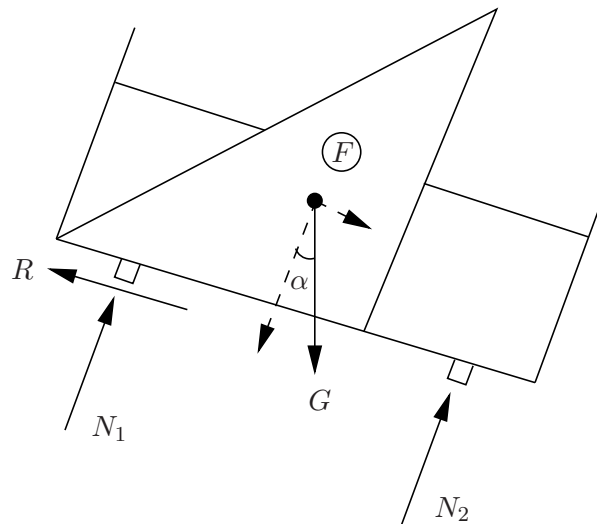


Aufgabe 11. Freischnitt2. Schwerpunkt

$$\begin{aligned}
 (a) \quad \hat{x}_{2F} &= \frac{a \cdot \frac{1}{2}(3a)^2 + 4a \cdot \frac{1}{2}(6a)^2 + 7,5a(3a^2)}{\frac{1}{2}(3a)^2 + \frac{1}{2}(6a)^2 + (3a)^2} = \frac{144a^3}{31,5a^2} = \frac{32}{7}a \\
 \hat{x}_{3F} &= \frac{2a \cdot \frac{9}{2}a^2 + 2a \cdot 18a^2 + 1,5a \cdot 9a^2}{31,5a^2} = \frac{58,5a^3}{31,5a^2} = \frac{13}{7}a \\
 (b) \quad \hat{x}_{2F} &= \frac{1,5a \cdot 9a^2 + 2a \cdot 4,5a^2 + 5a \cdot 18a^2}{31,5a^2} = \frac{112,5a^3}{31,5a^2} = \frac{25}{7}a \\
 \hat{x}_{3F} &= \frac{1,5a \cdot 9a^2 + 4a \cdot 4,5a^2 + 2a \cdot 18a^2}{31,5a^2} = \frac{67,5a^3}{31,5a^2} = \frac{15}{7}a
 \end{aligned}$$

3. Haftreibungskoeffizient

Kräfte-/ Momentengleichgewicht:

$$\Sigma \nearrow = 0 \quad N_1 + N_2 - G \cos \alpha = 0 \quad \rightarrow N_2 = -N_1 + G \cos \alpha \quad (1)$$

$$\Sigma \nwarrow = 0 \quad R - G \sin \alpha = 0 \quad \rightarrow R = G \sin \alpha \quad (2)$$

$$\odot \Sigma M_F = 0 \quad N_2(8a - \hat{x}_{2F}) - N_1(\hat{x}_{2F} - a) - R \cdot \hat{x}_{3F} = 0 \quad (3)$$

(1) & (2) in (3):

$$-N_1 8a + N_1 \hat{x}_{2F} + G \cos \alpha \cdot 8a - G \cos \alpha \cdot \hat{x}_{2F} -$$

$$-N_1 \hat{x}_{2F} + N_1 a - G \sin \alpha \cdot \hat{x}_{3F} = 0$$

$$N_1 7a = G \cos \alpha (8a - \hat{x}_{2F}) - G \sin \alpha \cdot \hat{x}_{3F}$$

$$N_1 = \frac{\cos \alpha (8a - \hat{x}_{2F}) - \sin \alpha \cdot \hat{x}_{3F}}{7a} \cdot G$$

Grenzfall der Haftreibung:

$$R \leq R_H^* = N_1 \cdot \mu_H$$

$$G \sin \alpha \leq \frac{\cos \alpha (8a - \hat{x}_{2F}) - \sin \alpha \cdot \hat{x}_{3F}}{7a} \cdot G \cdot \mu_H$$

$$\mu_H \geq \frac{\sin \alpha \cdot 7a}{\cos \alpha (8a - \hat{x}_{2F}) - \sin \alpha \cdot \hat{x}_{3F}}$$

$$(a) \quad \hat{x}_{2F} = \frac{32}{7}a, \quad \hat{x}_{3F} = \frac{13}{7}a, \quad \alpha = 10$$

$$\mu_H \geq \frac{\sin 10^\circ \cdot 7a}{\cos(10^\circ)(8a - \frac{32}{7}a) - \sin(10^\circ) \cdot \frac{13}{7}a} = 0,398$$

$$(b) \quad \hat{x}_{2F} = \frac{25}{7}a, \quad \hat{x}_{3F} = \frac{15}{7}a$$

$$\mu_H \geq \frac{\sin 10^\circ \cdot 7a}{\cos(10^\circ)(8a - \frac{25}{7}a) - \sin(10^\circ) \cdot \frac{15}{7}a} = 0,305$$

4. Haftreibungskoeffizient bei masselosem quadr. Körper

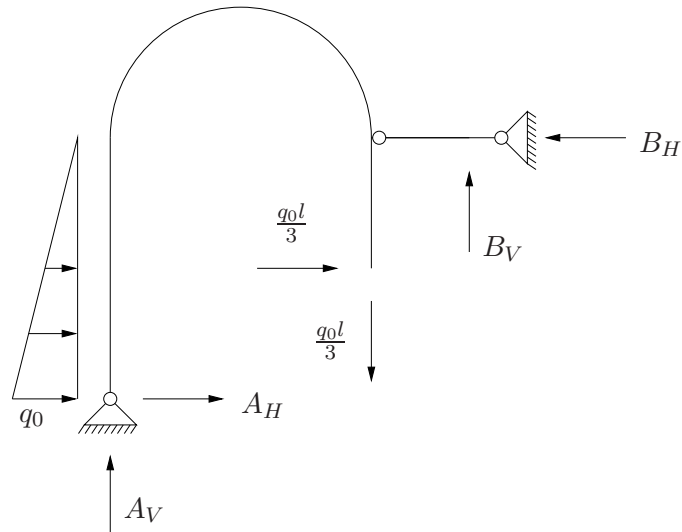
$$\hat{x}_{2F} = \frac{a \cdot 4,5a^2 + 4a \cdot 18a^2}{4,5a^2 + 18a^2} = \frac{76,5a^3}{22,5a^2} = \frac{17}{5}a$$

$$\hat{x}_{3F} = \frac{2a \cdot 4,5a^2 + 2a \cdot 18a^2}{4,5a^2 + 18a^2} = \frac{45a^3}{22,5a^2} = 2a$$

$$\rightarrow \mu_H \geq \frac{\sin(10^\circ) \cdot 7a}{\cos(10^\circ)(8a - \frac{17}{5}a) - \sin(10^\circ) \cdot 2a} = 0,291$$

Aufgabe 21. Statische Bestimmtheit

$$a = 4 + 2 - 3 \cdot 2 = 0 \quad \checkmark$$

2. Auflagerkräfte

$$\circlearrowleft \Sigma M_{\text{Pendelstab}} = 0 \quad B_V \cdot l = 0$$

$$\boxed{B_V = 0}$$

$$\uparrow \Sigma V_{\text{ges}} = 0 \quad A_V - \frac{q_0 l}{3} = 0$$

$$\boxed{A_V = \frac{q_0 l}{3}}$$

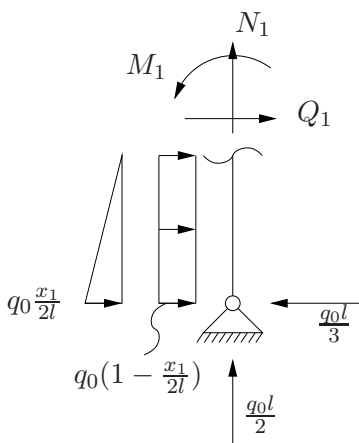
$$\circlearrowleft \Sigma M_{A, \text{ges}} = 0 \quad B_H \cdot 2l - \frac{q_0 l}{3} l - \frac{1}{2} q_0 \cdot 2l \cdot \frac{2l}{3} - \frac{q_0 l}{3} \cdot 2l = 0$$

$$B_H = \frac{1}{2} \left(\frac{q_0 l}{3} + \frac{2q_0 l}{3} + \frac{2q_0 l}{3} \right)$$

$$\boxed{B_H = \frac{5q_0 l}{6}}$$

$$\rightarrow \Sigma H_{\text{ges}} = 0 \quad \frac{q_0 l}{3} + A_H + q_0 l - \frac{5}{6} q_0 l = 0$$

$$\boxed{A_H = -\frac{q_0 l}{2}}$$

3. Schnittgrößen

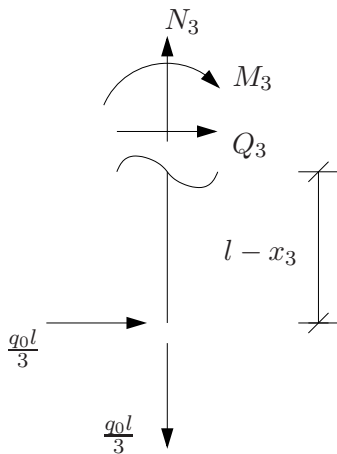
$$N_1(x_1) = -\frac{q_0 l}{3}$$

$$Q_1(x_1) = \frac{q_0 l}{2} - q_0 \left(1 - \frac{x_1}{2l}\right) x_1 - \frac{1}{2} q_0 \frac{x_1}{2l} x_1$$

$$Q_1(x_1) = q_0 \left(\frac{l}{2} - x_1 + \frac{x_1^2}{4l} \right)$$

$$M_1(x_1) = \frac{q_0 l}{2} x_1 - q_0 \left(1 - \frac{x_1}{2l}\right) \frac{x_1^2}{2} - q_0 \frac{x_1}{2l} \cdot \frac{1}{2} x_1 \cdot \frac{2x_1}{3}$$

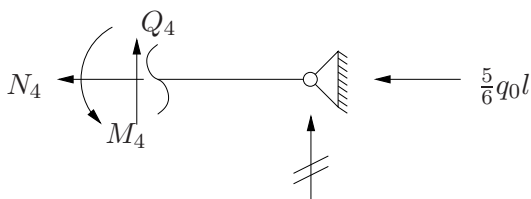
$$M_1(x_1) = q_0 \left(\frac{l}{2} x_1 - \frac{x_1^2}{2} + \frac{x_1^3}{12l} \right)$$



$$N_3(x_3) = \frac{q_0 l}{3}$$

$$Q_3(x_3) = -\frac{q_0 l}{3}$$

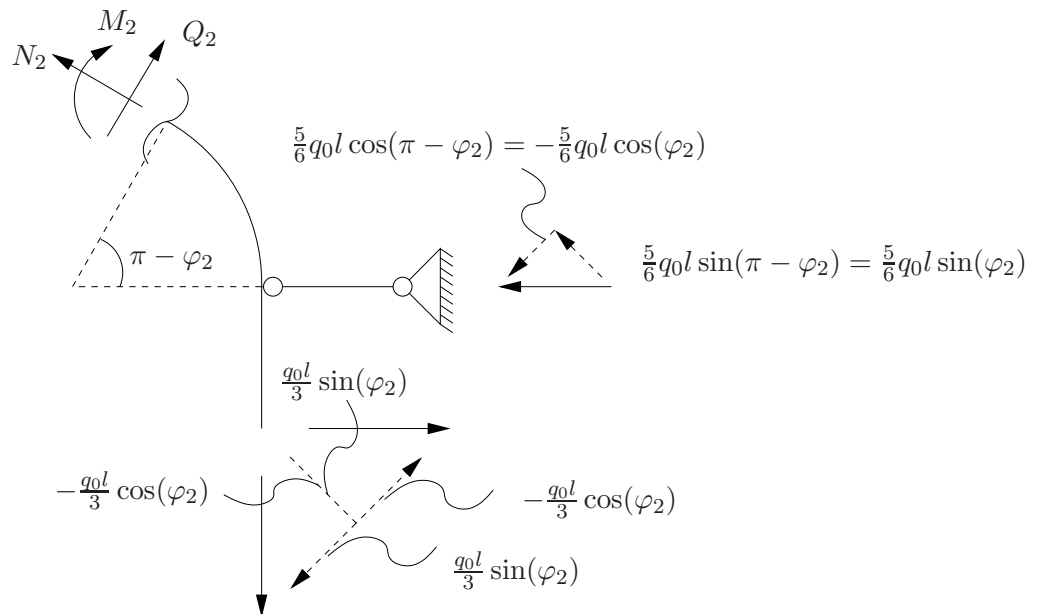
$$M_3(x_3) = \frac{q_0 l}{3}(l - x_3)$$



$$N_4(x_4) = -\frac{5}{6}q_0 l$$

$$Q_4(x_4) = 0$$

$$M_4(x_4) = 0$$

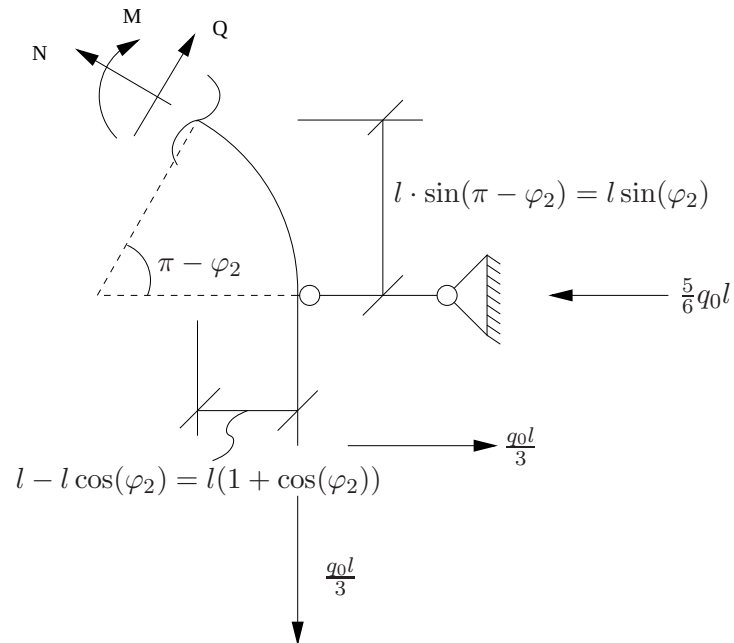


$$\Sigma F_{\text{tang}} = 0 \quad N_2 + \frac{5}{6}q_0 l \sin(\varphi_2) - \frac{q_0 l}{3} \sin(\varphi_2) + \frac{q_0 l}{3} \cos(\varphi_2) = 0$$

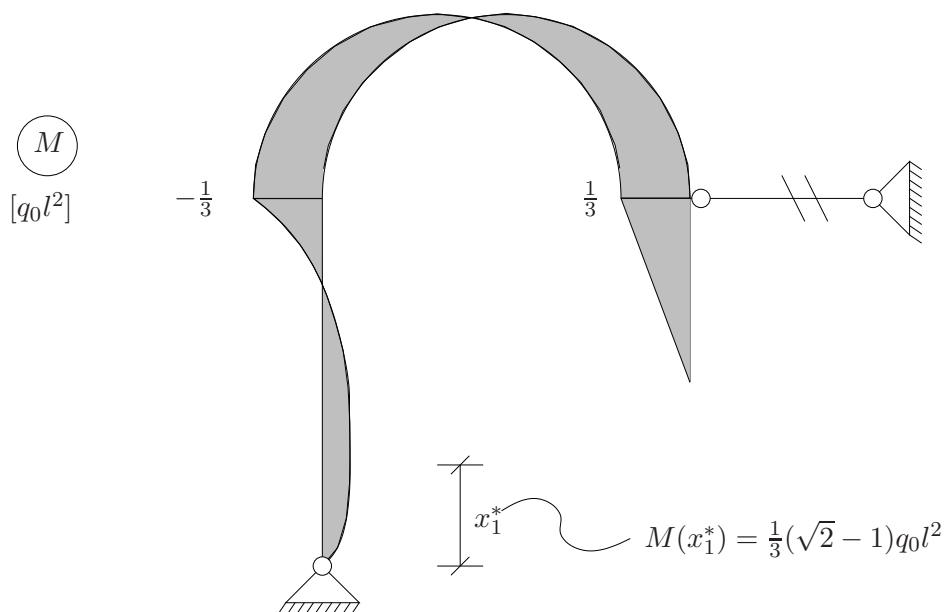
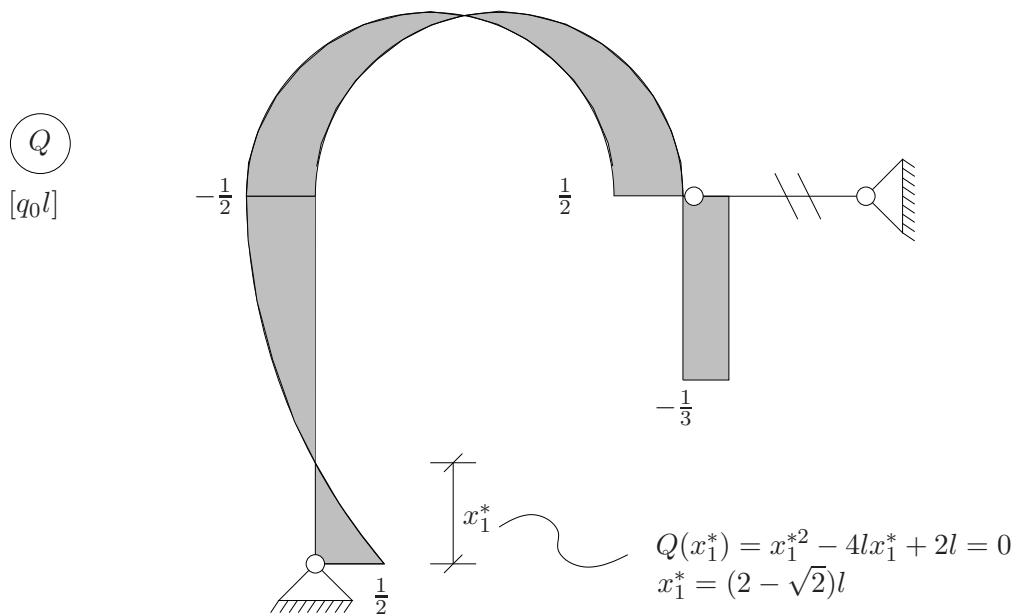
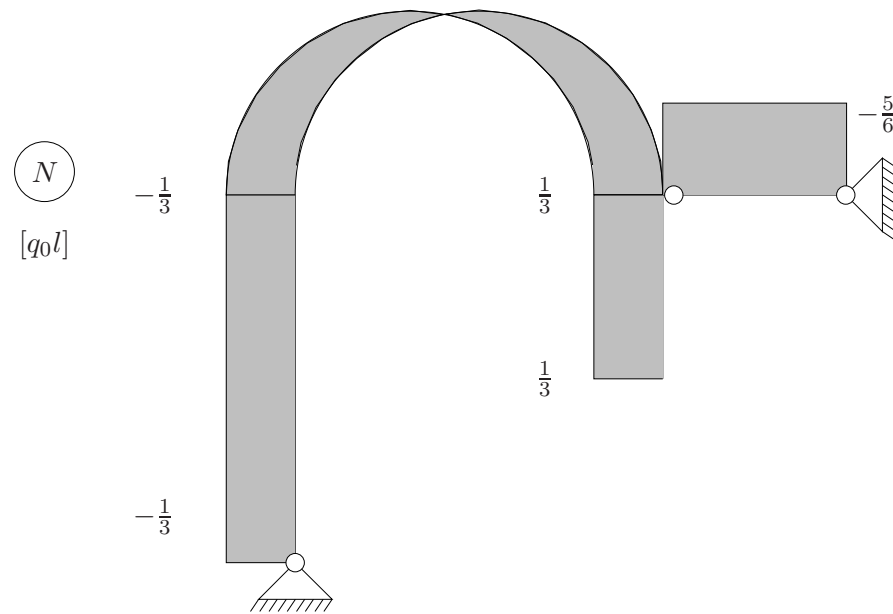
$$N_2(\varphi_2) = (-q_0 l) \left(\frac{1}{2} \sin(\varphi_2) + \frac{1}{3} \cos(\varphi_2) \right)$$

$$\Sigma F_{\text{radial}} = 0 \quad Q_2 - \left(-\frac{5}{6}q_0 l \cos(\varphi_2) \right) - \frac{q_0 l}{3} \cos(\varphi_2) - \frac{q_0 l}{3} \sin(\varphi_2) = 0$$

$$Q_2(\varphi_2) = q_0 l \left(\frac{1}{3} \sin(\varphi_2) - \frac{1}{2} \cos(\varphi_2) \right)$$



$$\begin{aligned} \Sigma M^{\odot+} &= 0 & M_2(\varphi_2) + \frac{5}{6}q_0 l \cdot l \sin(\varphi_2) - \frac{q_0 l}{3} \cdot l(1 + \sin(\varphi_2)) + \frac{q_0 l}{3} \cdot l \cdot (1 + \cos(\varphi_2)) &= 0 \\ M_2(\varphi_2) &= q_0 l^2 \left(\frac{1}{3} + \frac{1}{3} \sin(\varphi_2) - \frac{5}{6} \sin(\varphi_2) - \frac{1}{3} - \frac{1}{3} \cos(\varphi_2) \right) \\ M_2(\varphi_2) &= -q_0 l^2 \left(\frac{1}{2} \sin(\varphi_2) + \frac{1}{3} \cos(\varphi_2) \right) \end{aligned}$$



Aufgabe 31. Statische Bestimmtheit

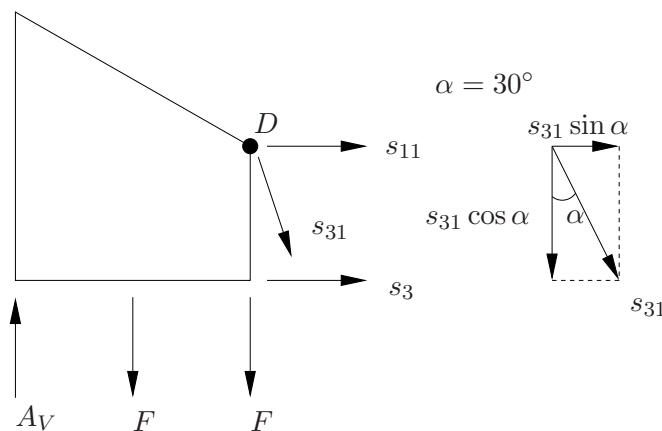
$$\begin{aligned}
 f &= 2 \cdot k - (r + s) \\
 &= 2 \cdot 20 - (3 + 37) = 0 \quad \checkmark \quad \text{statisch bestimmt}
 \end{aligned}$$

2. Sämtliche Auflagerreaktionen

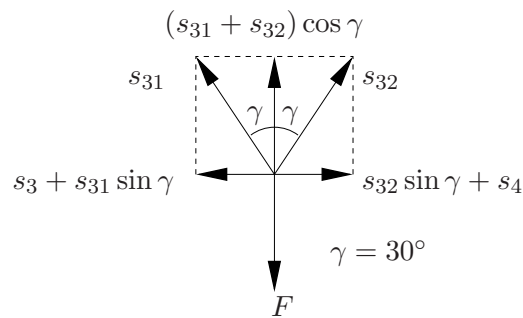
$$\begin{aligned}
 \rightarrow \Sigma F_H = 0 \quad B_H - F &= 0 & \boxed{B_H = F} \\
 \uparrow \Sigma F_V = 0 \quad -5F + B_V + A_V &= 0 & A_V + B_V = 5F \\
 \circlearrowleft \Sigma M_B = 0 \quad A_V \cdot (12\sqrt{3}a) - F \cdot (10\sqrt{3}a) - F \cdot (8\sqrt{3}a) - \\
 -F \cdot (6\sqrt{3}a) - F \cdot (4\sqrt{3}a) - F \cdot (2\sqrt{3}a) - F \cdot 8a &= 0 \\
 20,78A_V = 59,96F & & \boxed{A_V = 2,885F} \\
 & & \boxed{B_V = 2,114F}
 \end{aligned}$$

3. Direkt ersichtliche Nullstäbe

Nullstäbe: 1, 20, 26, 28, 30, 33, 35, 37

4. Stabkräfte 3, 11, 31 mit Ritterschnitts

$$\begin{aligned}
 \rightarrow \Sigma F_H = 0 \quad s_{31} \sin \alpha + s_{11} + s_3 &= 0 \\
 (0,5)s_{31} + s_{11} + s_3 &= 0 & (*) \\
 \uparrow \Sigma F_V = 0 \quad A_v - 2F - s_{31} \cos \alpha &= 0 \\
 2,885F - 2F = s_{31} \cdot \frac{\sqrt{3}}{2} \\
 \frac{2 \cdot (0,885F)}{\sqrt{3}} = s_{31} & & \boxed{s_{31} = 1,021F} \\
 \circlearrowleft \Sigma M_D = 0 \quad -s_3 \cdot 6a - F \cdot 2\sqrt{3}a + A_V \cdot 4\sqrt{3}a &= 0 \\
 -6as_3 - 3,46Fa + 19,98Fa &= 0 \\
 -6as_3 + 16,52Fa &= 0 & \boxed{s_3 = 2,75F} & (**) \\
 (*) \& (**) \quad (0,5)(1,021F) + s_{11} + 2,75F &= 0 & \boxed{s_{11} = -3,26}
 \end{aligned}$$

Stabkräfte mit Knotenschnittverfahren

$$\uparrow \Sigma F_V = 0 \quad (s_{31} + s_{32}) \frac{\sqrt{3}}{2} = F$$

$$\frac{\sqrt{3}}{2} s_{31} = F - \frac{\sqrt{3}}{2} (1,021) F$$

$$\boxed{s_{32} = 0,133 F}$$

$$\rightarrow \Sigma F_H = 0 \quad s_4 + s_{32} \sin \gamma - s_3 - s_{31} \sin \gamma = 0$$

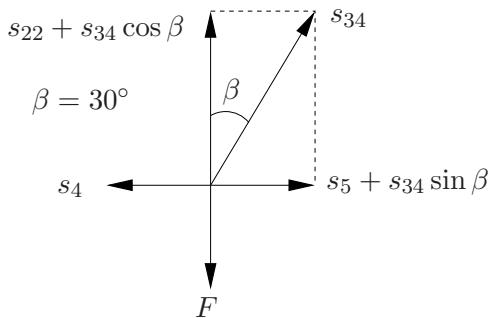
$$s_4 = 2,75 F + 0,51 F - 0,0665 F$$

$$\boxed{s_4 = 3,1935 F}$$

$$s_{11} \quad \leftarrow \bullet \rightarrow \quad s_{10}$$

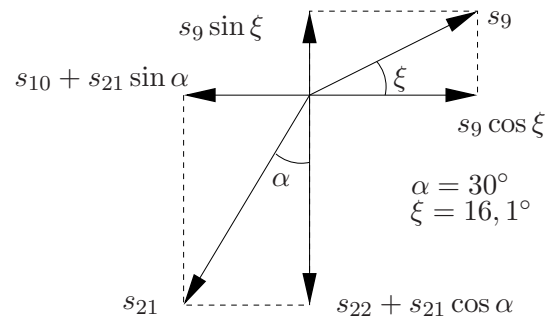
$$\rightarrow \Sigma F_H = 0 \quad s_{10} - s_{11} = 0$$

$$\boxed{s_{10} = -3,26 F}$$



$$\rightarrow \Sigma F_H = 0 \quad s_{34} \cdot (0,5) + s_5 = s_4$$

$$\uparrow \Sigma F_V = 0 \quad s_{34} \cdot \frac{\sqrt{3}}{2} + s_{22} = F$$



$$s_{21} = s_{32} = 0,133F$$

$$\rightarrow \Sigma F_H = 0 \quad s_9 \cos \xi = s_{21} \sin \alpha + s_{10}$$

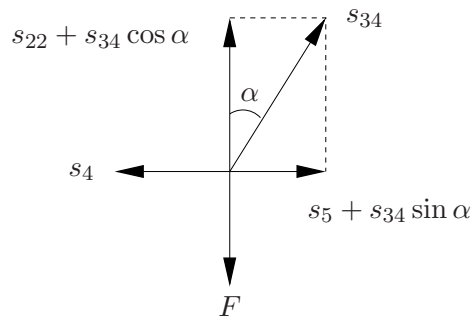
$$s_9 = \frac{-(0,133)(0,5)F + 0,326F}{0,96}$$

$$s_9 = -0,353F$$

$$\uparrow \Sigma F_V = 0 \quad s_9 \sin \xi - s_{21} \cos \alpha = s_{22}$$

$$(-0,353 \sin(16,1)) + 0,133F \cos(30) = s_{22}$$

$$s_{22} = 1,037F$$



$$\rightarrow \Sigma F_H = 0 \quad s_{34} \sin \alpha + s_5 - s_4 = 0$$

$$s_{34} \cdot 0,44 = 3,1935F - s_5$$

$$\uparrow \Sigma F_V = 0 \quad s_{22} + s_{34} \cos \alpha = F$$

$$-1,037F + s_{34} \cdot \frac{\sqrt{3}}{2} = F$$

$$s_{34} = 2,3F$$

$$s_5 = 2,189F$$