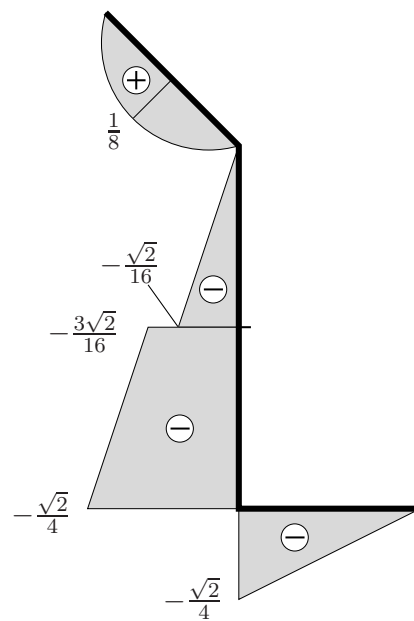
Normalkraftverlauf:



Querkraftverlauf:



Momentenverlauf:



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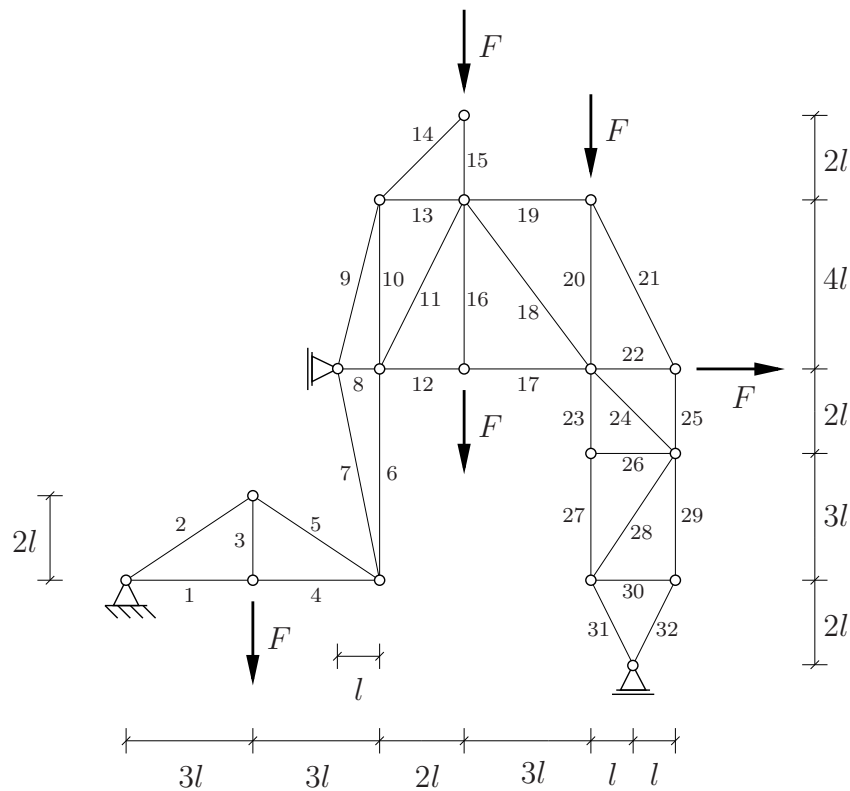
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Aufgabe 2 (25 Punkte)

Das dargestellte Fachwerk wird durch fünf Einzelkräfte belastet.

1. Überprüfen Sie das System auf statische Bestimmtheit.
2. Bestimmen Sie sämtliche Auflagerkräfte und Gelenkreaktionen.
3. Ermitteln Sie die Kräfte in den Stäben 11-13 mit Hilfe eines geeigneten Ritterschnittes.
4. Bestimmen Sie die Stabkräfte 1-10 sowie 27-32 mit dem Knotenpunktverfahren



Geg.: l , F .

Lösung:

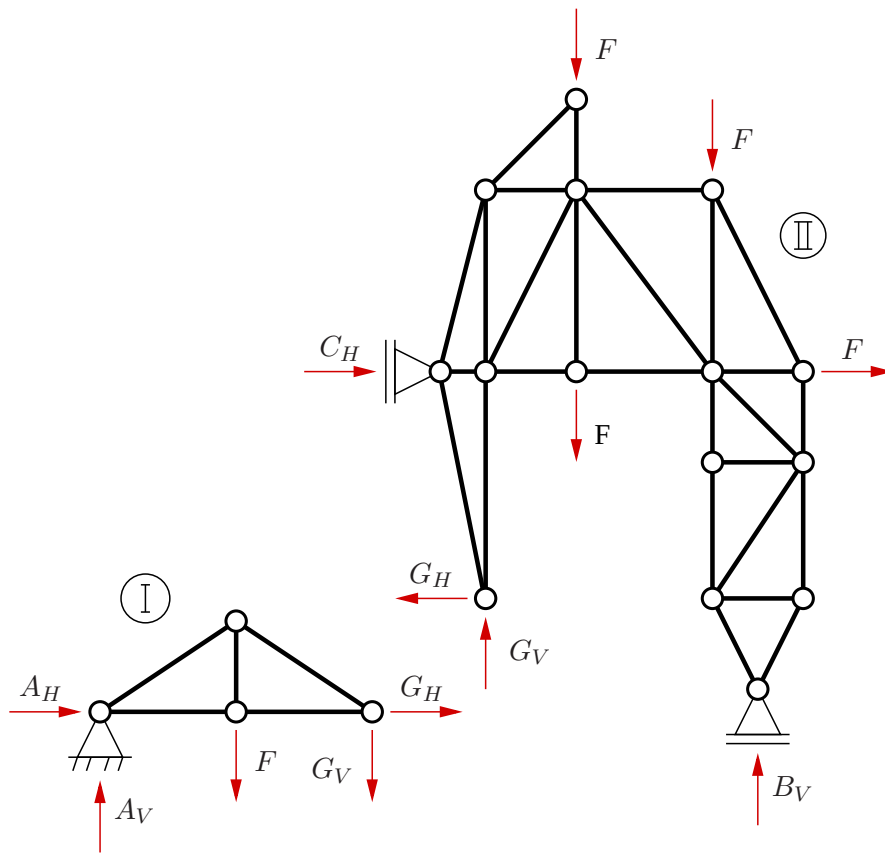
1. Statische Bestimmtheit

Äußere: $f = 3 \cdot 2 - (4 + 2) = 0$

Innere: $2 \cdot 18 - (32 + 4) = 0$

$\Rightarrow$ Das System ist statisch bestimmt.

2. Auflagerkräfte



TS 1:

$$\curvearrowright G: -A_V \cdot 6l + F \cdot 3l = 0 \Rightarrow A_V = \frac{1}{2}F$$

$$\curvearrowright A: -F \cdot 3l - G_V \cdot 6l = 0 \Rightarrow G_V = -\frac{1}{2}F$$

GS:

$$\uparrow \Sigma: A_V + B_V - F - F - F - F = 0$$

$$\Rightarrow B_V = 4F - \frac{1}{2}F = \frac{7}{2}F$$

TS2:

$$\curvearrowright \Sigma M_G: B_V \cdot 6l - C_H \cdot 5l - 2F \cdot 2l - F \cdot 5l - F \cdot 5l = 0$$

$$\Rightarrow C_H = \frac{1}{5l} \left(\frac{7}{2} \cdot 6Fl - 4Fl - 5Fl - 5Fl \right) = \frac{7}{5}F$$

GS:

$$\leftarrow \Sigma: A_H + C_H + F = 0$$

$$\Rightarrow A_H = -\frac{7}{5}F - F = -\frac{12}{5}F$$

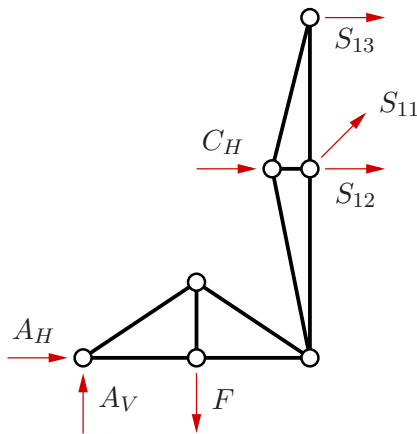
TS 1:

$$\uparrow \Sigma: G_H + A_H = 0$$

$$\Rightarrow G_H = \frac{12}{5}F$$

3. Stabkräfte

Nullstäbe: S_{14} .



$$\curvearrowright \Sigma I: -S_{13} \cdot 4l + F \cdot 3l - \frac{1}{2}F \cdot 6l + \left(-\frac{12}{5}F\right) \cdot 5l = 0$$

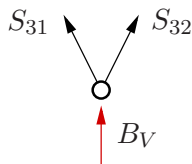
$$\Rightarrow S_{13} = -3F$$

$$\uparrow \Sigma V: S_{11} \cdot \frac{4l}{\sqrt{20}l} - F + \frac{1}{2}F = 0$$

$$\Rightarrow S_{11} = \frac{1}{2}F \cdot \frac{\sqrt{20}}{4} = \frac{\sqrt{5}}{4}F$$

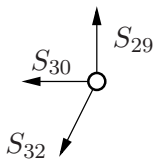
$$\rightarrow \Sigma H: S_{13} + S_{12} + \frac{7}{5}F - \frac{12}{5}F + S_{11} \frac{2}{\sqrt{20}} = 0$$

$$\Rightarrow S_{12} = 3F - \frac{7}{5}F + \frac{12}{5}F - \frac{1}{4}F = \frac{15}{4}F$$



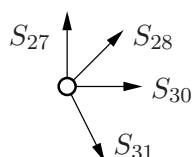
$$\rightarrow \Sigma H: -\frac{1}{\sqrt{5}} \cdot S_{31} + \frac{1}{\sqrt{5}} S_{32} = 0 \Rightarrow S_{31} = S_{32}$$

$$\uparrow \Sigma V: \frac{7}{\sqrt{5}} S_{31} \cdot 2 + \frac{7}{2}F = 0 \Rightarrow S_{31} = S_{32} = -\frac{7\sqrt{5}}{8}F$$



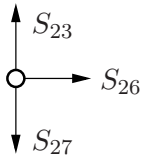
$$\uparrow \Sigma V: S_{29} - \frac{2}{\sqrt{5}} S_{32} = 0 \Rightarrow S_{29} = -\frac{7}{4}F$$

$$\rightarrow \Sigma H: S_{30} - \frac{1}{\sqrt{5}} S_{32} = 0 \Rightarrow S_{30} = \frac{7}{8}F$$



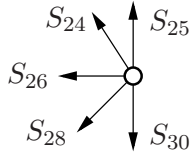
$$\rightarrow \Sigma H: \frac{2}{\sqrt{13}} S_{28} + S_{30} + \frac{1}{\sqrt{5}} S_{31} = 0 \Rightarrow S_{28} = 0$$

$$\uparrow \Sigma V: S_{27} - \frac{2}{\sqrt{5}} S_{31} = 0 \Rightarrow S_{27} = -\frac{7}{4}F$$



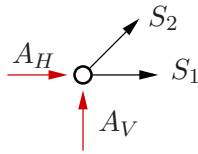
$$\uparrow \Sigma V : S_{23} - S_{27} = 0 \Rightarrow S_{23} = -\frac{7}{4}F$$

$$\rightarrow \Sigma H : \Rightarrow S_{26} = 0$$



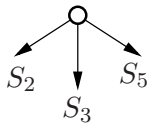
$$\rightarrow \Sigma H : \Rightarrow S_{24} = 0$$

$$\uparrow \Sigma V : \Rightarrow S_{25} = S_{30} = \frac{7}{8}F$$



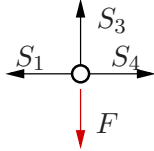
$$\uparrow \Sigma V : \frac{2}{\sqrt{13}}S_2 + \frac{1}{2}F = 0 \Rightarrow S_2 = -\frac{\sqrt{13}}{4}F$$

$$\rightarrow \Sigma H : S_1 - \frac{12}{5}F + \frac{3}{\sqrt{13}}S_2 = 0 \Rightarrow S_1 = \frac{63}{20}F$$

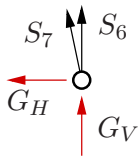


$$\rightarrow \Sigma H : -\frac{3}{\sqrt{13}}S_2 + \frac{3}{\sqrt{13}}S_5 = 0 \Rightarrow S_5 = -\frac{\sqrt{13}}{4}F$$

$$\downarrow \Sigma V : S_3 + \frac{2}{\sqrt{13}}S_2 + \frac{7}{\sqrt{13}}S_5 = 0 \Rightarrow S_3 = F$$

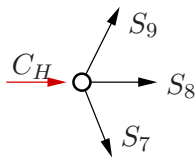


$$\rightarrow \Sigma H : -S_1 + S_4 = 0 \Rightarrow S_4 = S_1 = \frac{63}{20}F$$



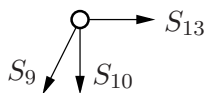
$$\rightarrow \Sigma H : -\frac{1}{\sqrt{26}}S_7 - \frac{12}{5}F = 0 \Rightarrow S_7 = -\frac{12\sqrt{26}}{5}F$$

$$\downarrow \Sigma V : S_6 + \frac{5}{\sqrt{26}}S_7 - \frac{1}{2}F = 0 \Rightarrow S_6 = \frac{25}{2}F$$



$$\uparrow \Sigma V : \frac{4}{\sqrt{17}}S_9 - \frac{5}{\sqrt{26}}S_7 = 0 \Rightarrow S_9 = -3\sqrt{17}F$$

$$\rightarrow \Sigma H : \frac{7}{5}D + S_8 + \frac{1}{\sqrt{17}}S_9 + \frac{1}{\sqrt{26}}S_7 = 0 \Rightarrow S_8 = 4F$$



$$\downarrow \Sigma V : S_{10} + \frac{4}{\sqrt{17}}S_9 = 0 \Rightarrow S_{10} = 12F$$

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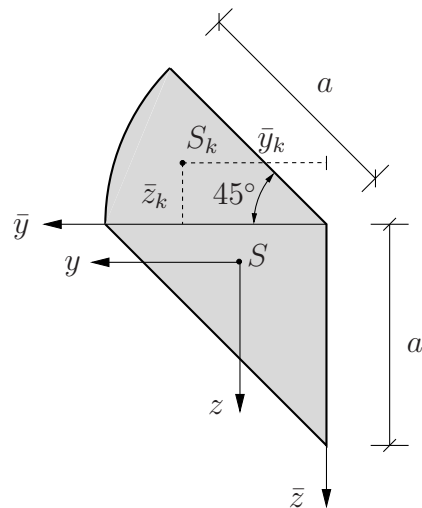
Aufgabe 3 (25 Punkte)

Berechnen Sie für den abgebildeten Querschnitt

1. den Schwerpunkt S bzgl. des $\bar{y}, \bar{z}$ - Koordinatensystems, wobei der Schwerpunkt S_k des Achtelkreises gegeben ist,
2. die Flächenträgheitsmomente $I_{\bar{y}\bar{y}}, I_{\bar{z}\bar{z}}, I_{\bar{y}\bar{z}}$ bzgl. des $\bar{y}, \bar{z}$ - Koordinatensystems,
3. das Flächenträgheitsmoment I_{yy} bzgl. des y, z - Koordinatensystems.

Hinweis: Verwenden Sie die Integrale

- $\int \sin^2 \varphi \, d\varphi = \frac{1}{2}(\varphi - \frac{1}{2} \sin 2\varphi)$
- $\int \cos^2 \varphi \, d\varphi = \frac{1}{2}(\varphi + \frac{1}{2} \sin 2\varphi)$
- $\int \sin \varphi \cos \varphi \, d\varphi = -\frac{1}{4} \cos 2\varphi$



Geg.: $a, \bar{y}_k = \frac{4\sqrt{2}}{3\pi}a, \bar{z}_k = -\frac{4}{3\pi}(2 - \sqrt{2})$.

Lösung:

1. Schwerpunkt

	A_i	$\bar{y}_i A_i$	$\bar{z}_i A_i$
	$\frac{\pi a^2}{8}$	$\frac{\sqrt{2}}{6} a^3$	$\frac{a^3}{3} \frac{\sqrt{2}-2}{2}$
	$\frac{a^2}{2}$	$\frac{\sqrt{2}}{6} a^3$	$\frac{a^3}{6}$
Σ	$\frac{4+\pi}{8} a^2$	$\frac{1+\sqrt{2}}{6} a^3$	$\frac{a^3}{3} \frac{\sqrt{2}-1}{2} a^3$

$$\bar{z}_s = \frac{\sum \bar{z}_i A_i}{A} = \frac{\frac{\sqrt{2}-1}{6} a^3}{\frac{4+\pi}{8} a^2} = \frac{4a}{3(4+\pi)} (\sqrt{2}-1)$$

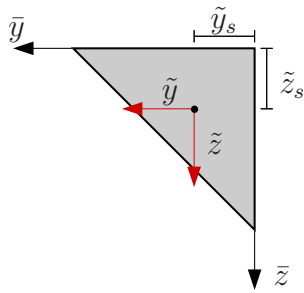
$$\bar{y}_s = \frac{\sum \bar{y}_i A_i}{A} = \frac{\frac{1+\sqrt{2}}{6} a^3}{\frac{4+\pi}{8} a^2} = \frac{4a}{3(4+\pi)} (\sqrt{2}+1)$$

2. Flächenträgheitsmoment bzgl. $\bar{y}$, $\bar{z}$

1: Achtelkreis

$$\begin{aligned} I_{\bar{y}\bar{y}}^1 &= \int_A \bar{z}^2 dA = \int_0^{\pi/4} \int_0^a r^3 \sin^2 \phi dr d\phi \\ &= \frac{a^4}{4} \int_0^{\pi/4} \frac{1}{2} (1 - \cos(2\phi)) d\phi = \frac{a^4}{8} \left[\phi - \frac{1}{2} \sin(2\phi) \right]_0^{\pi/4} \\ &= \frac{a^4}{8} \left(\frac{\pi}{4} - \frac{1}{2} \right) = \frac{a^4(\pi-2)}{32} \\ I_{\bar{z}\bar{z}}^1 &= \int_A \bar{y}^2 dA = \int_0^{\pi/4} \int_0^a r^3 \cos^2 \phi dr d\phi \\ &= \frac{a^4}{4} \int_0^{\pi/4} \frac{1}{2} (1 + \cos(2\phi)) d\phi = \frac{a^4}{8} \left[\phi + \frac{1}{2} \sin(2\phi) \right]_0^{\pi/4} \\ &= \frac{a^4}{8} \left(\frac{\pi}{4} + \frac{1}{2} \right) = \frac{a^4(\pi+2)}{32} \\ I_{\bar{y}\bar{z}}^1 &= \int_A \bar{y}\bar{z} dA = \int_0^{\pi/4} \int_0^a r^3 \sin \phi \cos \phi dr d\phi \\ &= \frac{a^4}{4} \int_0^{\pi/4} \frac{1}{2} \sin(2\phi) d\phi = \frac{a^4}{8} \left[-\frac{1}{2} \cos(2\phi) \right]_0^{\pi/4} \\ &= \frac{a^4}{16} \end{aligned}$$

2: Dreieck



$$I_{\bar{y}\bar{y}}^2 = \frac{a^4}{36} + \left(-\frac{a}{3}\right)^2 \cdot \frac{a^2}{2} = \frac{a^4}{12}$$

$$I_{\bar{z}\bar{z}}^2 = \frac{a^4}{36} + \left(-\frac{a}{3}\right)^2 \cdot \frac{a^2}{2} = \frac{a^4}{12}$$

$$I_{\bar{y}\bar{z}}^2 = \frac{a^4}{72} - \left(-\frac{a}{3}\right) \cdot \left(-\frac{a}{3}\right) \cdot \frac{a^2}{2} = -\frac{a^4}{24}$$

Gesamt: 1 + 2

$$I_{\bar{y}\bar{y}} = \frac{a^4(\pi - 2)}{32} + \frac{a^4}{12}$$

$$I_{\bar{z}\bar{z}} = \frac{a^4(\pi + 2)}{32} + \frac{a^4}{12}$$

$$I_{\bar{y}\bar{z}} = \frac{a^4}{16} - \frac{a^4}{24} = \frac{a^4}{48}$$

3. Flächenträgheitsmoment I_{yy} bzgl. y, x

$$I_{yy} = I_{\bar{y}\bar{y}} - \bar{z}_s^2 \cdot A$$

$$\approx 0,119a^4 - 0,077^2a^2 \cdot 0,893a^2$$

$$\approx 0,114a^4$$