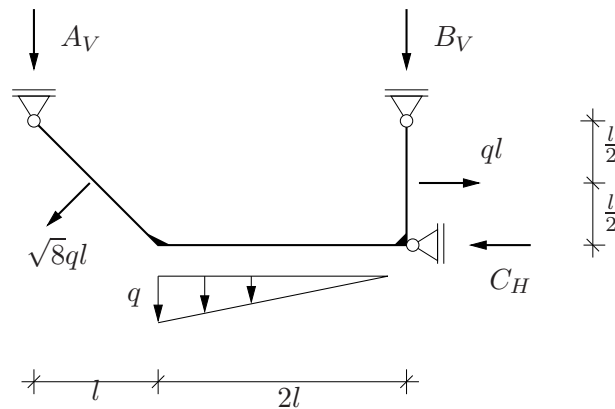


Lösung

a)



1.) statische Bestimmtheit

$$a = 3 + 0 - 3 \cdot 1 = 0 \quad \checkmark$$

2.) Auflagerreaktionen

$$\rightarrow \Sigma H : \quad ql - \sqrt{8}ql \cdot \frac{1}{\sqrt{2}} - C_H = 0$$

$$\underline{C_H = -ql}$$

$$\curvearrowright \Sigma M_C : \quad ql \frac{l}{2} + A_V \cdot 3l - 2ql \cdot \frac{5}{2}l - 2ql \frac{l}{2} = 0$$

$$-q \cdot 2l \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot 2l = 0$$

$$A_V \cdot 3l = -\frac{ql^2}{2} + 5ql^2 + ql^2 + \frac{4}{3}ql^2$$

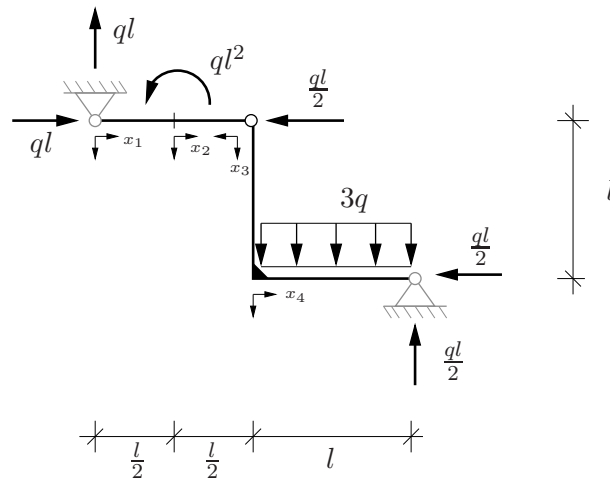
$$\underline{A_V = \frac{41}{18}ql}$$

$$\uparrow \Sigma V : \quad A_V + B_V - 2ql - q \cdot 2l \cdot \frac{1}{2} = 0$$

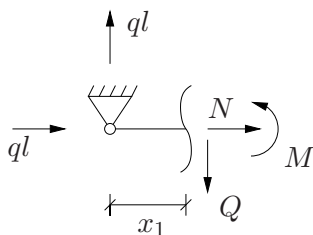
$$B_V = ql(3 - \frac{41}{18})$$

$$\underline{B_V = \frac{13}{18}ql}$$

b)

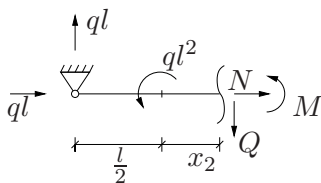


1. Schnitt



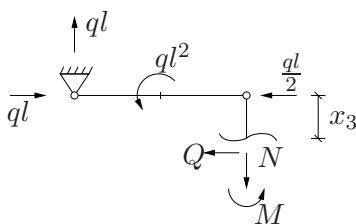
$$\begin{aligned} N(x_1) &= -ql \\ Q(x_1) &= ql \\ M(x_1) &= ql \cdot x_1 \\ \rightarrow M(1) &= 0 \quad M\left(\frac{l}{2}\right) = \frac{ql^2}{2} \end{aligned}$$

2. Schnitt



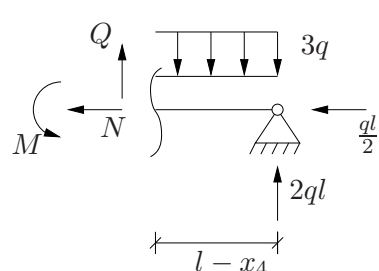
$$\begin{aligned} N(x_2) &= -ql \\ Q(x_2) &= ql \\ \curvearrowright \Sigma M : \quad 0 &= M(x_2) + ql^2 - ql\left(\frac{l}{2} + x_2\right) \\ M(x_2) &= -\frac{ql^2}{2} + ql \cdot x_2 \\ \rightarrow M(0) &= -\frac{ql^2}{2} \quad M\left(\frac{l}{2}\right) = 0 \end{aligned}$$

3. Schnitt



$$\begin{aligned} N(x_3) &= ql \\ Q(x_3) &= \frac{ql}{2} \\ \curvearrowright \Sigma M : \quad 0 &= M(x_3) + \frac{ql}{2}x_3 + ql^2 - ql \cdot l - ql \cdot x_3 \\ M(x_3) &= \frac{ql}{2}x_3 \\ \rightarrow M(0) &= 0 \quad M(l) = \frac{ql^2}{2} \end{aligned}$$

4. Schnitt



$$N(x_4) = -\frac{ql}{2}$$

$$\uparrow \Sigma V : \quad 0 = Q(x_4) + 2ql - 3q(l - x_4)$$

$$0 = Q(x_4) - ql + 3q \cdot x_4$$

$$Q(x_4) = ql - 3q \cdot x_4$$

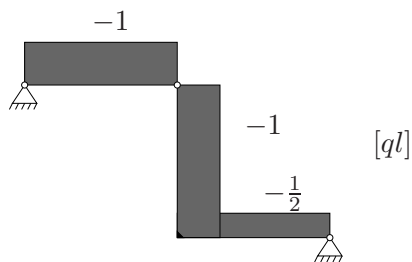
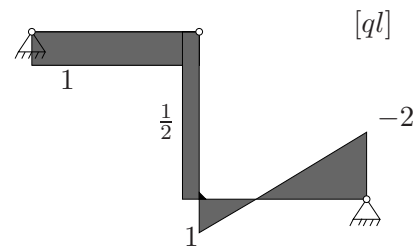
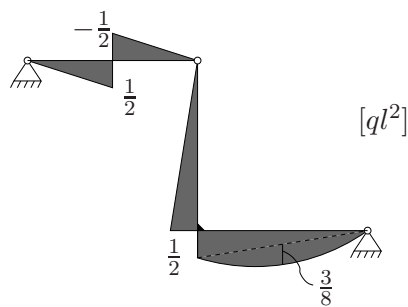
$$\rightarrow Q(0) = ql \quad Q(l) = -2ql$$

$$\curvearrowright \Sigma M : \quad 0 = -M(x_4) + 2ql(l - x_4) - 3q \frac{(l - x_4)^2}{2}$$

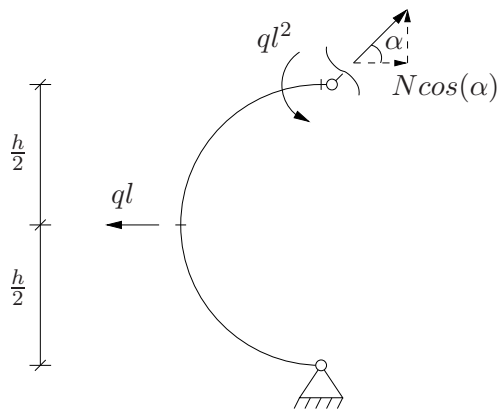
$$0 = -M(x_4) + 2ql^2 - \frac{3}{2}ql^2 - 2ql \cdot x_4 + 3ql \cdot x_4 - \frac{3}{2}q \cdot x_4^2$$

$$M(x_4) = \frac{1}{2}ql^2 + ql \cdot x_4 - \frac{3}{2}q \cdot x_4^2$$

$$\rightarrow M(0) = \frac{1}{2}ql^2 \quad M(l) = 0$$

NormalkraftverlaufQuerkraftverlaufMomentenverlauf

c) 1 Schnitt und keine Berechnung der Auflagerkräfte notwendig:



$$\curvearrowright \Sigma M : \quad 0 = N \cos(\alpha) \cdot h - ql^2 - ql \cdot \frac{h}{2}$$

$$N = \frac{ql^2 + \frac{qlh}{2}}{\cos(\alpha) \cdot h}$$

Parametrisierung des Verhältnisses $\frac{h}{l}$ und α :

$$t := \frac{h}{l} \quad \rightarrow \quad h = t \cdot l, \quad \alpha = \text{atan}(t)$$

$$N = N(t) = \frac{ql^2 + \frac{1}{2}ql^2t}{t \cdot l} \sqrt{t^2 + 1} = ql \left(\frac{1}{t} + \frac{1}{2} \right) \sqrt{t^2 + 1}$$

$$N'(t) = ql \left(-\frac{\sqrt{t^2+1}}{t^2} + \frac{1+\frac{t}{2}}{\sqrt{t^2+1}} \right) = ql \frac{\frac{t^3}{2} - 1}{t^2 \sqrt{t^2+1}} \stackrel{!}{=} 0$$

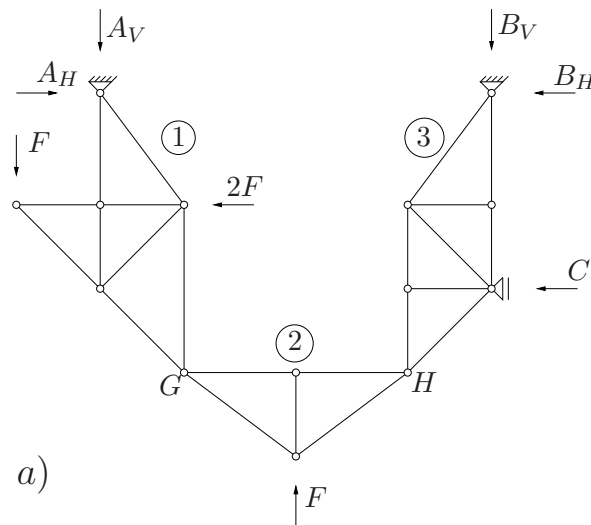
$$\rightarrow \frac{t^3}{2} - 1 = 0, \quad \underline{t = \sqrt[3]{2}}$$

Überprüfung des Randwerts $t = 2$

$$N(t=2) = ql \left(\frac{1}{2} + \frac{1}{2} \right) \sqrt{4+1} = \sqrt{5}ql$$

$$N(t = \sqrt[3]{2}) \approx 2,08ql < \sqrt{5}ql$$

Lösung



1. Stat Bestimmtheit (innere und äußere)

$$\text{innere :} \quad f = 2 \cdot k - (a + s) = 2 \cdot 14 - (5 + 23) = 0 \quad \checkmark$$

$$\text{äußere :} \quad f = 3 \cdot k - (a + g) = 3 \cdot 3 - (5 + 4) = 0 \quad \checkmark$$

2. Berechnen Sie alle Auflagerkräfte

$$\text{TS 1 :} \quad \curvearrowright \Sigma G : \quad A_V \cdot 3l - A_H \cdot 10l + 2F \cdot 6l + F \cdot 6l = 0$$

$$A_H = \frac{1}{10}(A_V \cdot 3 + 12F + 6F) = \frac{1}{10}(3A_V + 18F)$$

$$\text{TS 1\&2 :} \quad \curvearrowright \Sigma H : \quad A_V \cdot 11l - A_H \cdot 10l + 2F \cdot 6l + F \cdot 14l + F \cdot 4l = 0$$

$$A_V \cdot 11l - 3l \cdot A_V - 18Fl + 12Fl + 14Fl + 4Fl = 0$$

$$8l \cdot A_V = -12Fl$$

$$A_V = -\frac{3}{2}F$$

$$A_H = \frac{27}{20}F$$

$$\downarrow \Sigma : \quad A_V + B_V + 2F = 0$$

$$B_V = -\frac{1}{2}F$$

$$\curvearrowright \Sigma A : \quad F \cdot 3l - 2F \cdot 4l - F \cdot 7l - C \cdot 7l - B_V \cdot 14l = 0$$

$$C = -\frac{5}{7}F$$

$$\rightarrow \Sigma : \quad A_H - B_H - C - 2F = 0$$

$$B_H = \frac{9}{140}F$$

Auflager:

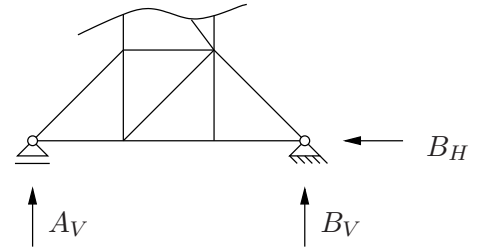
$$\curvearrowleft \Sigma M_A : \quad B_V \cdot 9l + P \cdot 12l - 2P \cdot 9l = 0$$

$$B_V = \frac{2}{3}P$$

$$\rightarrow \Sigma : \quad B_H = 0$$

$$\uparrow \Sigma : \quad A_V + B_V - 3P = 0$$

$$A_V = 3P - B_V = \frac{7}{3}P$$



Stabkräfte:

Nullstäbe: S27, S35

$$\text{Knoten 1 : } \downarrow \Sigma : \quad P - \frac{1}{\sqrt{2}}S_1 = 0 \quad S_1 = \sqrt{2}P$$

$$\rightarrow \Sigma : \quad \frac{1}{\sqrt{2}}S_1 + S_2 = 0 \quad S_2 = -P$$

$$\text{Knoten 2 : } \rightarrow \Sigma : \quad S_4 - \frac{1}{\sqrt{2}}S_1 = 0 \quad S_4 = P$$

$$\uparrow \Sigma : \quad -\frac{1}{\sqrt{2}}S_1 - S_3 = 0 \quad S_3 = -P$$

$$\text{Knoten 3 : } \uparrow \Sigma : \quad S_3 + \frac{1}{\sqrt{2}}S_5 = 0 \quad S_5 = \sqrt{2}P$$

$$\rightarrow \Sigma : \quad -S_2 + S_6 + \frac{1}{\sqrt{2}}S_5 = 0 \quad S_6 = -2P$$

Reibungsfreie Umlenkrolle $\rightarrow$ Seilkraft: $S = 2P$

$$\text{Knoten 5 : } \rightarrow \Sigma : \quad -S_4 + S_8 + \frac{15}{3\sqrt{26}}S - \frac{1}{\sqrt{2}}S_5 = 0$$

$$S_8 = P - \frac{15}{3\sqrt{26}}2P + P = \frac{26-5\sqrt{26}}{13}P$$

$$\uparrow \Sigma : \quad -\frac{1}{\sqrt{2}}S_5 - S_7 + \frac{3}{3\sqrt{26}}2P = 0$$

$$S_7 = P + \frac{6}{3\sqrt{26}}P = \frac{13+\sqrt{26}}{13}P$$

Ritterschnitt:

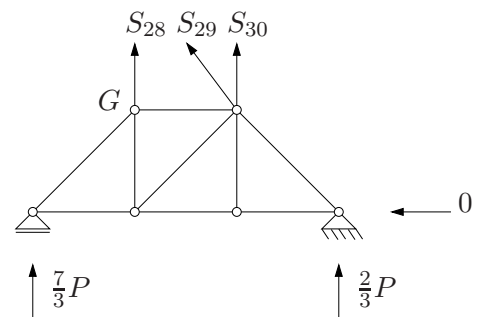
$$\rightarrow \Sigma : \quad S_{29} = 0$$

$$\curvearrowleft \Sigma G : \quad \frac{2}{3}P \cdot 6l - \frac{7}{3}P \cdot 3l + S_{30} \cdot 3l = 0$$

$$S_{30} = (7P - 4P)\frac{1}{3} = P$$

$$\uparrow \Sigma : \quad S_{28} + S_{30} + \frac{7}{3}P + \frac{2}{3}P = 0$$

$$S_{28} = -P - \frac{7}{3}P - \frac{2}{3}P = -4P$$



Lösung1) Seillinie

$$\text{DGL : } y_1''(x_1) = \frac{q_0}{H} = \text{const.}$$

$$y_1'(x_1) = \frac{q_0}{H}x_1 + c_1$$

$$y_1(x_1) = \frac{q_0}{H} \frac{x_1^2}{2} + c_1 \cdot x_1 + c_2$$

$$\text{RB : } y_1(0) = 0 \quad \rightarrow c_2 = 0$$

$$y_1(l) = 0 \quad \rightarrow \frac{q_0 l^2}{2H} + c_1 l = 0$$

$$c_1 = -\frac{q_0 l}{2H}$$

$$\rightarrow \underline{y_1(x_1) = \frac{q_0}{2H}(x_1^2 - x_1 l) = \frac{q_0}{2H}x_1(x_1 - l)}$$

2) Horizontalzug H_1 , Lagerkräfte in A

$$S_1 = H_1 \sqrt{1 + (y_1')^2} \quad \rightarrow S_{1,max} = S_1(y_{1,max}') = S_1(y_1'(x_1 = 0))$$

$$S_{1,max} = H_1 \sqrt{1 + [y_1'(0)]^2} = H_1 \sqrt{1 + c_1^2}$$

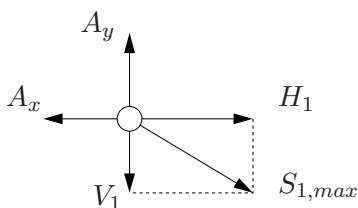
$$= H_1 \sqrt{1 + \left(-\frac{q_0 l}{2H_1}\right)^2}$$

aufösen nach H_1 :

$$S_{1,max}^2 = H_1^2 \left(1 + \left(\frac{q_0 l}{2H_1}\right)^2\right) = H_1^2 + \frac{(q_0 l)^2}{4}$$

$$\underline{H_1 = \sqrt{S_{1,max}^2 - \frac{(q_0 l)^2}{4}}} = \frac{1}{2} \sqrt{(2S_{1,max})^2 - (q_0 l)^2}$$

$\rightarrow$ Auflagerreaktion:



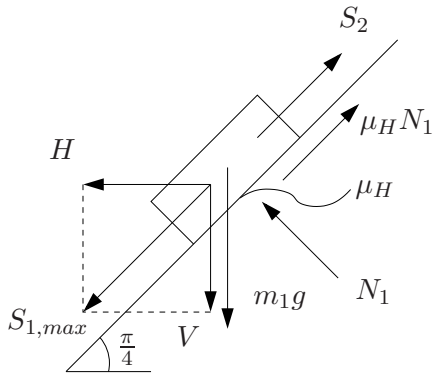
$$\rightarrow A_x = H_1 = \sqrt{S_{1,max}^2 - \left(\frac{q_0 l}{2}\right)^2}$$

$$A_y^2 = S_{1,max}^2 - H_1^2$$

$$\rightarrow \underline{A_y = \sqrt{\left(\frac{q_0}{2}\right)^2} = \frac{q_0 l}{2} = V_1}$$

3) Grenzen für m_2 a) m_2 gerade groß genug, dass System in Ruhe bleibt

Teilsystem I:

Symmetrie: $\rightarrow H(0) = H(L); V(0) = V(l)$ 

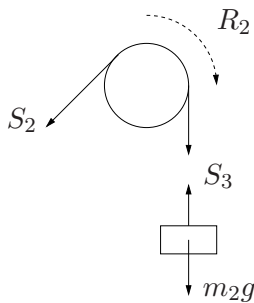
$$\nearrow \Sigma: S_2 - H \frac{\sqrt{2}}{2} - V \frac{\sqrt{2}}{2} + \mu_H \cdot N_1 - m_1 g \frac{\sqrt{2}}{2} = 0$$

$$\nwarrow \Sigma: N_1 + H \frac{\sqrt{2}}{2} - V \frac{\sqrt{2}}{2} - m_1 g \frac{\sqrt{2}}{2} = 0$$

$$N_1 = \frac{\sqrt{2}}{2}(m_1 g + V - H)$$

$$\underline{S_2 = \frac{\sqrt{2}}{2}(H + V + m_1 g - \mu(m_1 g + V - H))}$$

Teilsystem II:

 $\rightarrow$ an "unterer Grenze" für m_2 gilt $S_3 < S_2$ 

$$S_3 = m_2 g$$

$$S_3 = \frac{S_2}{e^{\mu\varphi}}$$

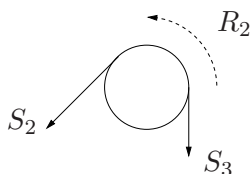
$$\rightarrow m_2 = \frac{S_2}{g e^{\mu \frac{3\pi}{4}}}$$

b) obere Grenze:

Teilsystem I:

$$\nearrow \Sigma: \rightarrow \underline{S_2 = \frac{\sqrt{2}}{2}(H + V + m_1 g + \mu_H(m_1 g + V - H))}$$

Teilsystem II:



$$m_2 g = S_2 e^{\mu\varphi}$$

$$\rightarrow \underline{m_2 = \frac{S_2 e^{\mu \frac{3\pi}{4}}}{g}}$$